Model Selection and Feature Selection

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CS5350/6350: Machine Learning

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Note: Usually considered in supervised learning contexts but unsupervised learning too faces this issue (e.g., "how many clusters" when doing clustering)

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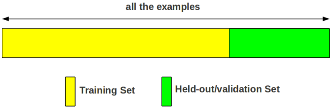

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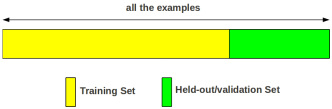

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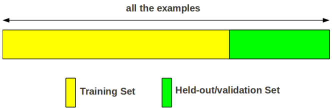

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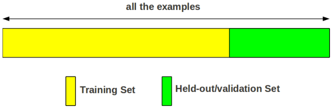

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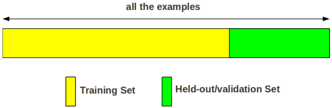

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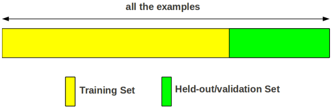

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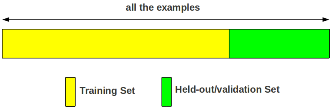

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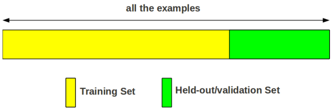

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    - Can ameliorate unfortunate splits by repeated random subsampling

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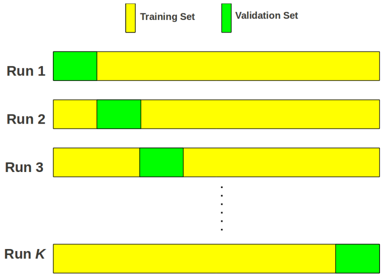

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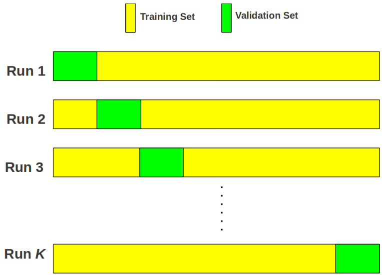

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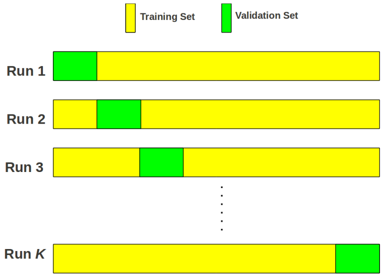

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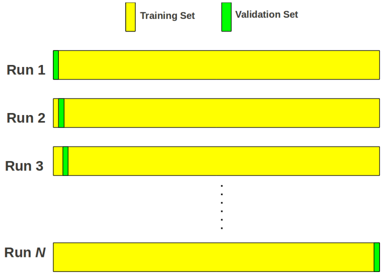

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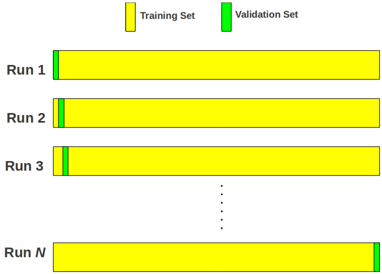

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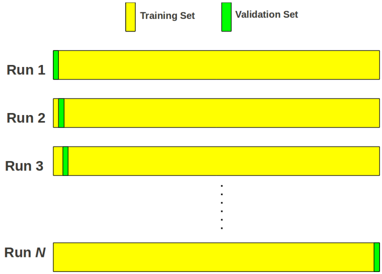

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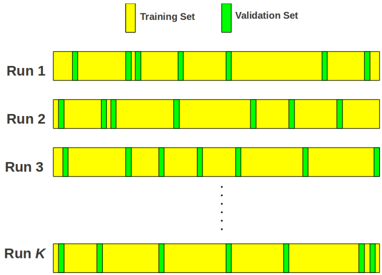

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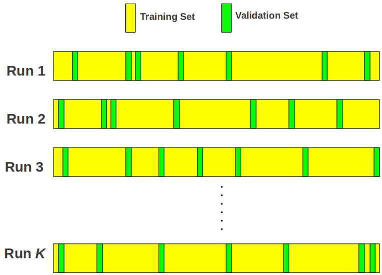

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- **Note:** the above estimate may still be bad if we overfit and have $e_{training-examples} = 0$. Why?

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- Can be used even for model selection in unsupervised learning

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- Note: MDL criteria is kind of equivalent to preferring the best regularized model

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- Note: Feature Selection is different from Feature Extraction
    - The latter transforms original features to get a small set of new features
    - More on feature extraction when we cover **Dimensionality Reduction**

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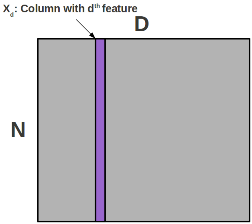

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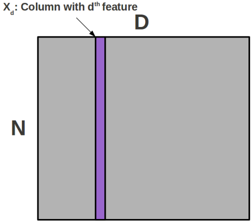

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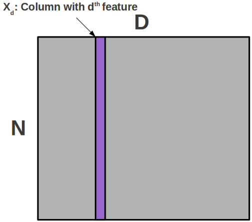

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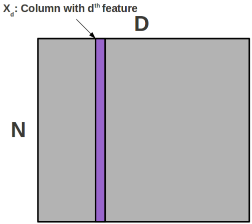

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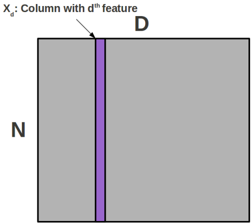

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