Institut
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d'apprentissage
de Montréal

MILA

Deep Generative Models

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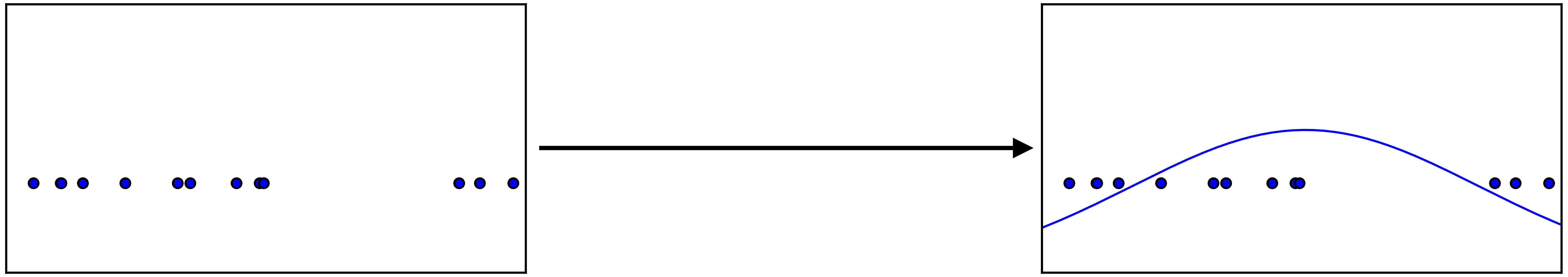
6.S191: Introduction to Deep Learning

MIT, Jan 30th, 2018

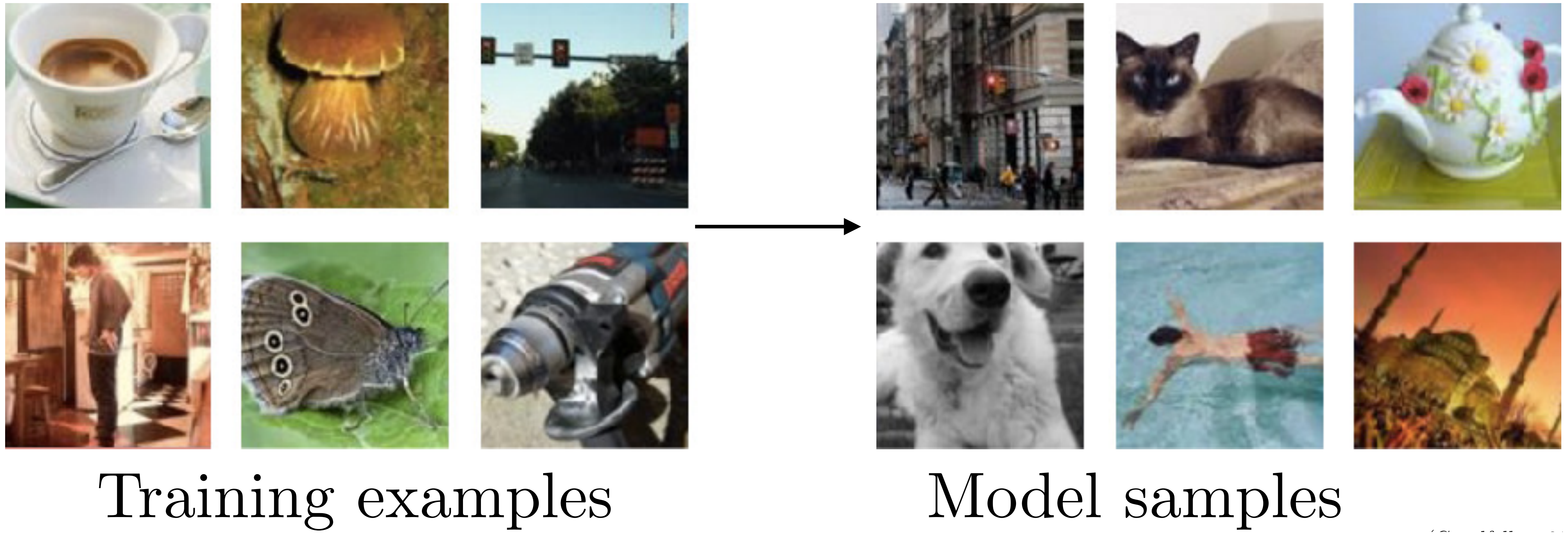
Generative modeling

- Generative models take training samples from some data distribution and learn a model that represents that distribution.

- Density estimation:



- Sample generation:



images taken from Goodfellow (2017)

Why generative models?

- Many tasks require structured output
  - Eg. Machine translation

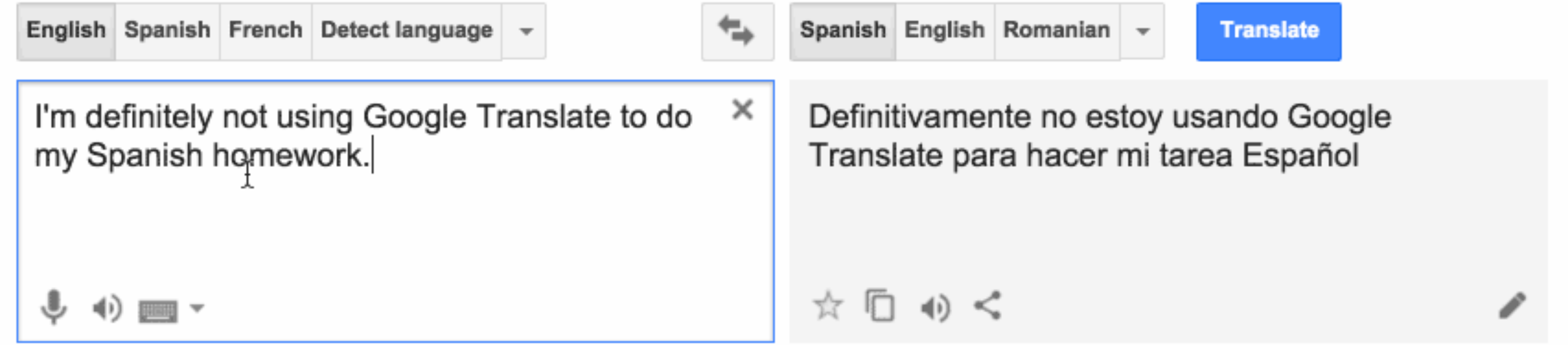
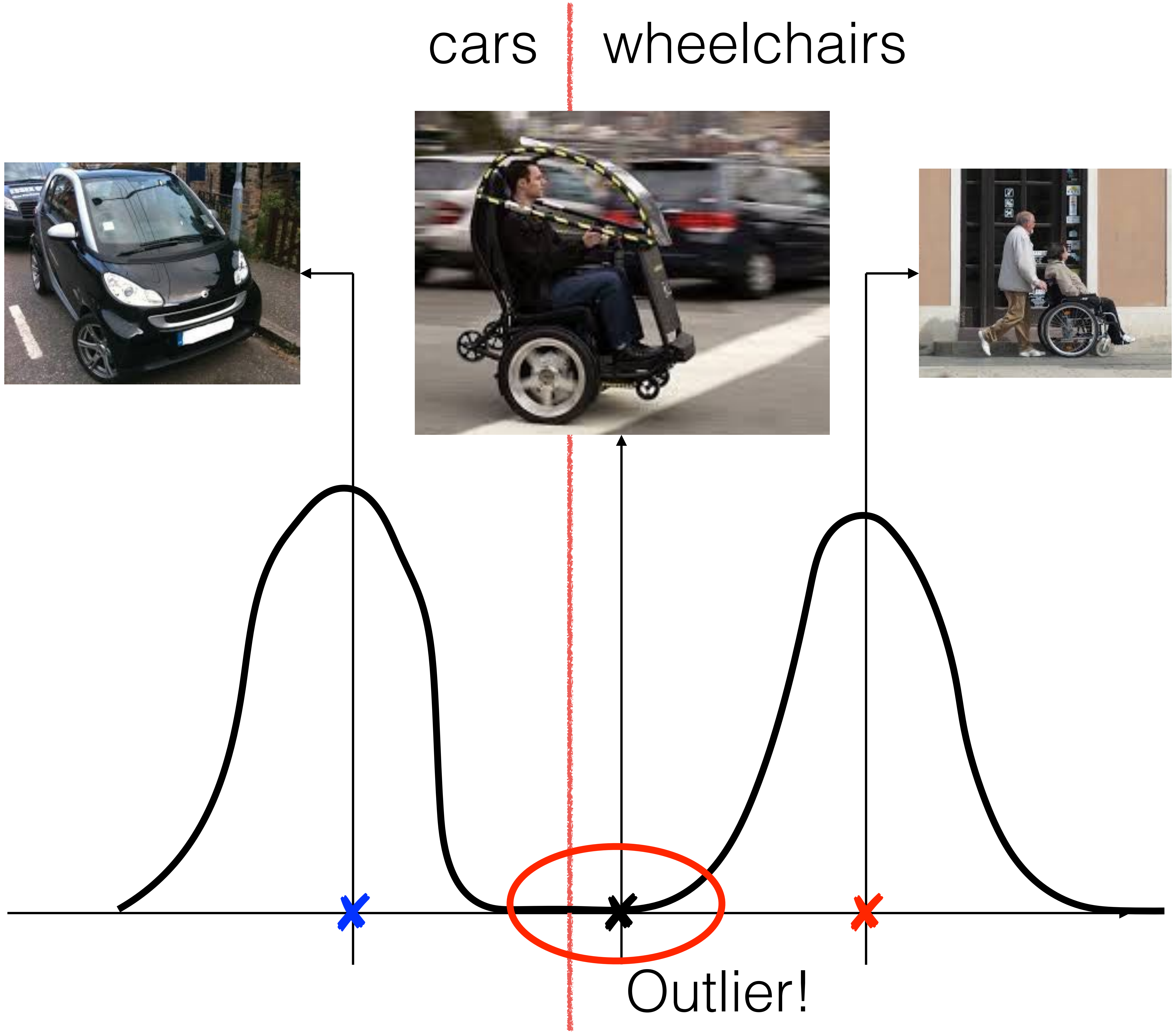


image credit: Adam Geitgey blog (2016) *Machine Learning is Fun Part 5: Language Translation with Deep Learning and the Magic of Sequences*

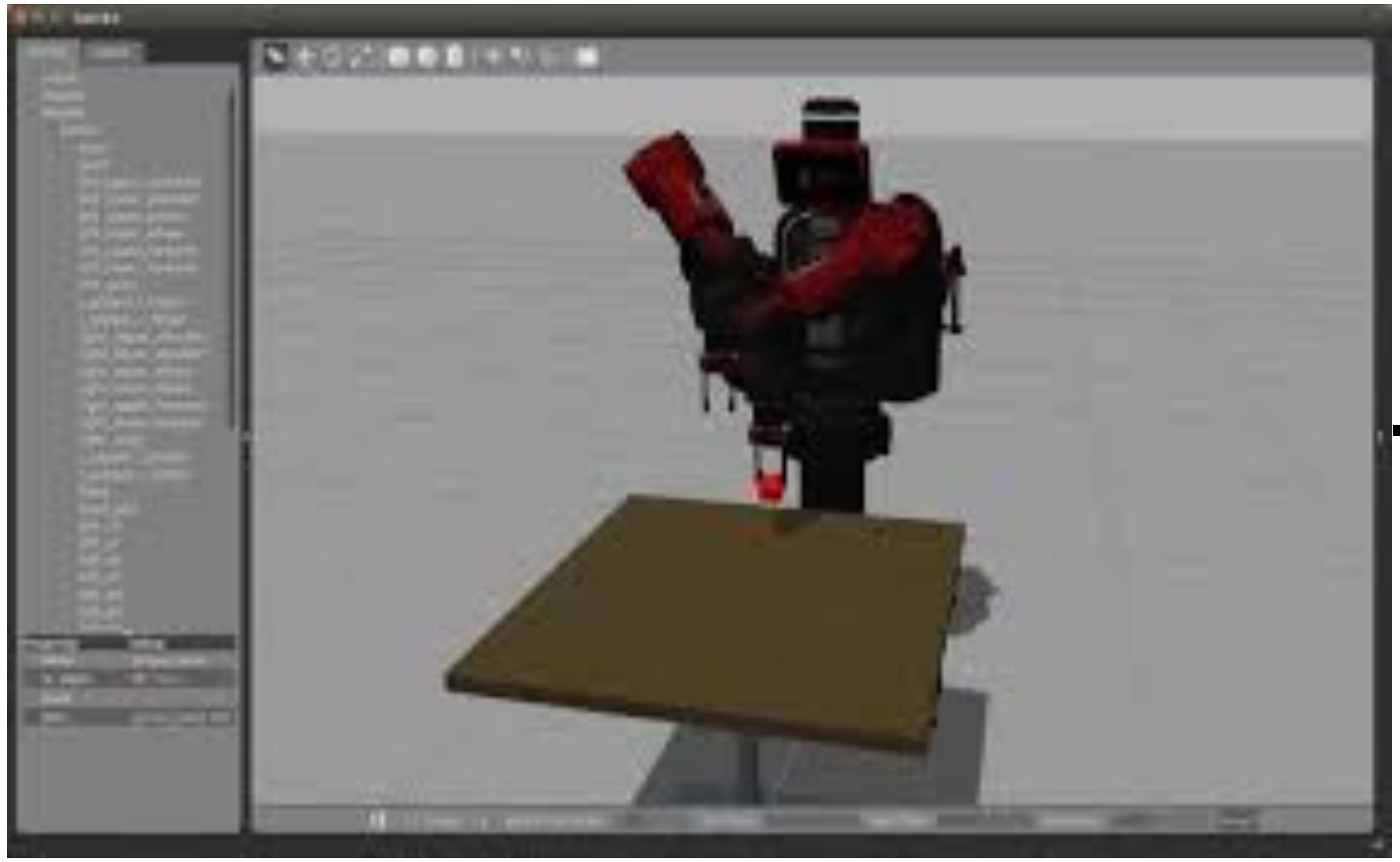
Why Generative Models? Outlier detection

- Large-scale deployment of CNN-based perception systems is becoming a reality.
- How do we detect when we encounter something new or rare (i.e. not appearing in the training data)?
- Goal: detect these outliers (anomalies) to avoid dangerous misclassification.
- Strategy: Leverage generative models of the training distribution to detect outliers.



Why Generative Models? Generation for Simulation

- Supports Reinforcement Learning for Robotics: Make simulations sufficiently realistic that learned policies can readily transfer to real-world application



Generative model →

Photo from IEEE Spectrum

Deep Generative Models: Outline

Autoregressive models

- Deep NADE, PixelRNN, PixelCNN, WaveNet, Video Pixel Network, etc.

Latent variable models

- Variational Auto encoders
- Generative Adversarial Networks

our focus today

Latent Variable Models

- The Variational Autoencoder model:
  - Kingma and Welling, *Auto-Encoding Variational Bayes*, *International Conference on Learning Representations (ICLR)* 2014.
  - Rezende, Mohamed and Wierstra, *Stochastic back-propagation and variational inference in deep latent Gaussian models*. ICML 2014.

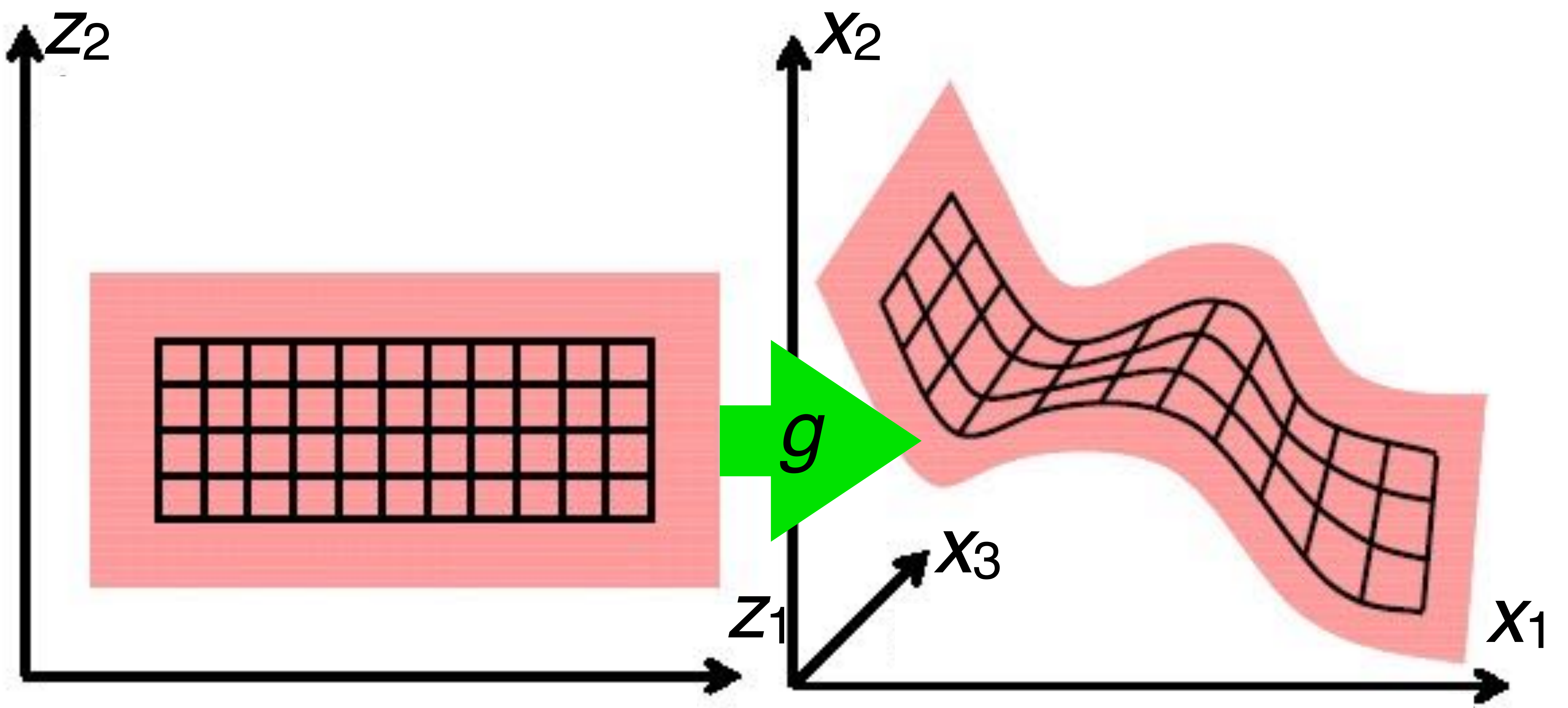
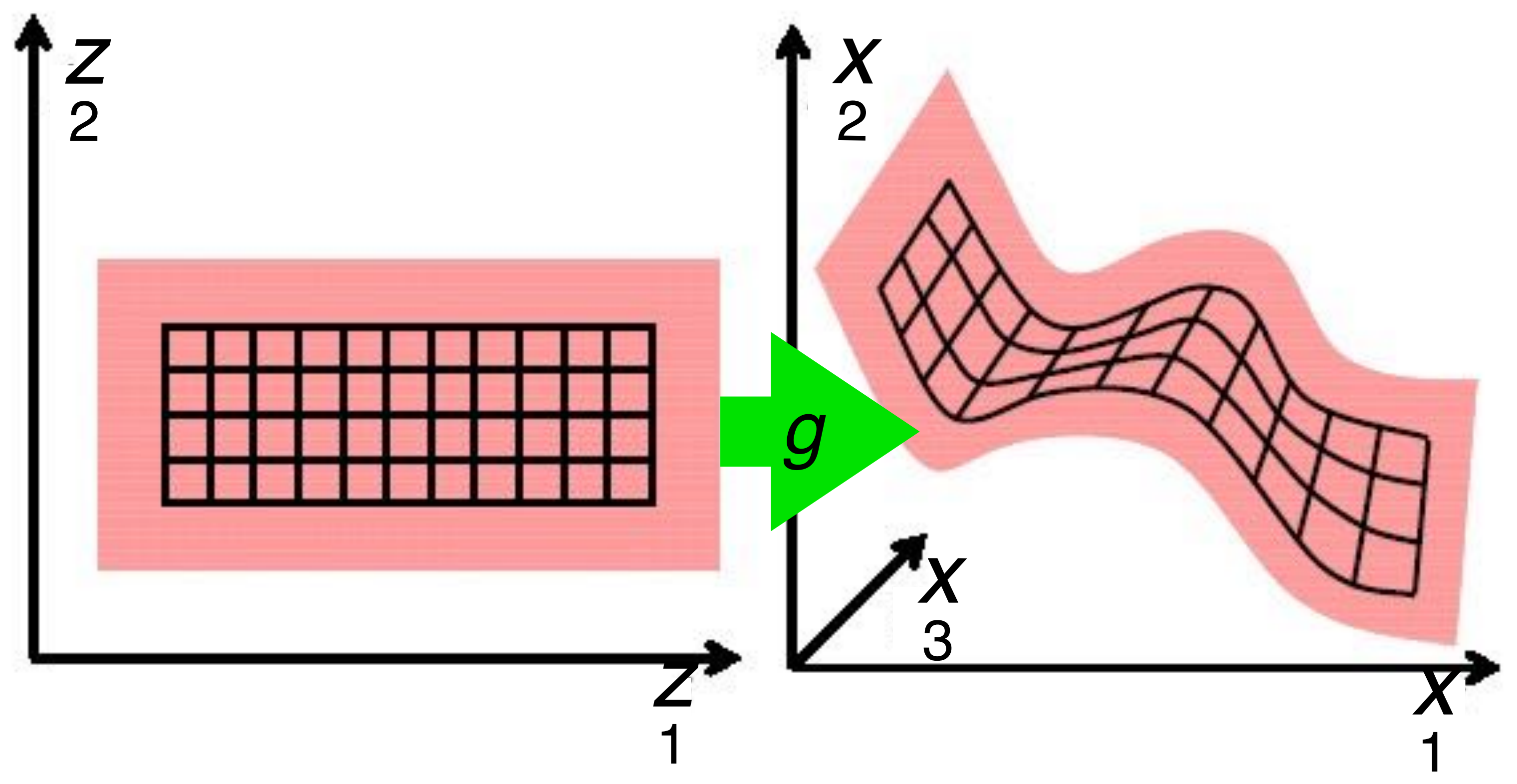
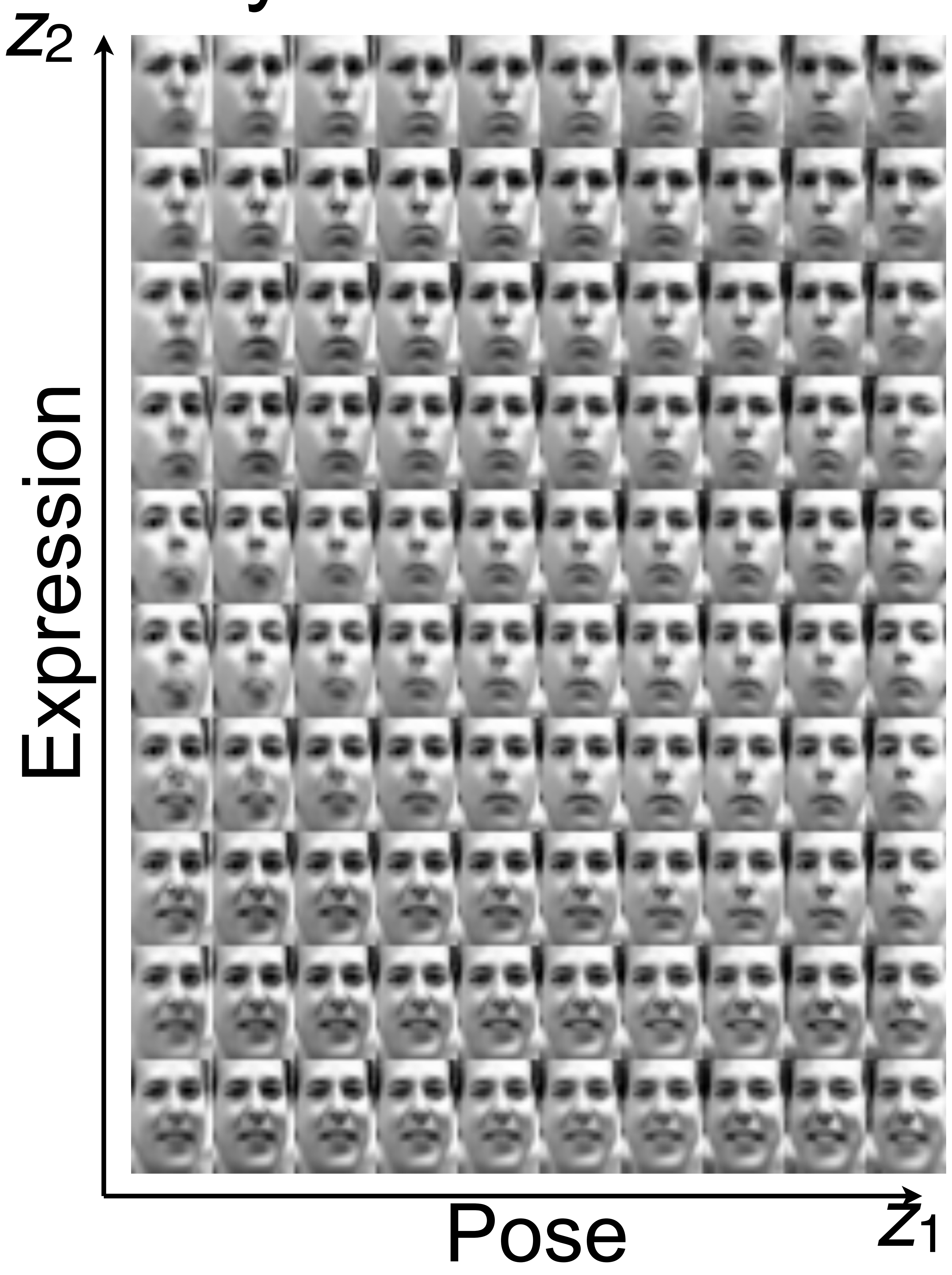


Image from: Ward, A. D., Hamarneh, G.: **3D Surface Parameterization Using Manifold Learning for Medial Shape Representation**, *Conference on Image Processing, Proc. of SPIE Medical Imaging*, 2007

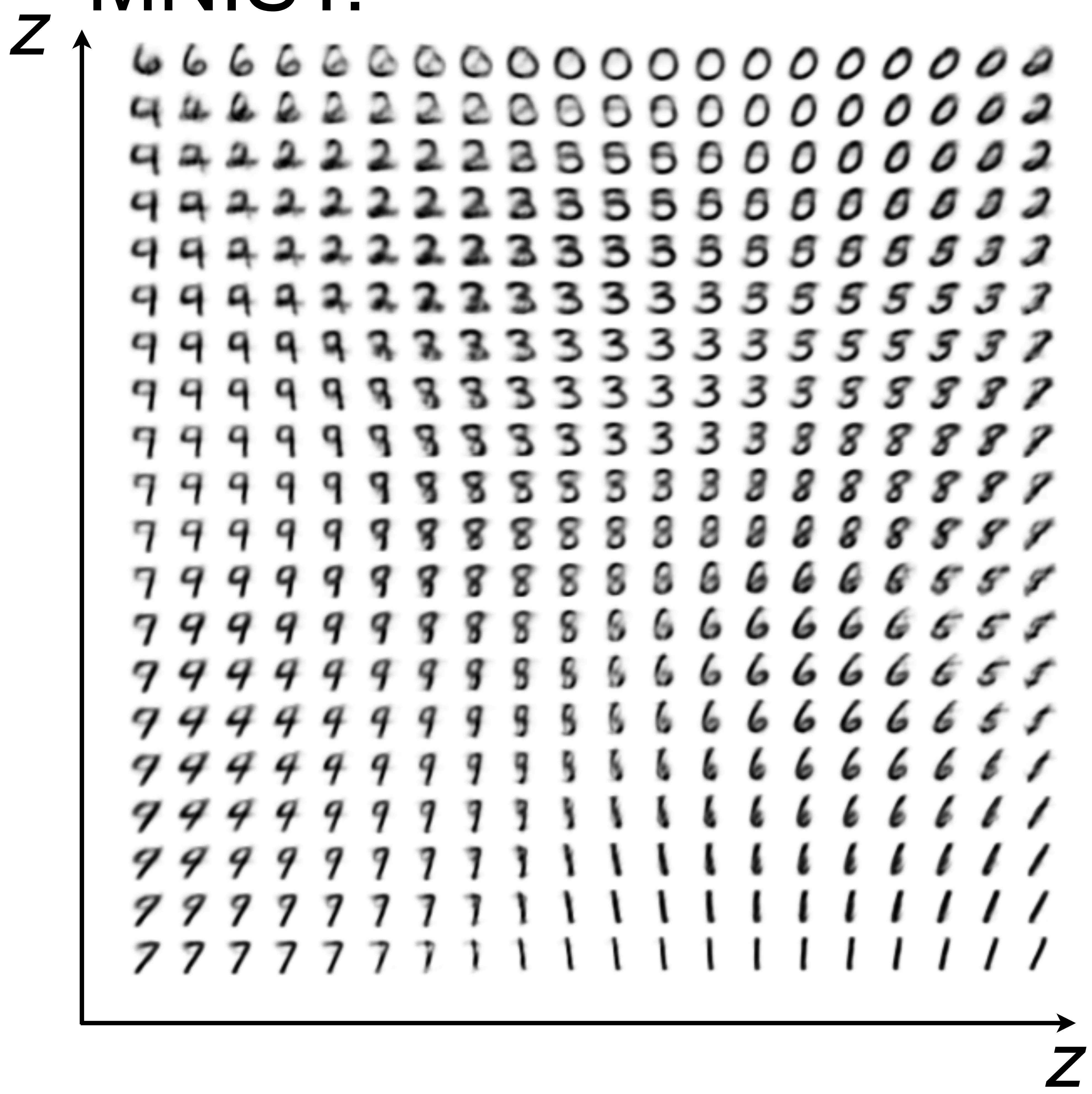
Latent Variable Models



Frey Faces:



MNIST:



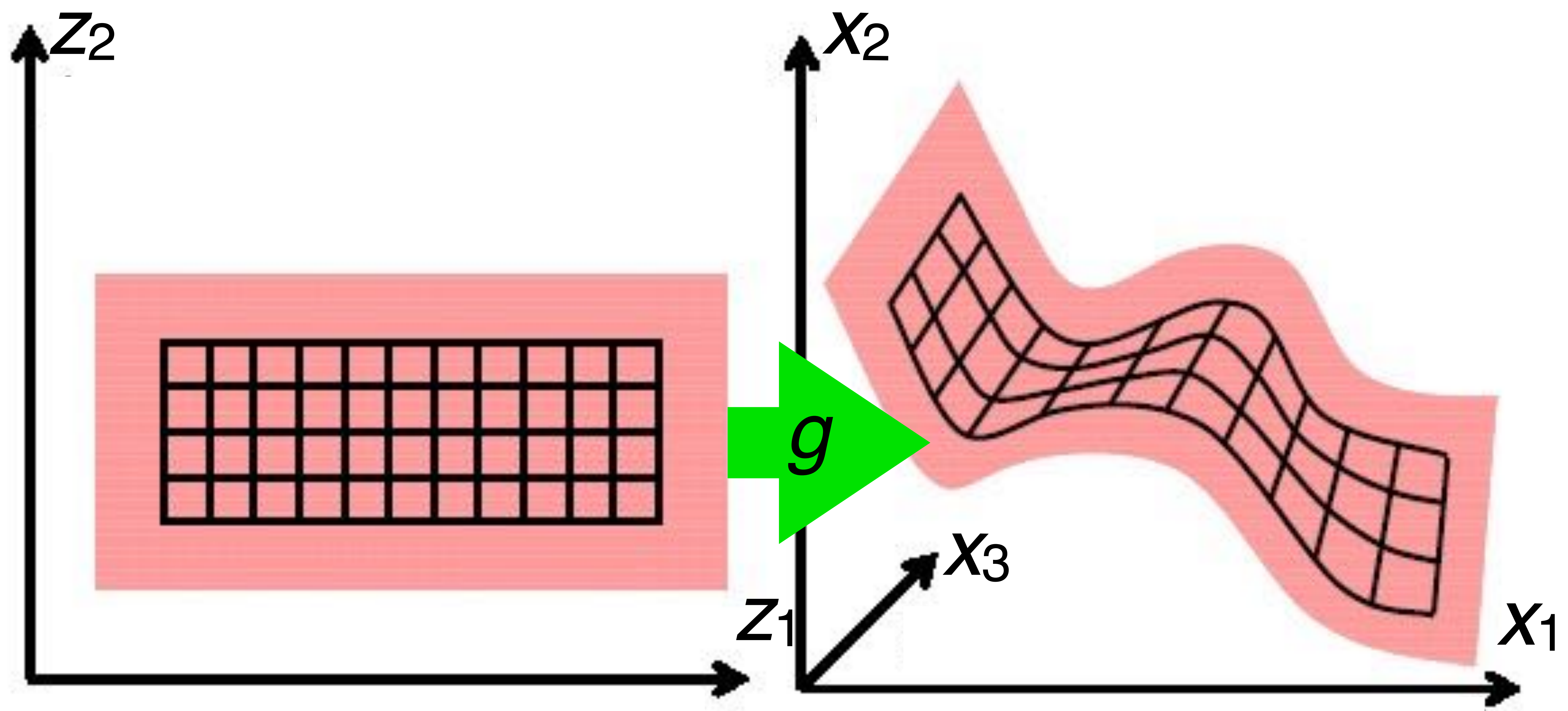
Latent Variable Models

- latent variable model: learn a mapping from some latent variable z to a complicated distribution on x.

$$p(x) = \int p(x,z)\ dz \qquad \text{where} \quad p(\boldsymbol{x},\boldsymbol{z}) = p(\boldsymbol{x} \mid \boldsymbol{z})p(\boldsymbol{z})$$

Prior $$p(\boldsymbol{z}) = \text{something simple} \qquad p(\boldsymbol{x} \mid \boldsymbol{z}) = g(\boldsymbol{z})$$

- Can we learn to decouple the true explanatory factors underlying the data distribution? E.g. separate identity and expression in face images



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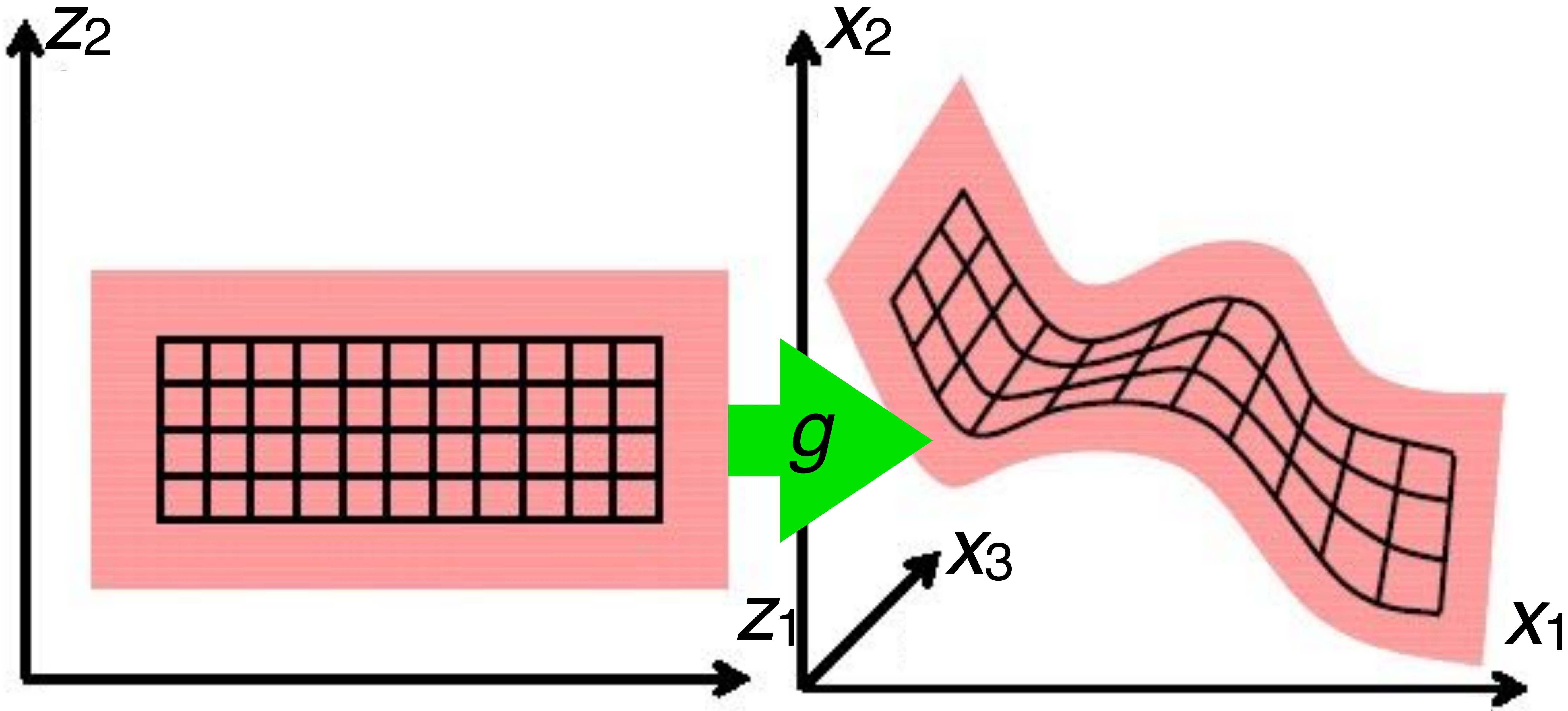


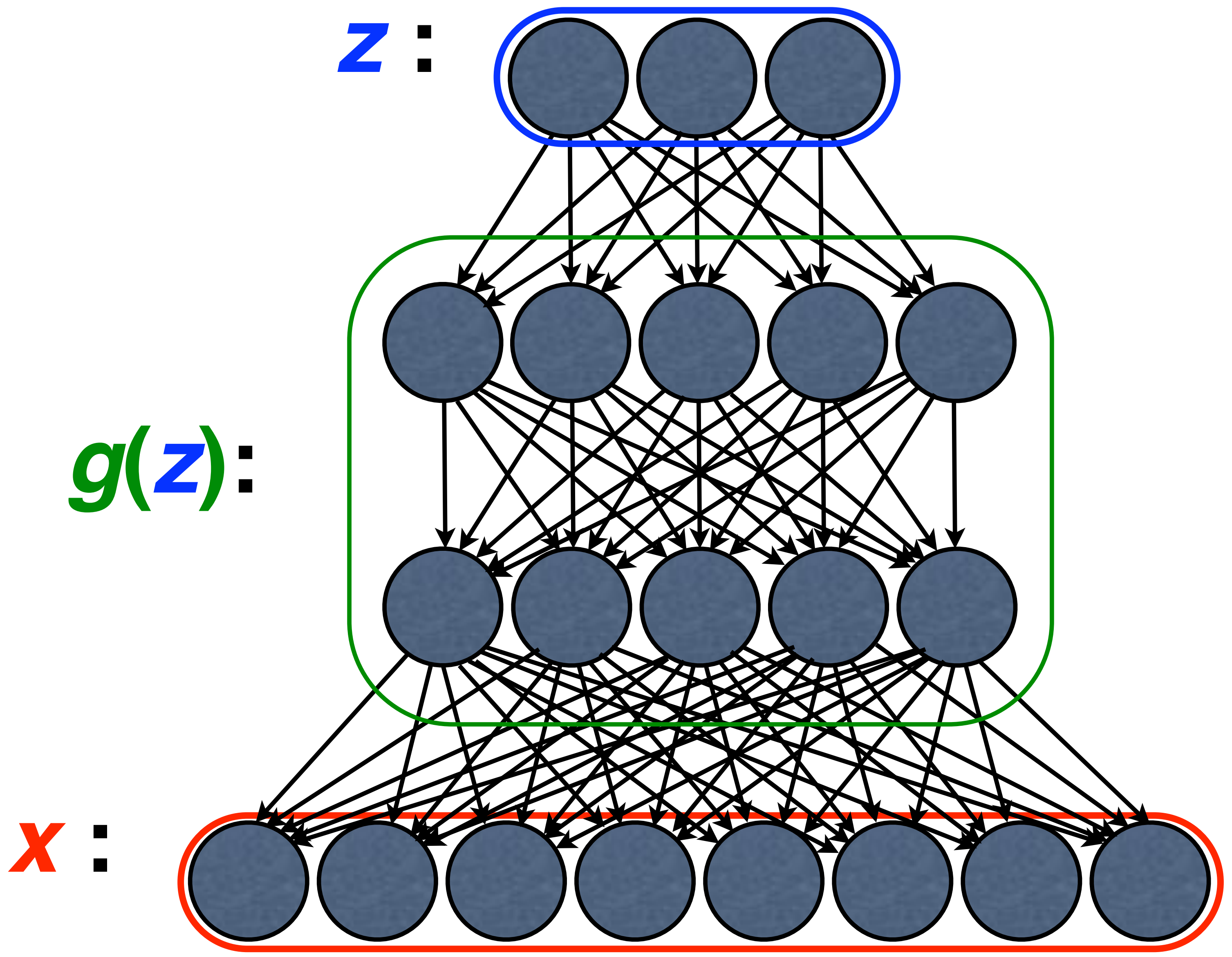


Image from: Ward, A. D., Hamarneh, G.: **3D Surface Parameterization Using Manifold Learning for Medial Shape Representation**, *Conference on Image Processing, Proc. of SPIE Medical Imaging*, 2007

Variational Auto-Encoder (VAE)

- Where does ***z*** come from? — The classic DAG problem.
- The VAE approach: introduce an inference machine $q_\phi(z \mid x)$ that learns to approximate the posterior $p_\theta(z \mid x)$.
- Define a variational lower bound on the data likelihood: $\log p_\theta(x) \geq \mathcal{L}(\theta, \phi, x)$

$$
\begin{aligned}
\log p_\theta(x) &\geq \log p_\theta(x) - D_{\mathrm{KL}}\left[q_\phi(z|x) \| p_\theta(z|x)\right] \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log p_\theta(x) + \log p_\theta(z|x) - \log q_\phi(z|x)\right] \\
&= \mathcal{L}(\theta, \phi, x) \\
&= \mathbb{E}_{q_\phi(z|x)}\left[\log p_\theta(x|z) + \log p_\theta(z) - \log q_\phi(z|x)\right] \\
&= -D_{\mathrm{KL}}\left[q_\phi(z|x) \| p_\theta(z)\right] + \mathbb{E}_{q_\phi(z|x)}\left[\log p_\theta(x|z)\right]
\end{aligned}
$$

- What is $q_\phi(z \mid x)$?

***regularization* term** ***reconstruction* term**

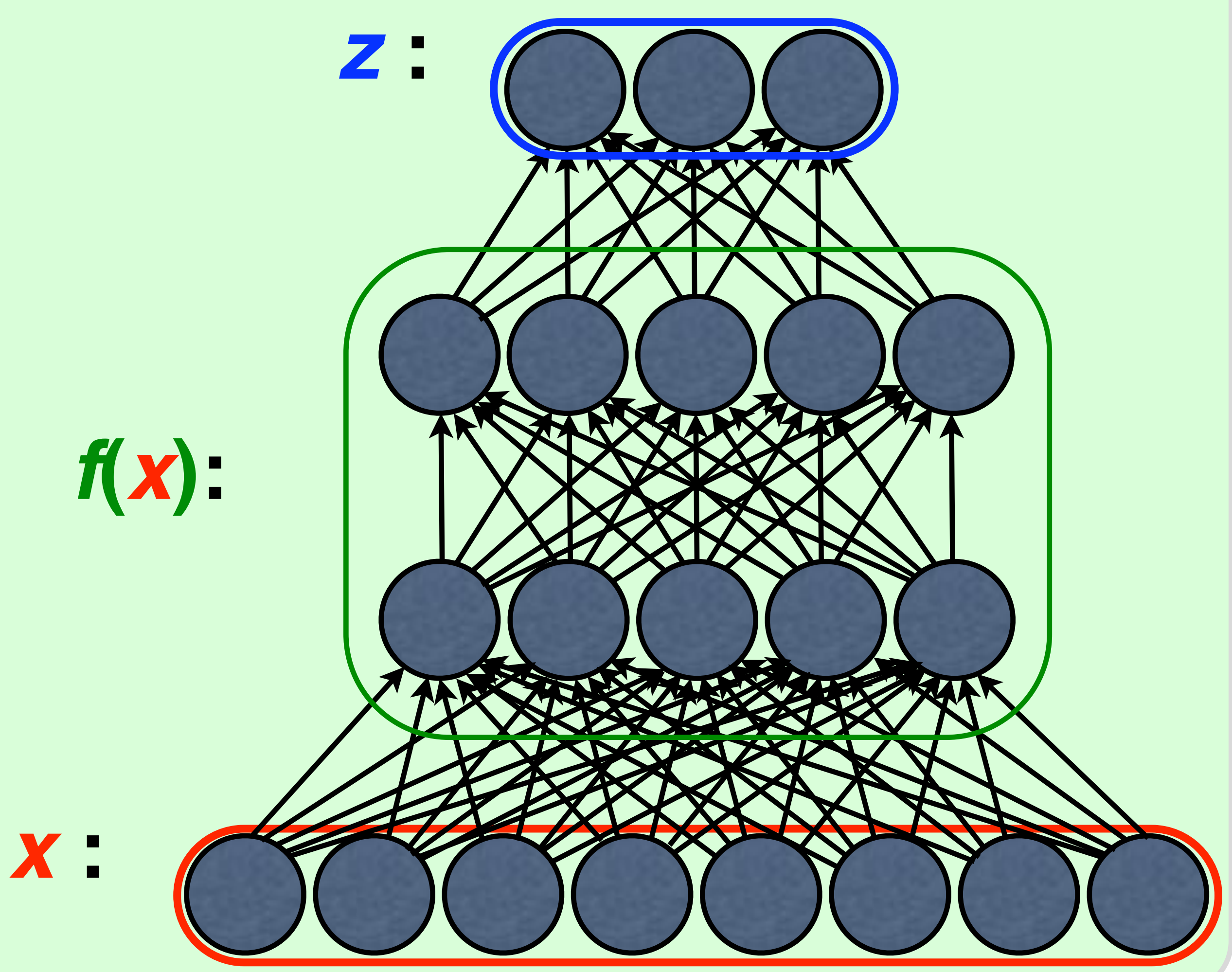
VAE Inference model

- The VAE approach: introduce an inference model $q_\phi(z \mid x)$ that learns to approximates the intractable posterior $p_\theta(z \mid x)$ by optimizing the variational lower bound:

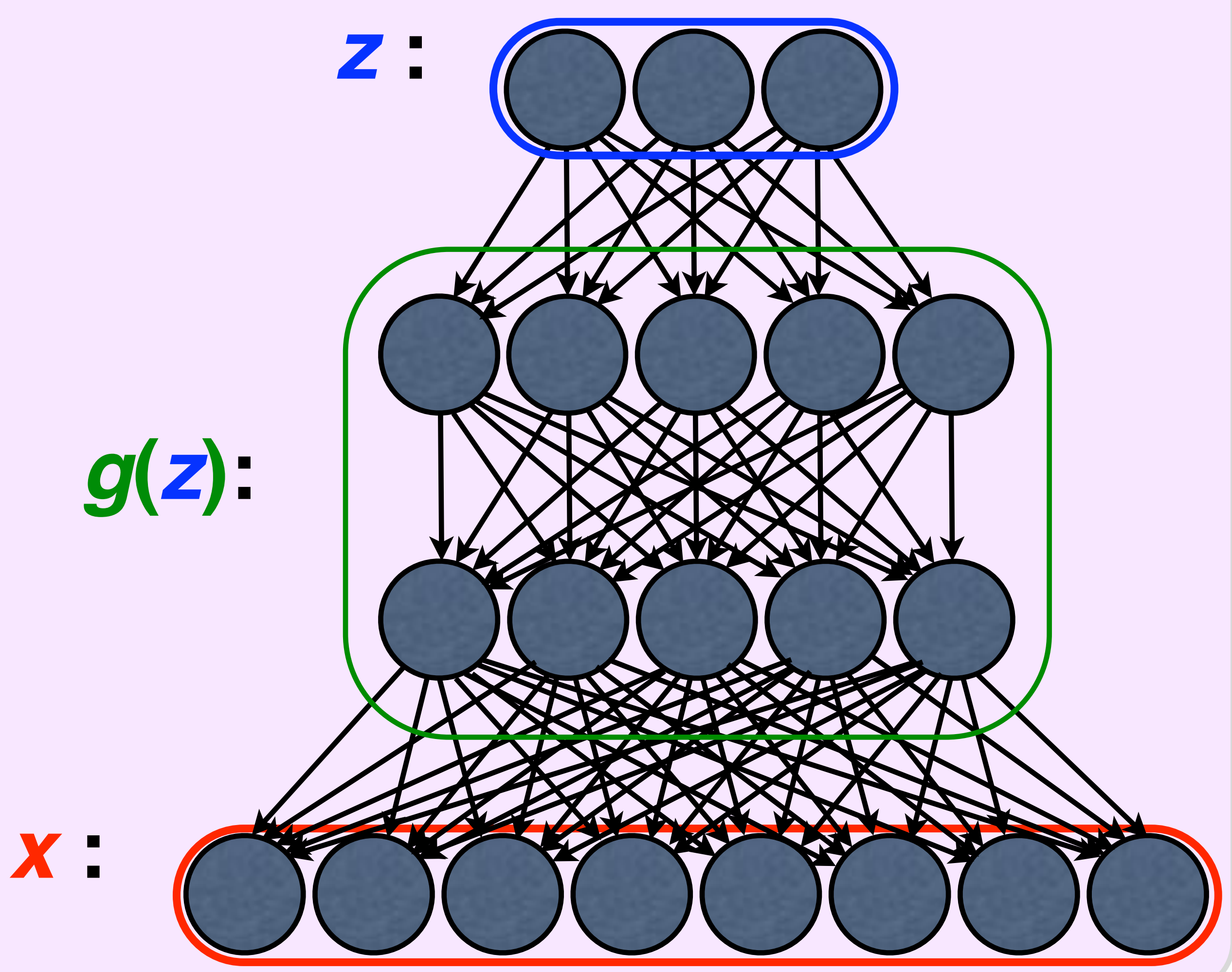
$$\mathcal{L}(\theta, \phi, x) = -D_{\mathrm{KL}}\left(q_\phi(z \mid x) \| p_\theta(z)\right) + \mathbb{E}_{q_\phi(z \mid x)}\left[\log p_\theta(x \mid z)\right]$$

- We parameterize $q_\phi(z \mid x)$ with another neural network:

$$q_\phi(z \mid x) = q(z; f(x, \phi))$$

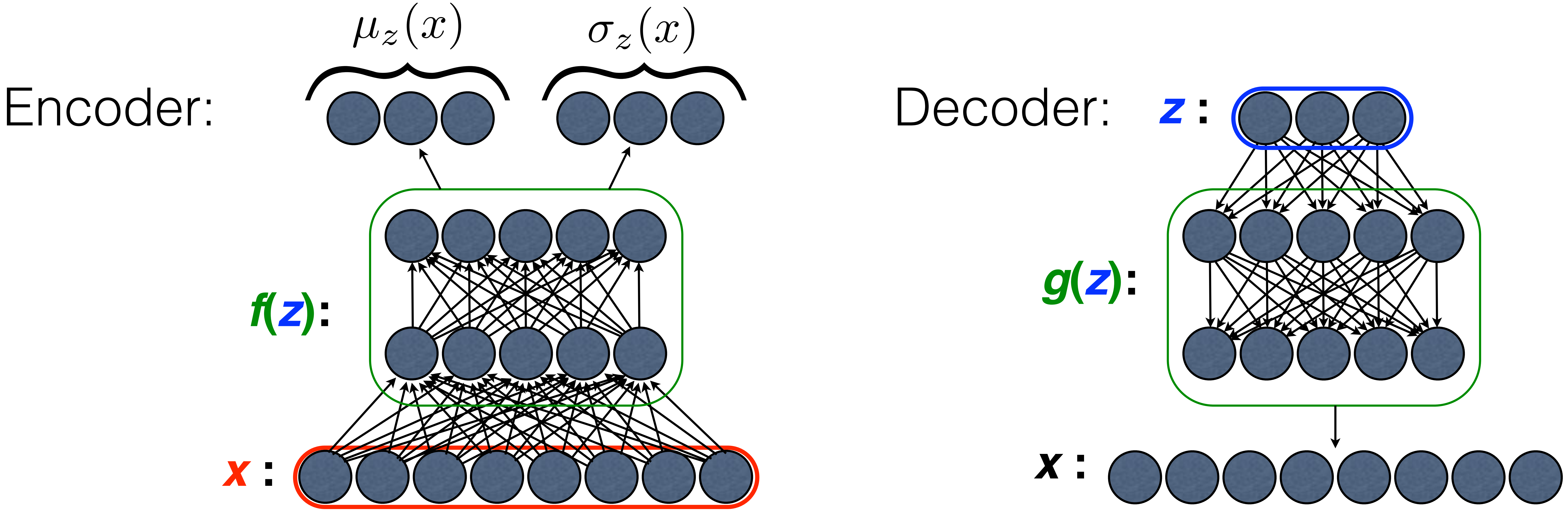


$$p_\theta(x \mid z) = p(x; g(z, \theta))$$



Reparametrization trick

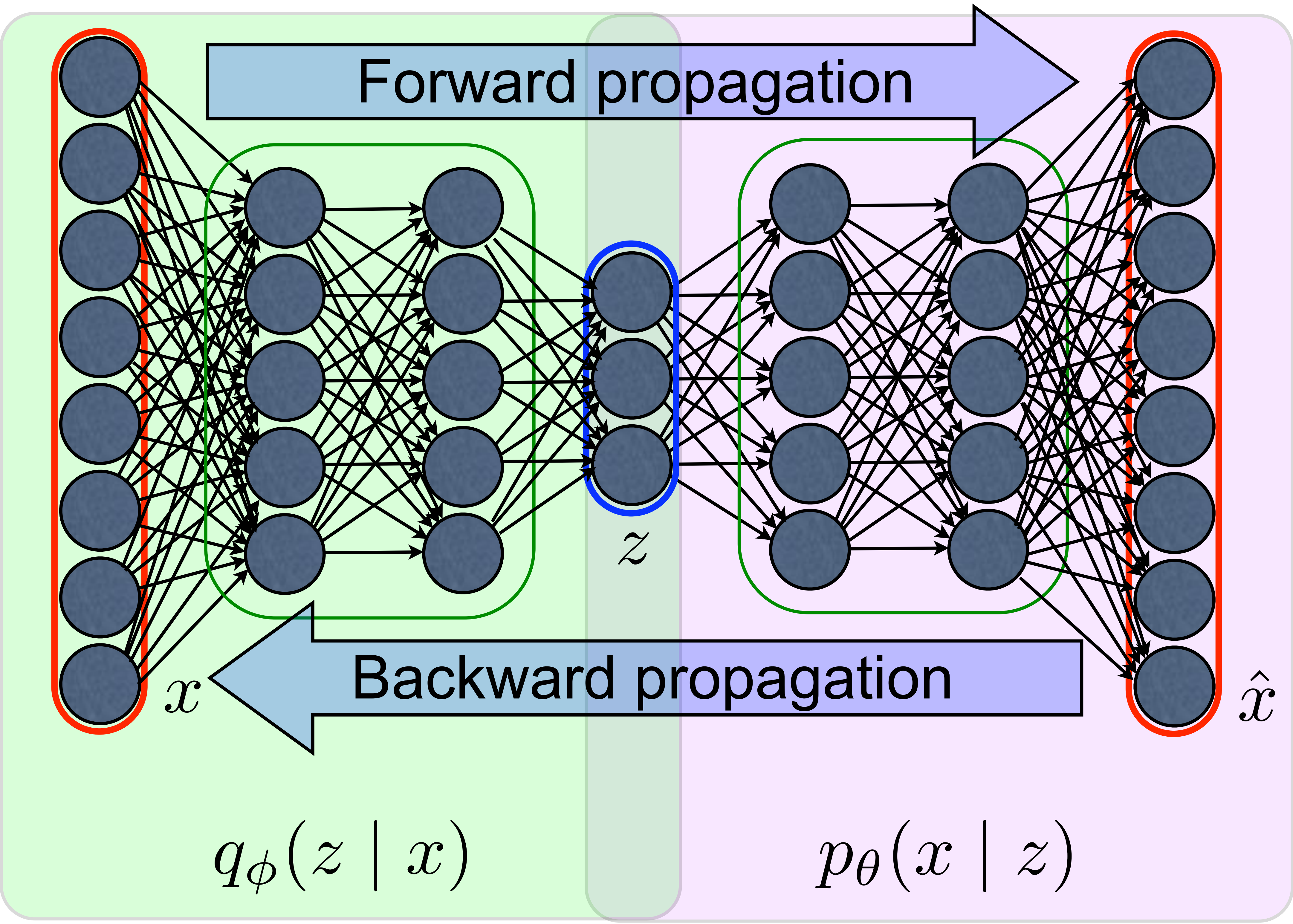
- Adding a few details + one really important trick
- Let's consider ***z*** to be real and $q_\phi(z \mid x) = \mathcal{N}(z; \mu_z(x), \sigma_z(x))$
- Parametrize ***z*** as $z = \mu_z(x) + \sigma_z(x)\epsilon_z$ where $\epsilon_z = \mathcal{N}(0, 1)$



Training with backpropagation!

- Due to a reparametrization trick, we can simultaneously train both the generative model $p_\theta(x \mid z)$ and the inference model $q_\phi(z \mid x)$ by optimizing the variational bound using gradient backpropagation.

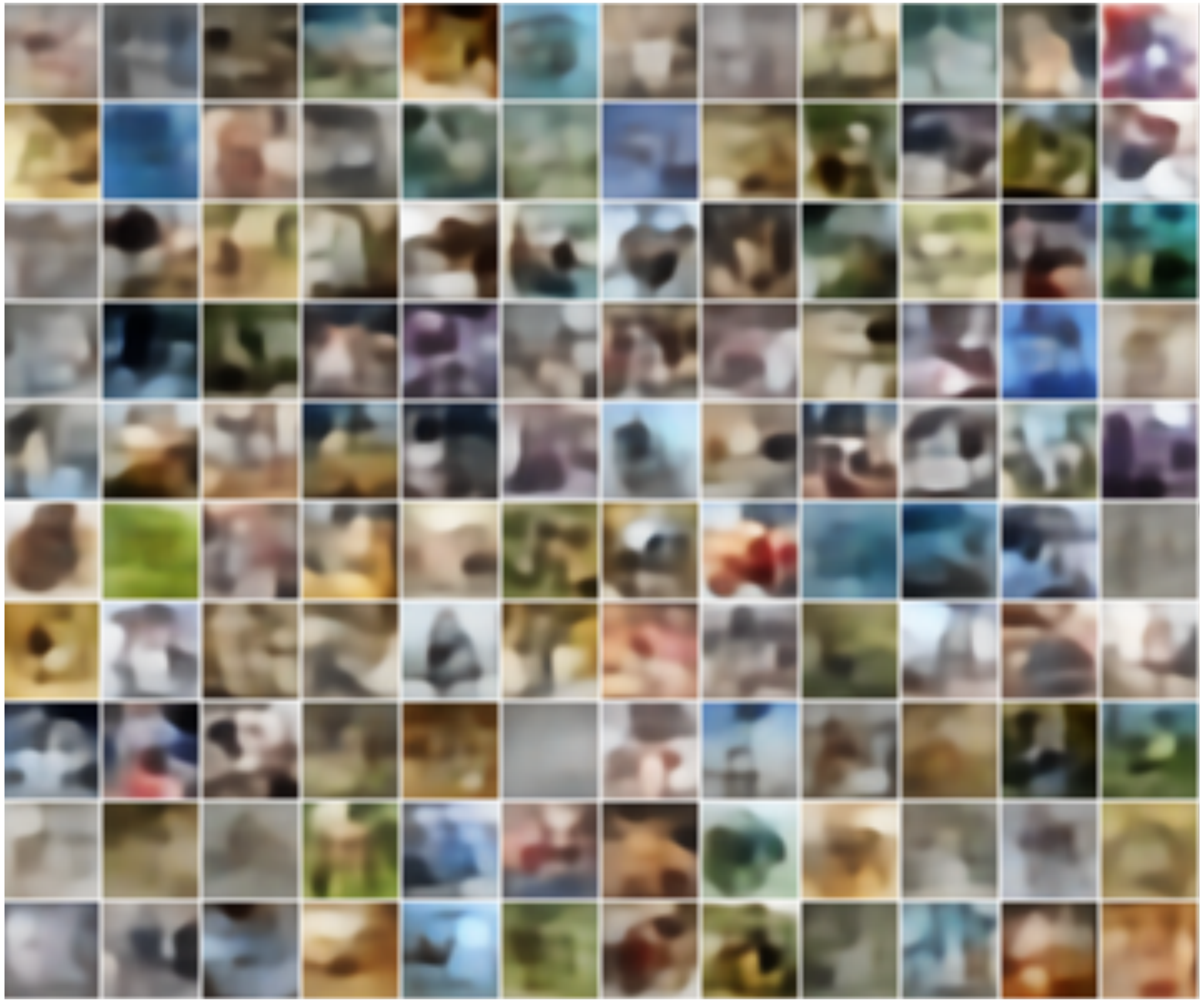
Objective function: $\mathcal{L}(\theta, \phi, x) = -D_{\mathrm{KL}}\left(q_\phi(z \mid x) \| \, p_\theta(z)\right) + \mathbb{E}_{q_\phi(z|x)}\left[\log p_\theta(x \mid z)\right]$



vanilla VAE samples

Impressive …
… at the time

Labelled Faces in the Wild (LFW)



ImageNet (small)

PixelVAE

Ishaan Gulrajani, Kundan Kumar, Faruk Ahmed Adrien Ali Taiga, Francesco Visin, David Vazquez, Aaron Courville. ICLR 2017

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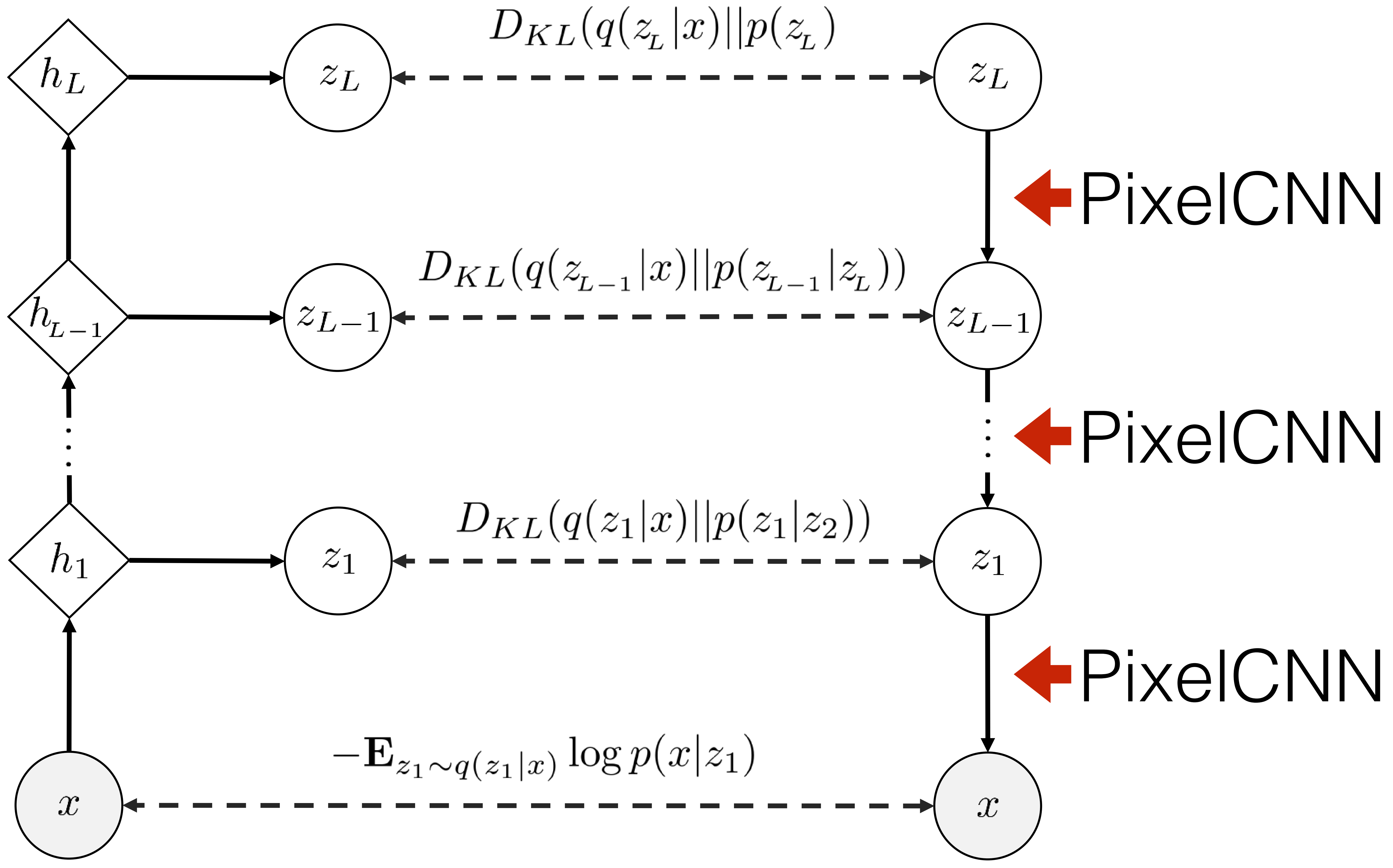
- Uses a PixelCNN in the VAE decoder to help avoid the blurring caused by the standard VAE assumption of independent pixels.



PixelVAE Samples (Gulrajani et al. 2017)

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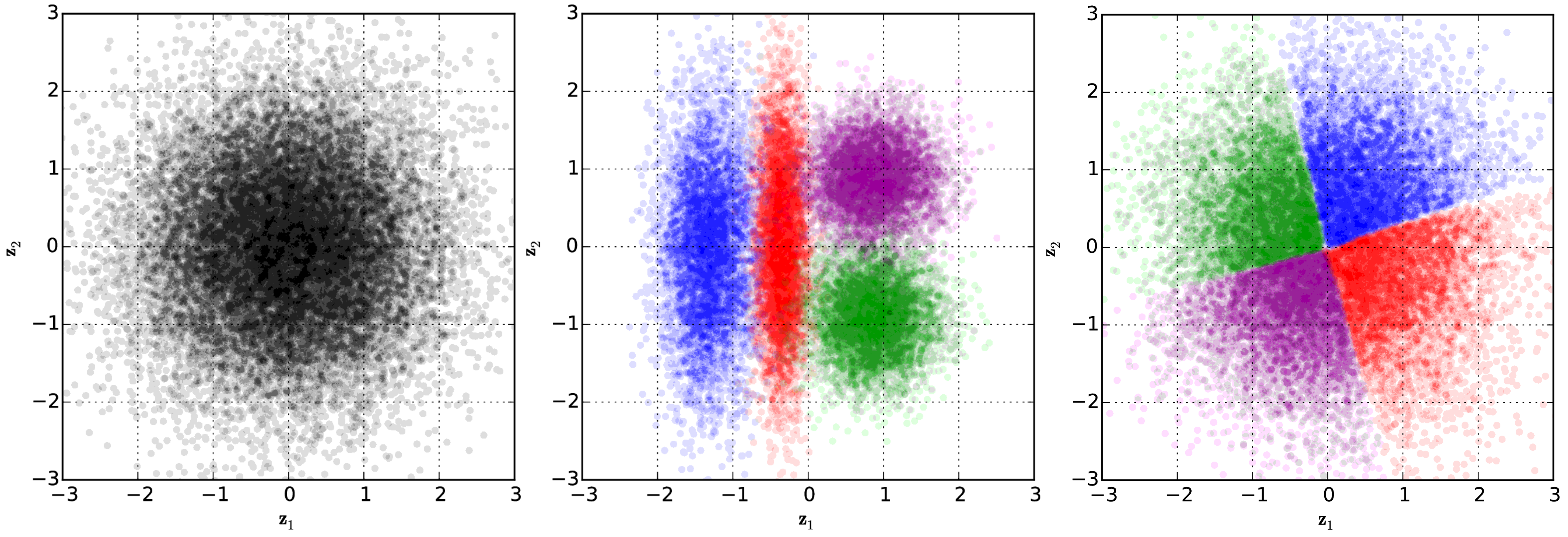
LSUN bedroom scenes (64x64)

ImageNet (64x64)

Inverse Autoregressive Flow (Kingma et al., NIPS 2016)



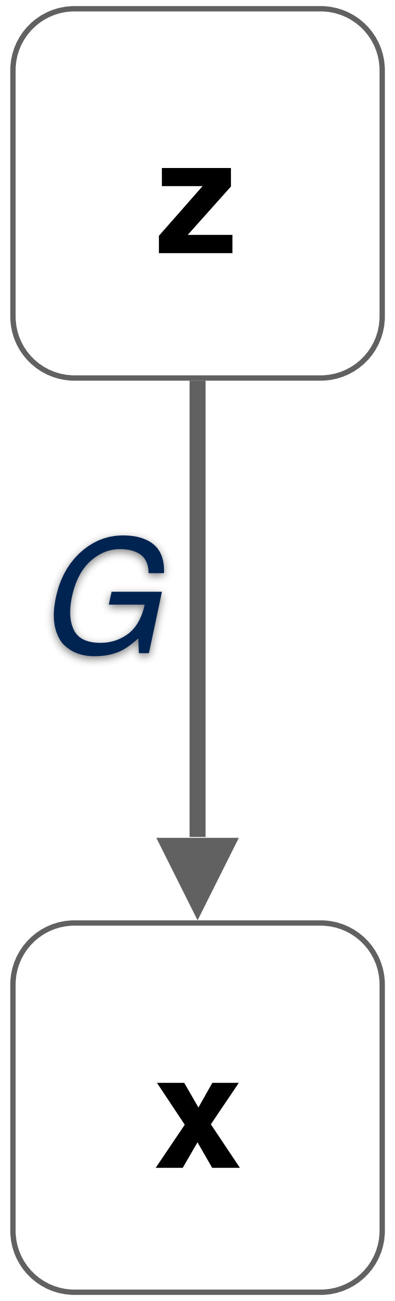
(a) Prior distribution (b) Posteriors in standard VAE (c) Posteriors in VAE with IAF

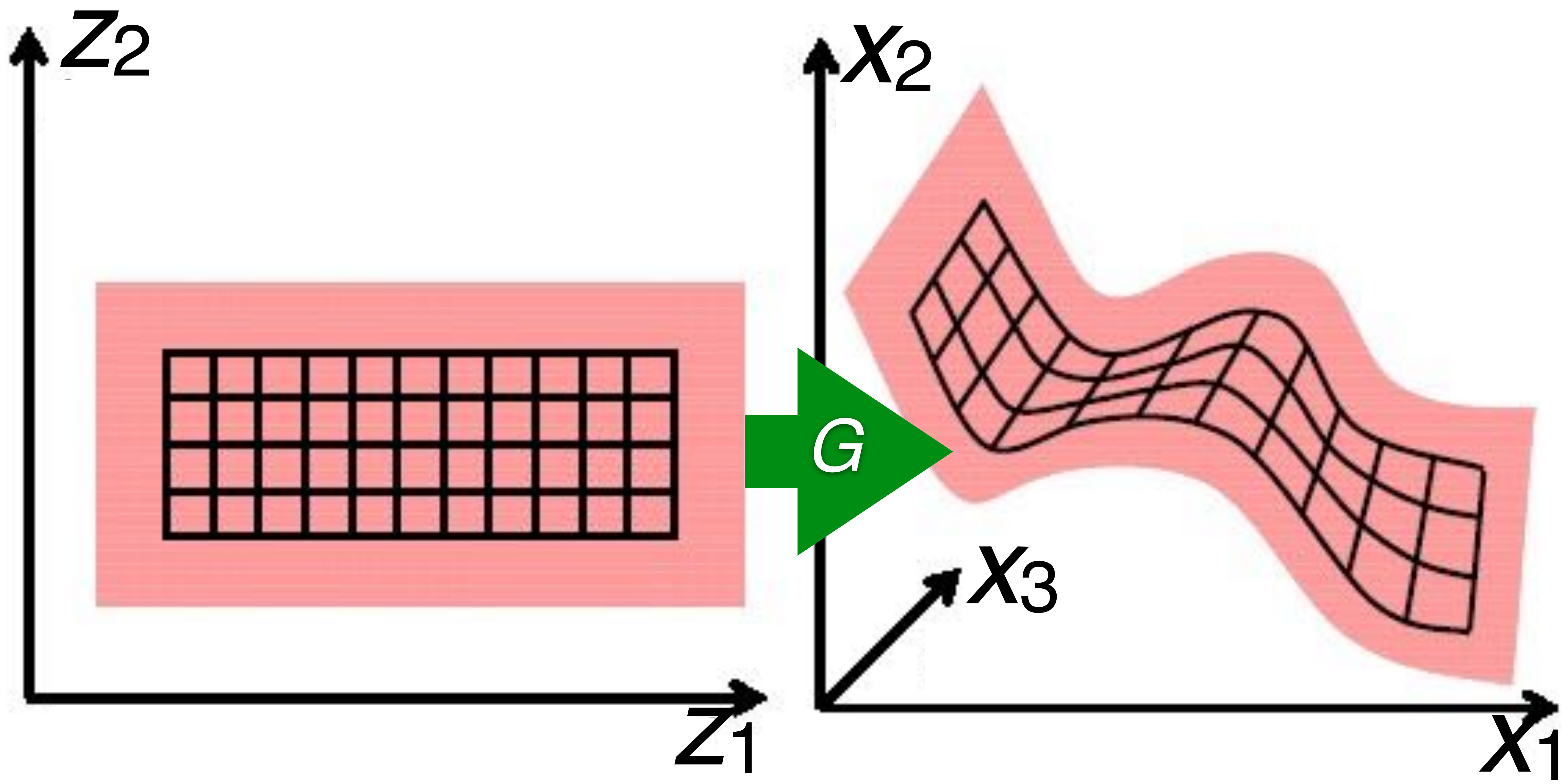
- Standard VAE posteriors are factorized - limiting how well they can (marginally) fit the prior.
- IAF greatly improves the flexibility of the posterior distributions, and allows for a much better fit between the posteriors and the prior.

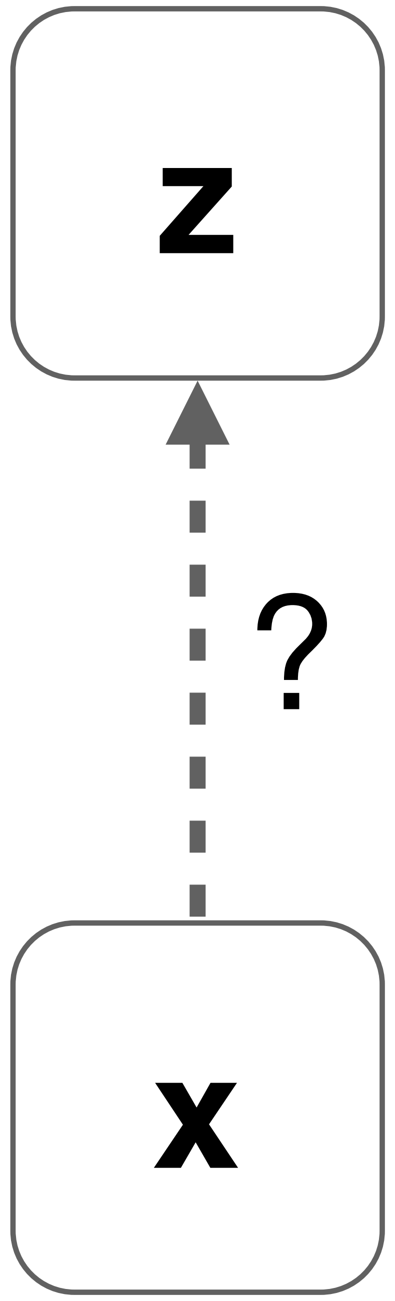
Another way to train a latent variable model?

inference

Latent variables

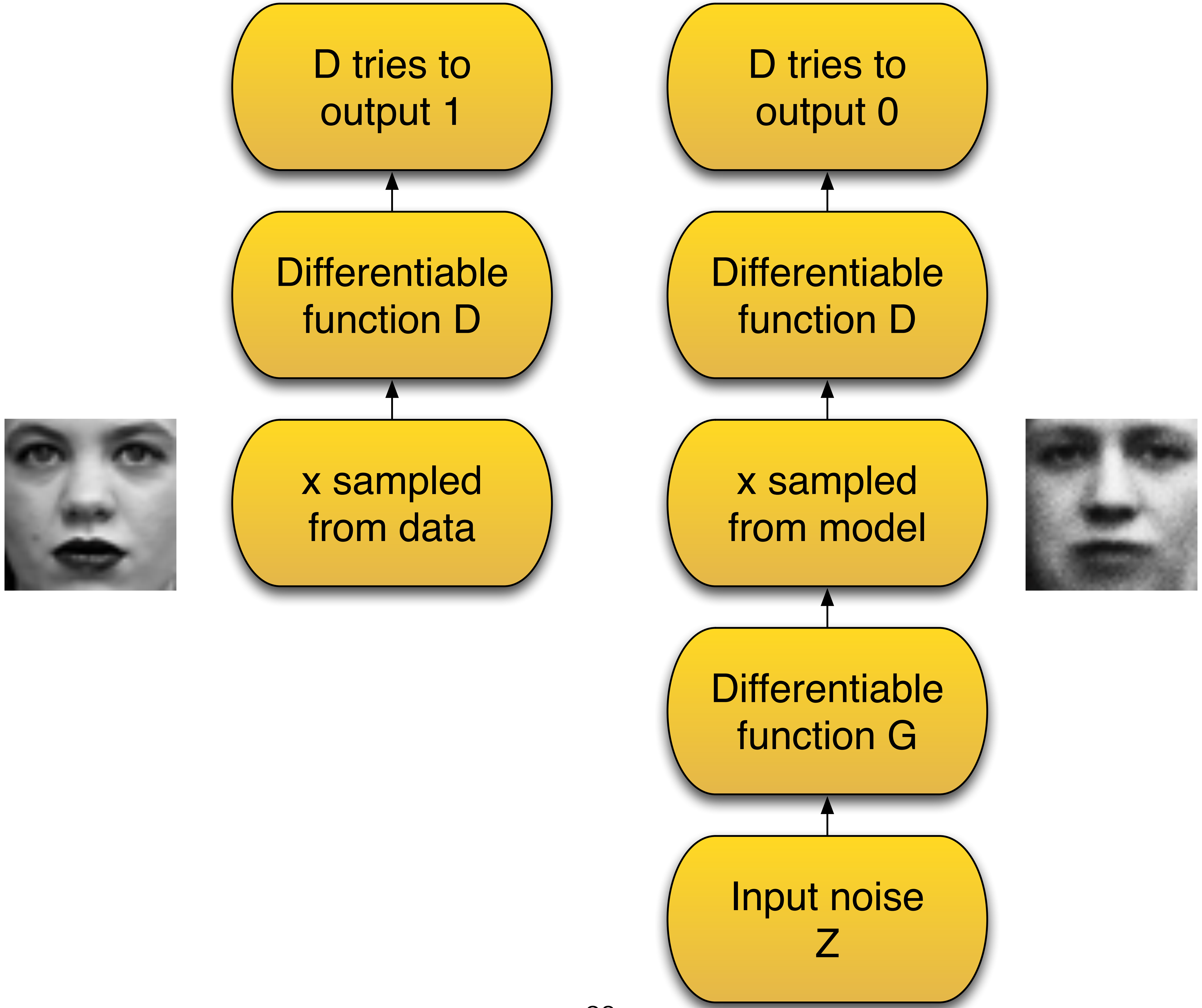






Observed variables

Generative Adversarial Networks



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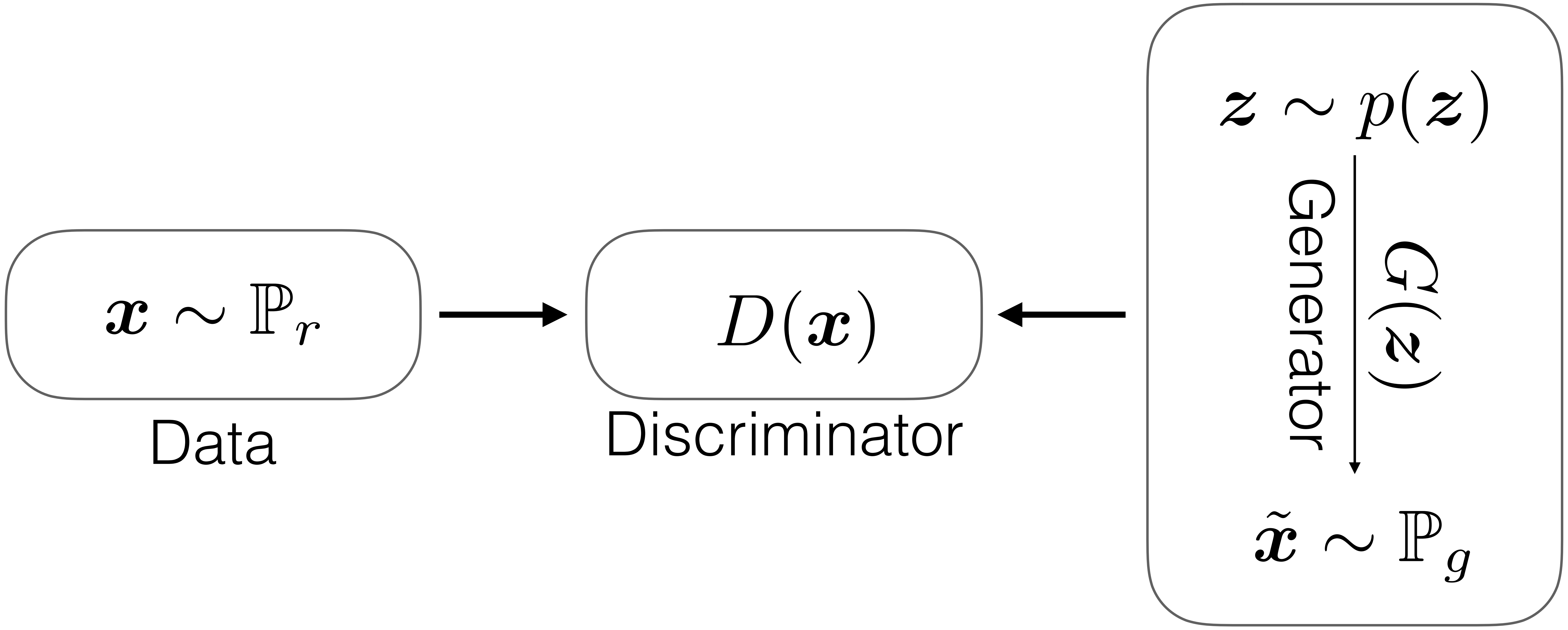

GAN Objective

- Formally, express the game between discriminator D and generator G with the minimax objective:

$$\min_{G} \max_{D} \mathop{\mathbb{E}}_{\boldsymbol{x} \sim \mathbb{P}_r} [\log(D(\boldsymbol{x}))] + \mathop{\mathbb{E}}_{\tilde{\boldsymbol{x}} \sim \mathbb{P}_g} [\log(1 - D(\tilde{\boldsymbol{x}}))].$$

where:

- $\mathbb{P}_r$ is the data distribution
- $\mathbb{P}_g$ is the model distribution implicitly defined by:

$$\tilde{\boldsymbol{x}} = G(\boldsymbol{z}), \quad \boldsymbol{z} \sim p(\boldsymbol{z})$$

- the generator input $\boldsymbol{z}$ is sampled from some simple noise distribution, (e.g. uniform or Gaussian).

GAN Theory

- Optimal (nonparametric) discriminator:

$$D^*(\boldsymbol{x}) = \frac{p_r(\boldsymbol{x})}{p_r(\boldsymbol{x}) + p_g(\boldsymbol{x})}$$

- Under an ideal discriminator, the generator minimizes the Jensen-Shannon divergence between $\mathbb{P}_r$ and $\mathbb{P}_g$.

$$\mathrm{JS}(\mathbb{P}_r \| \mathbb{P}_g) = \mathrm{KL}\left(\mathbb{P}_r \middle\| \frac{\mathbb{P}_r + \mathbb{P}_g}{2}\right) + \mathrm{KL}\left(\mathbb{P}_g \middle\| \frac{\mathbb{P}_r + \mathbb{P}_g}{2}\right)$$

where $\mathrm{KL}(\mathbb{P}_r \| \mathbb{P}_g) = \int \log\left(\frac{p_r(x)}{p_g(x)}\right) p_r(x) d\mu(x)$

GAN Theory … in practice

- The minimax objective leads to vanishing gradients as the discriminator saturates.

- In practice, Goodfellow et al (2014) advocate the heuristic training objective:

$$\max_{D} \underset{\boldsymbol{x}\sim\mathbb{P}_r}{\mathbb{E}}[\log(D(\boldsymbol{x}))] + \underset{\tilde{\boldsymbol{x}}\sim\mathbb{P}_g}{\mathbb{E}}[\log(1 - D(\tilde{\boldsymbol{x}}))].$$

$$\max_{G} \underset{\tilde{\boldsymbol{x}}\sim\mathbb{P}_g}{\mathbb{E}}[\log(D(\tilde{\boldsymbol{x}}))].$$

- However, this modified loss function can still misbehave in the presence of a good discriminator.

GAN samples

Least-Squares GAN

Xudong Mao, Qing Li†, Haoran Xie, Raymond Y.K. Lau and Zhen Wang, ArXiv, Feb. 2017

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128x128 LSUN bedroom scenes

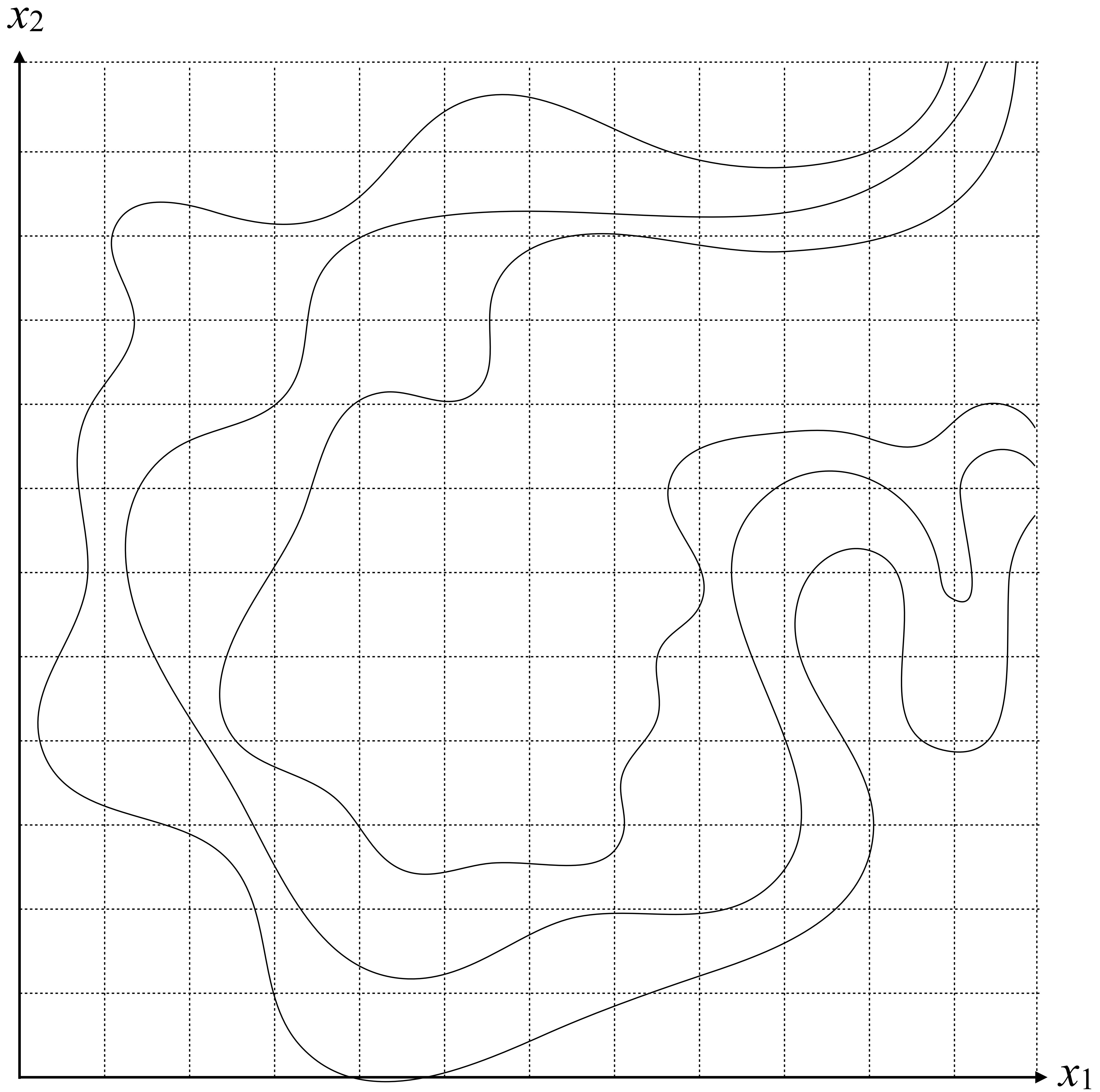
DCGAN samples (Radford, Metz and Chintala; 2016)

Z-space interpolations

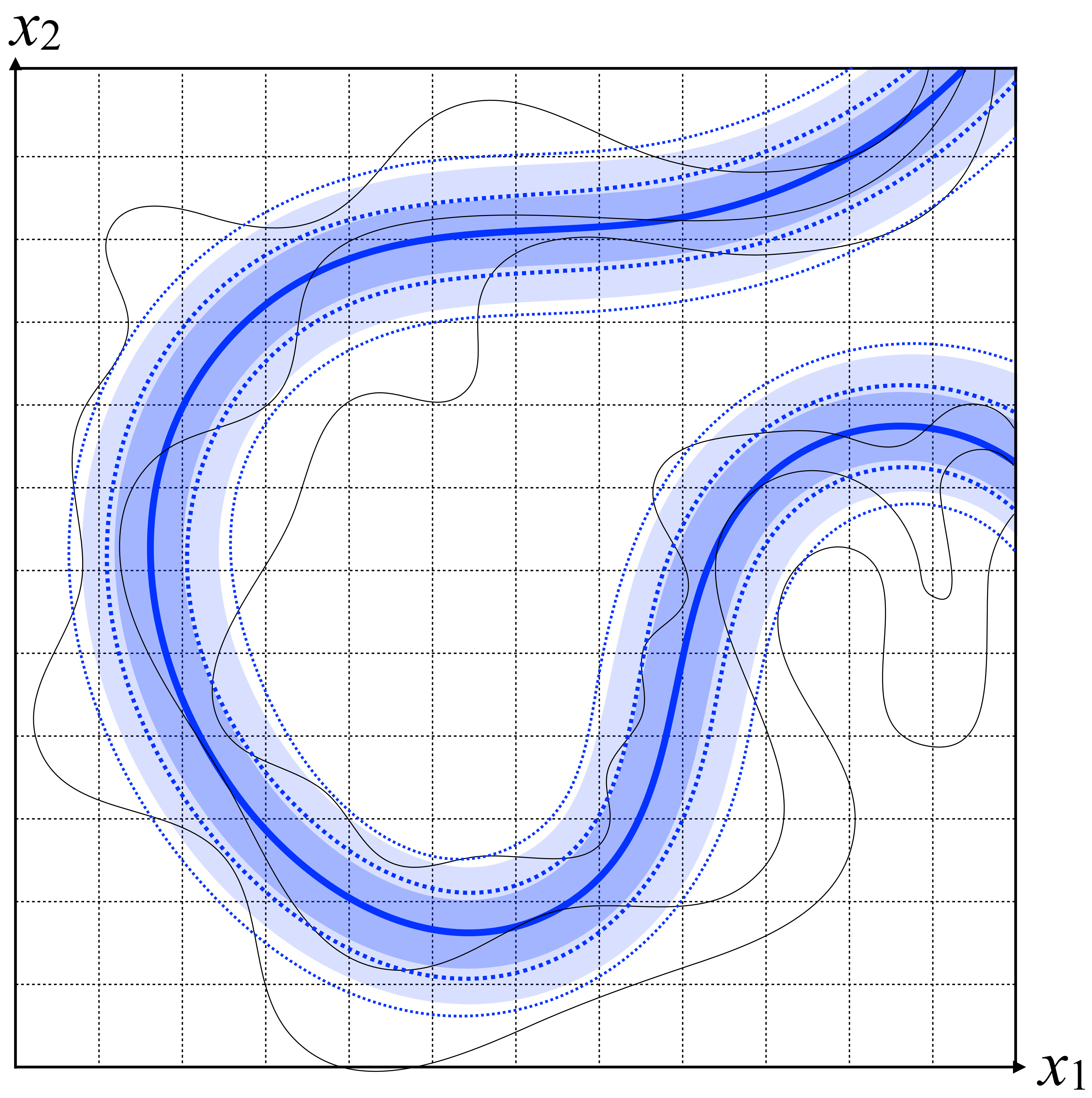
LSUN bedroom scenes

What makes GANs special?

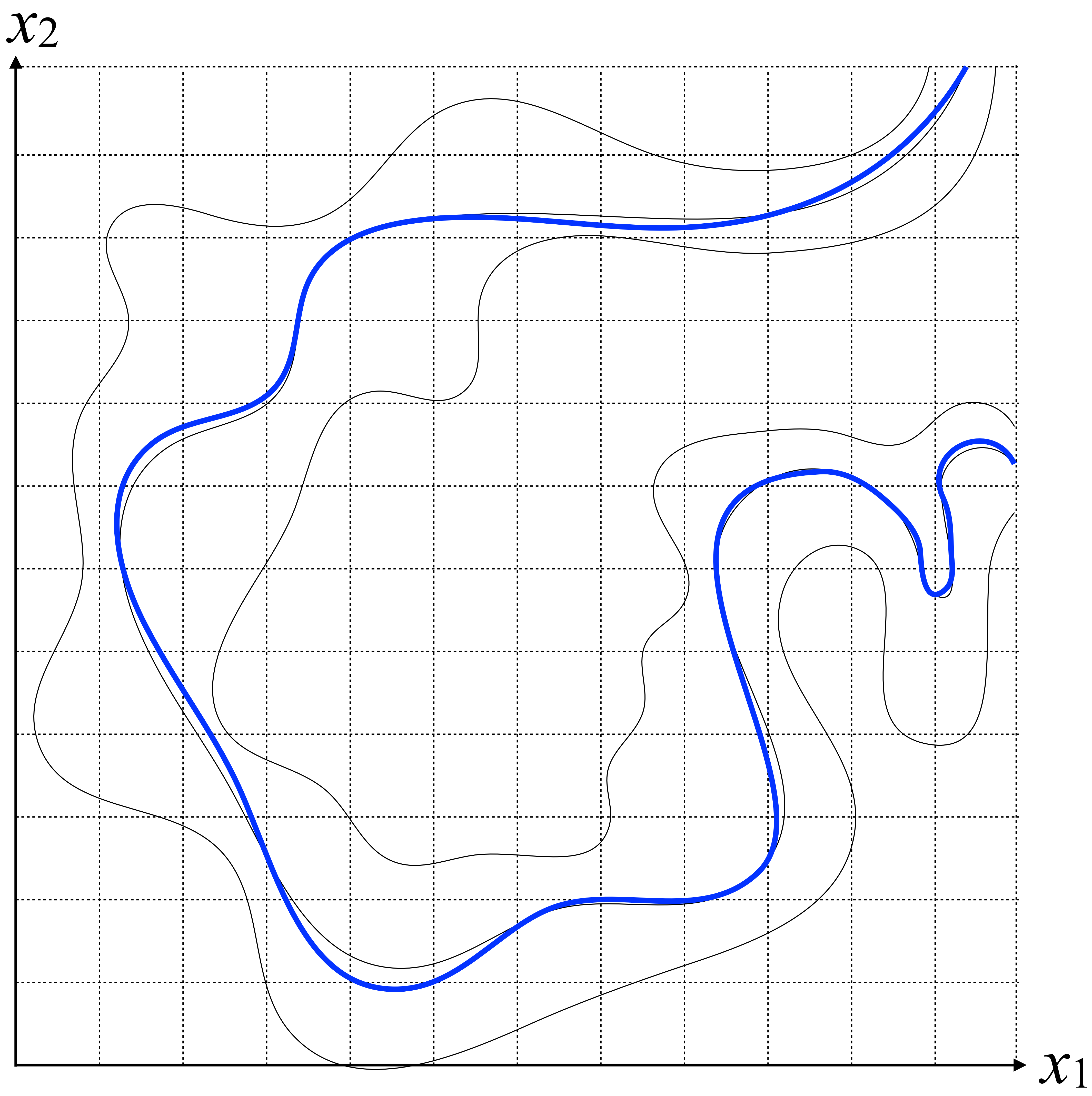
Cartoon of the Image manifold:



What makes GANs special?



more traditional max-likelihood approach

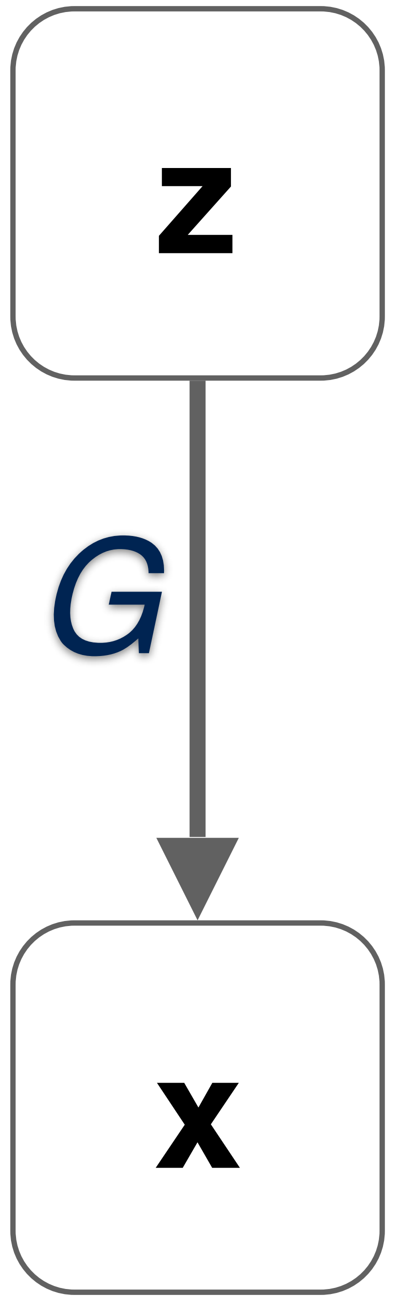


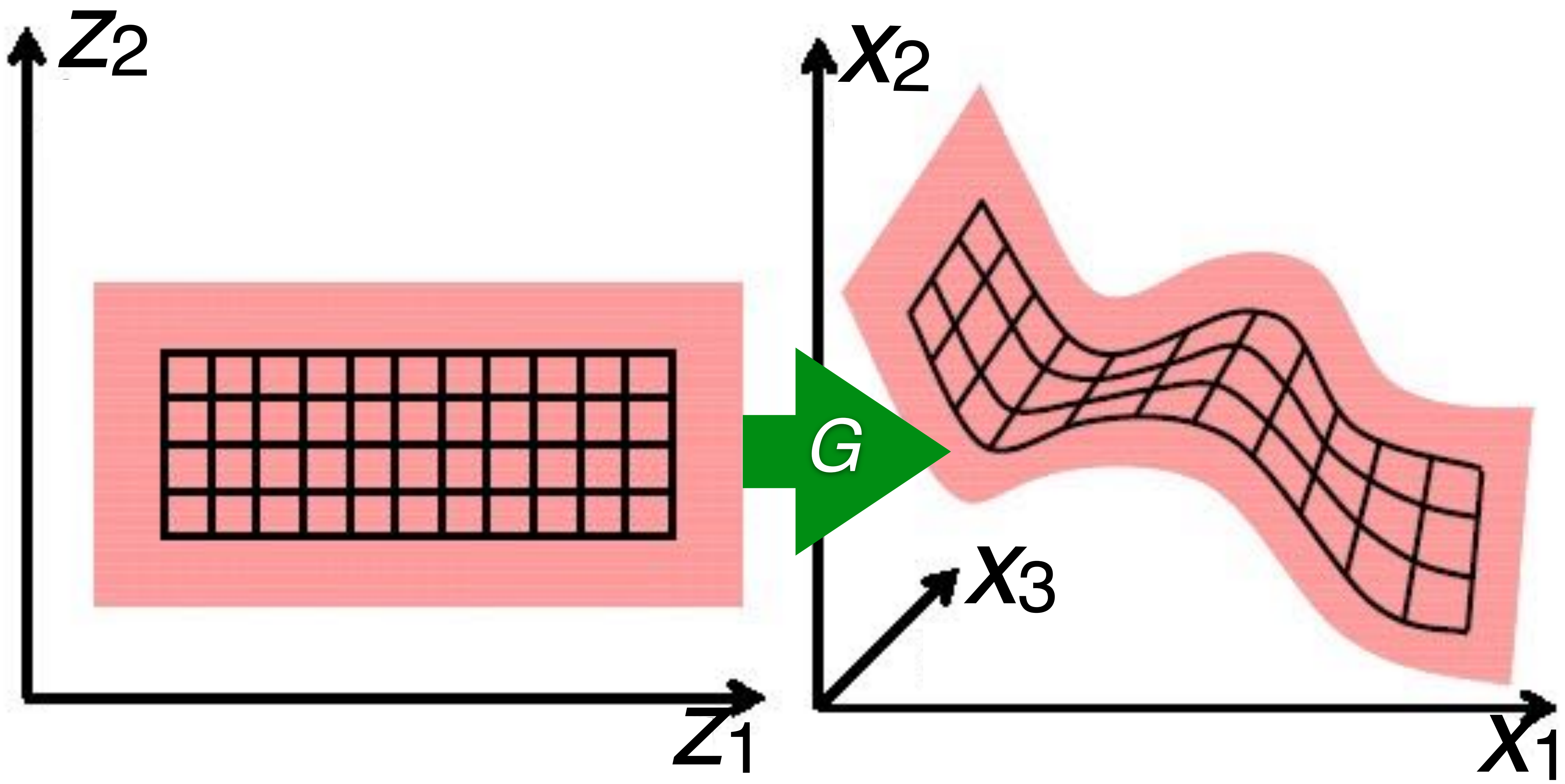
GAN

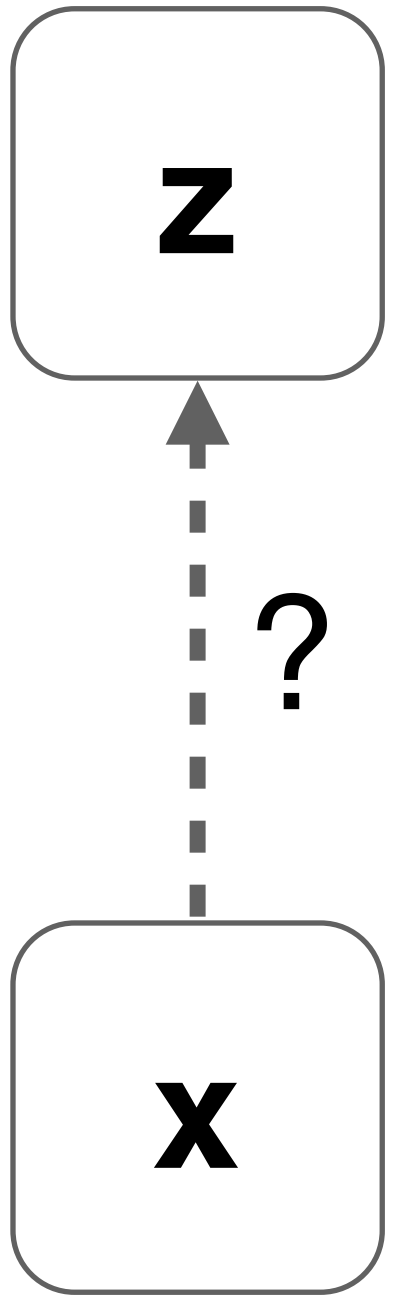
But what about inference...

- Can we incorporate an inference mechanism into GANs?

Latent variables

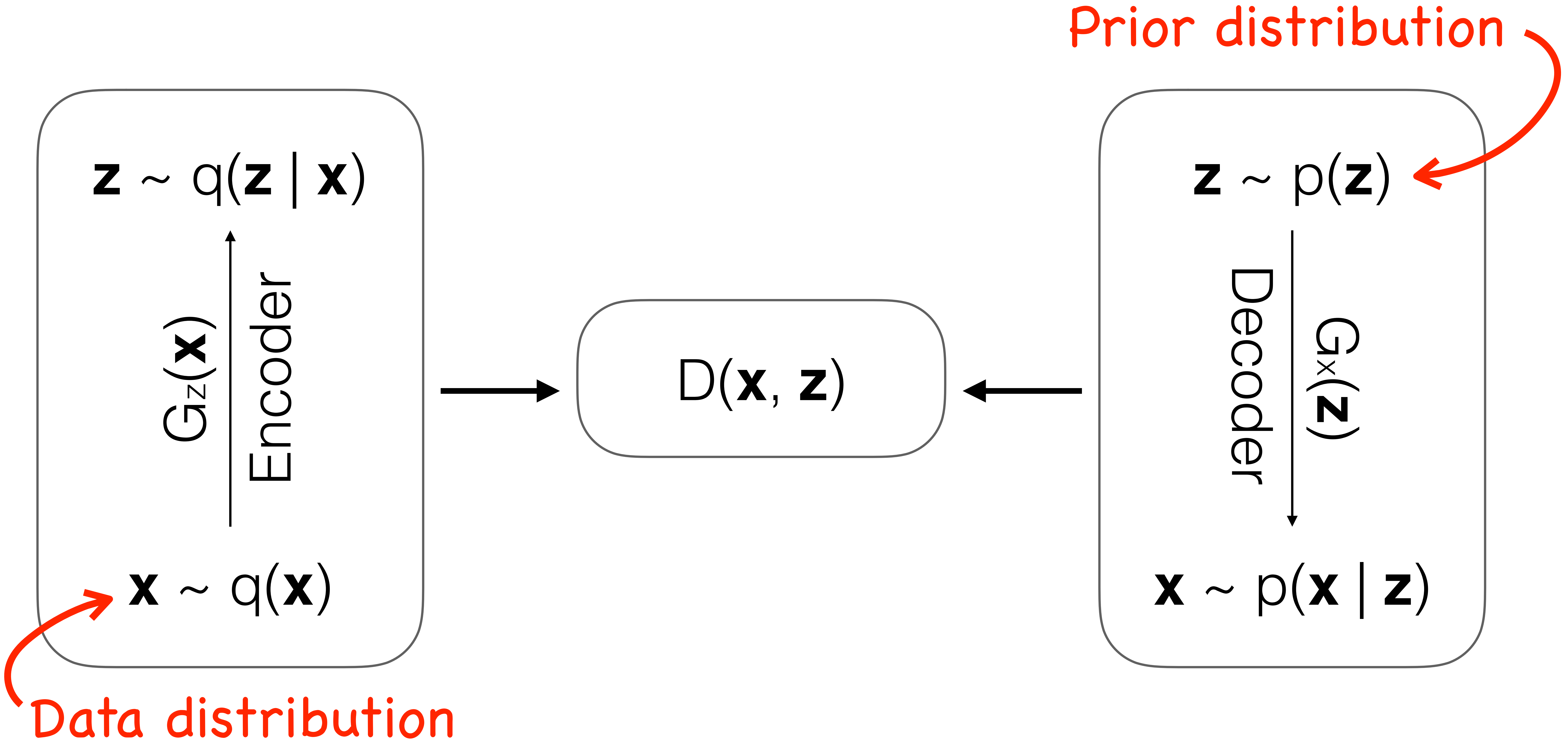






Observed variables

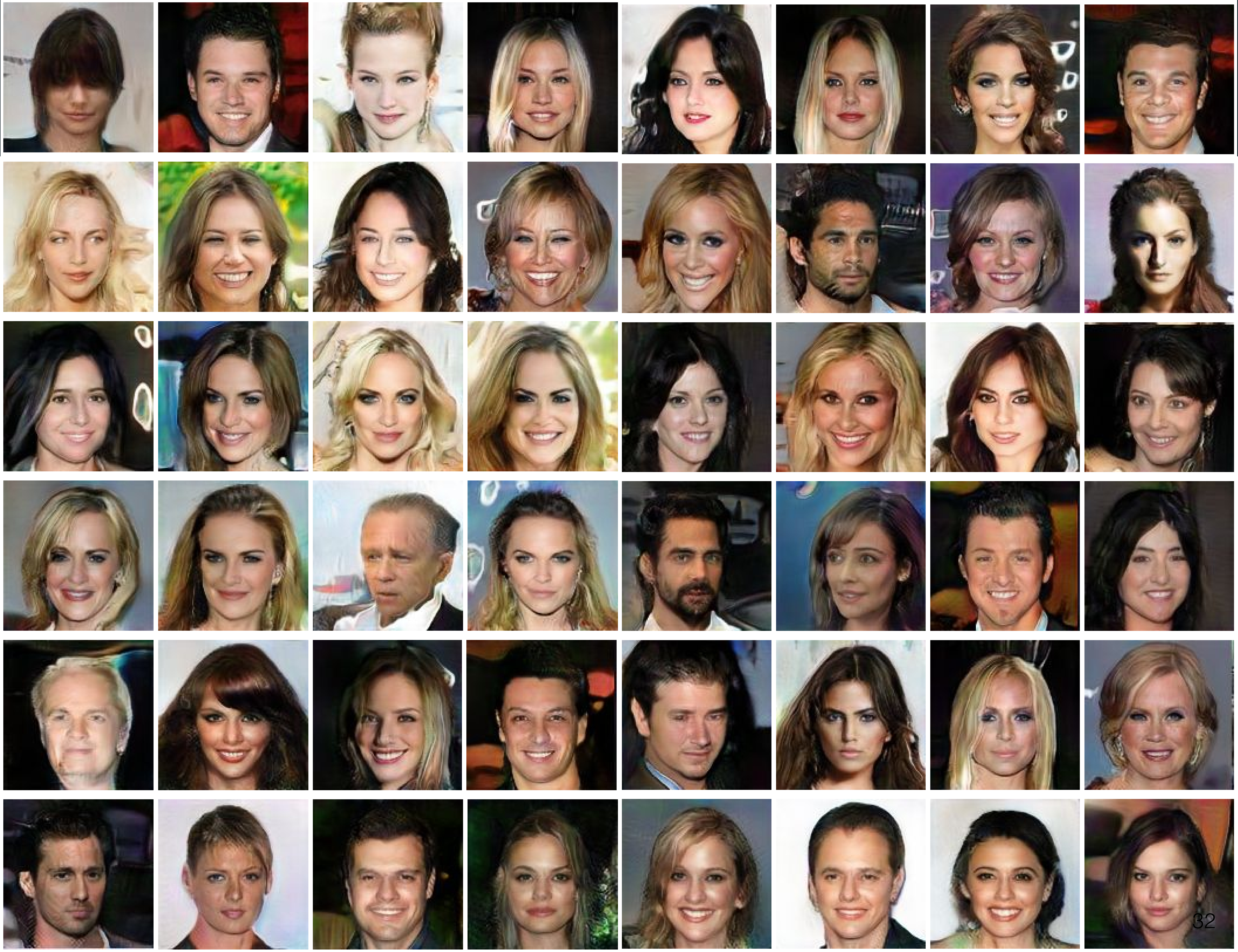
ALI / BiGAN: model diagram



- **ALI**: Vincent Dumoulin, Ishmael Belghazi, Olivier Mastropietro, Ben Poole, Alex Lamb, Martin Arjovsky (2016) *ADVERSARIALLY LEARNED INFERENCE*, arXiv:1606.00704, ICLR 2017
- **BiGAN**: Donahue, Krähenbühl and Darrell (2016), *ADVERSARIAL FEATURE LEARNING*, arXiv:1605.09782, ICLR 2017

Hierarchical ALI

CelebA-128X128



Model samples

Hierarchical ALI: CelebA-128X128

Reconstructions given z_1, z_2

Reconstructions given z_2

cycleGAN: Adversarial training of domain transformations

(Zhu et al. ICCV 2017)

- CycleGAN learns transformations across domains with unpaired data.
- Combines GAN loss with "cycle-consistency loss": L1 reconstruction.

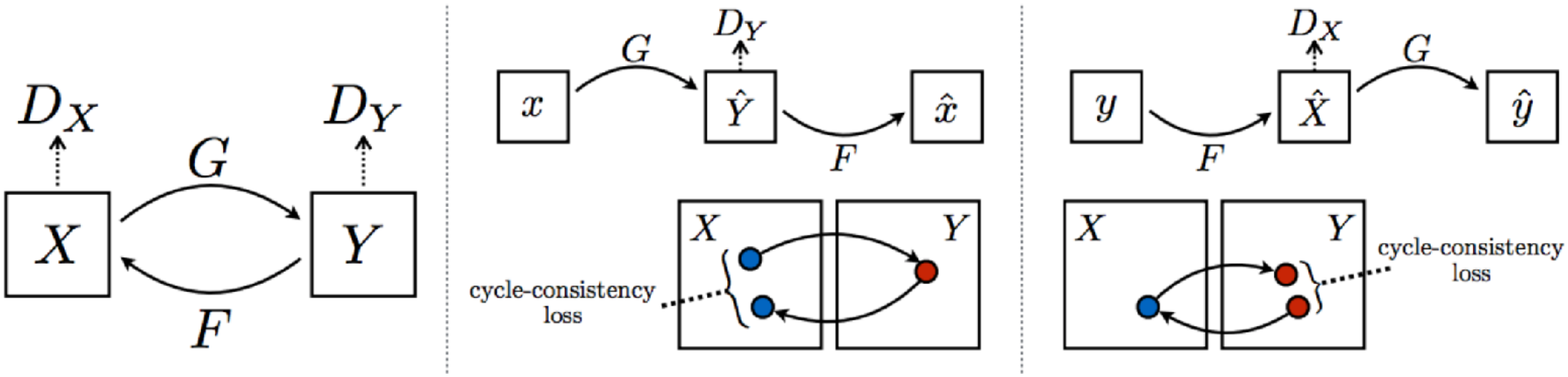


Image credits: Jun-Yan Zhu*, Taesung Park*, Phillip Isola, and Alexei A. Efros. "Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks", in IEEE International Conference on Computer Vision (ICCV), 2017.

CycleGAN for unpaired data

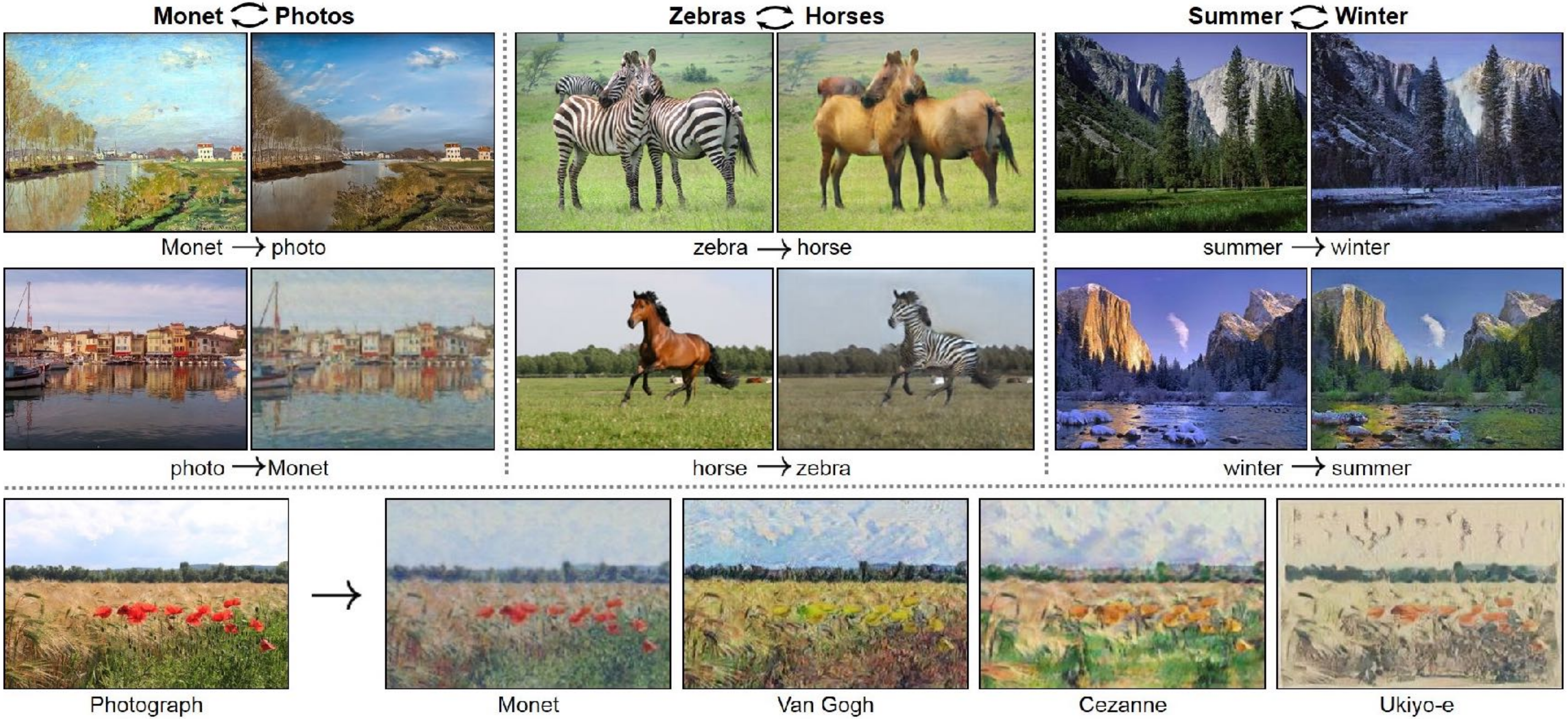


Image credits: Jun-Yan Zhu*, Taesung Park*, Phillip Isola, and Alexei A. Efros. "Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks", in IEEE International Conference on Computer Vision (ICCV), 2017.

PROGRESSIVE GROWING OF GANS FOR IMPROVED QUALITY, STABILITY, AND VARIATION (Kerras et al. from NVIDIA, 2017)

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- Recent work from NVIDIA.
- Improves image quality by growing the model size throughout training.
- Samples from a model trained on the CelebA face dataset.

1024x1024 model samples

PROGRESSIVE GROWING OF GANS FOR IMPROVED QUALITY, STABILITY, AND VARIATION (Kerras et al. from NVIDIA, 2017)

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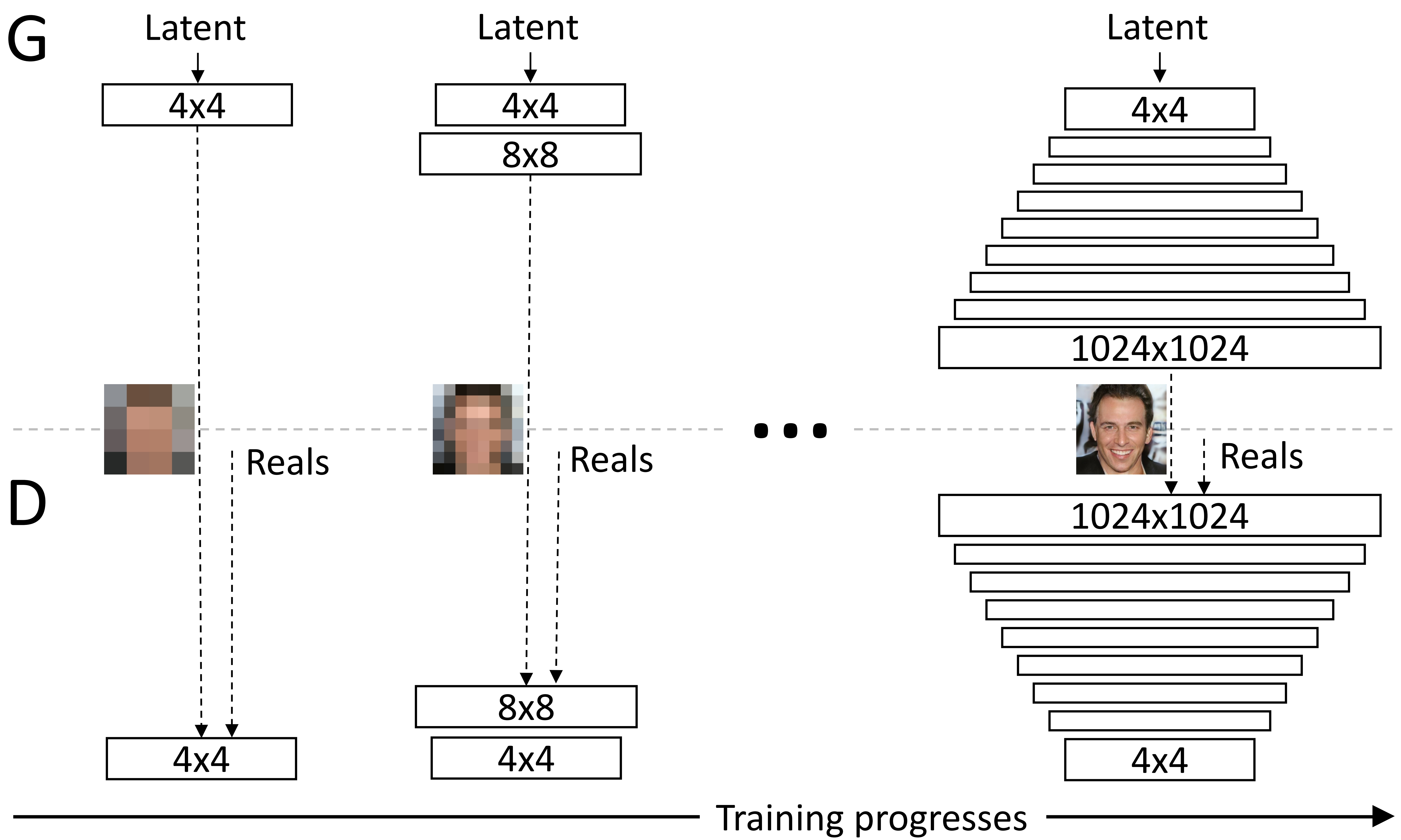

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MILA

- Recent work from NVIDIA.
- Improves image quality by growing the model size throughout training.
- Conditional samples from a model trained on the LSUN dataset

Model samples