Data Mining for Business Analytics

Lecture 10: Similarity and Nearest Neighbors

Stern School of Business
New York University
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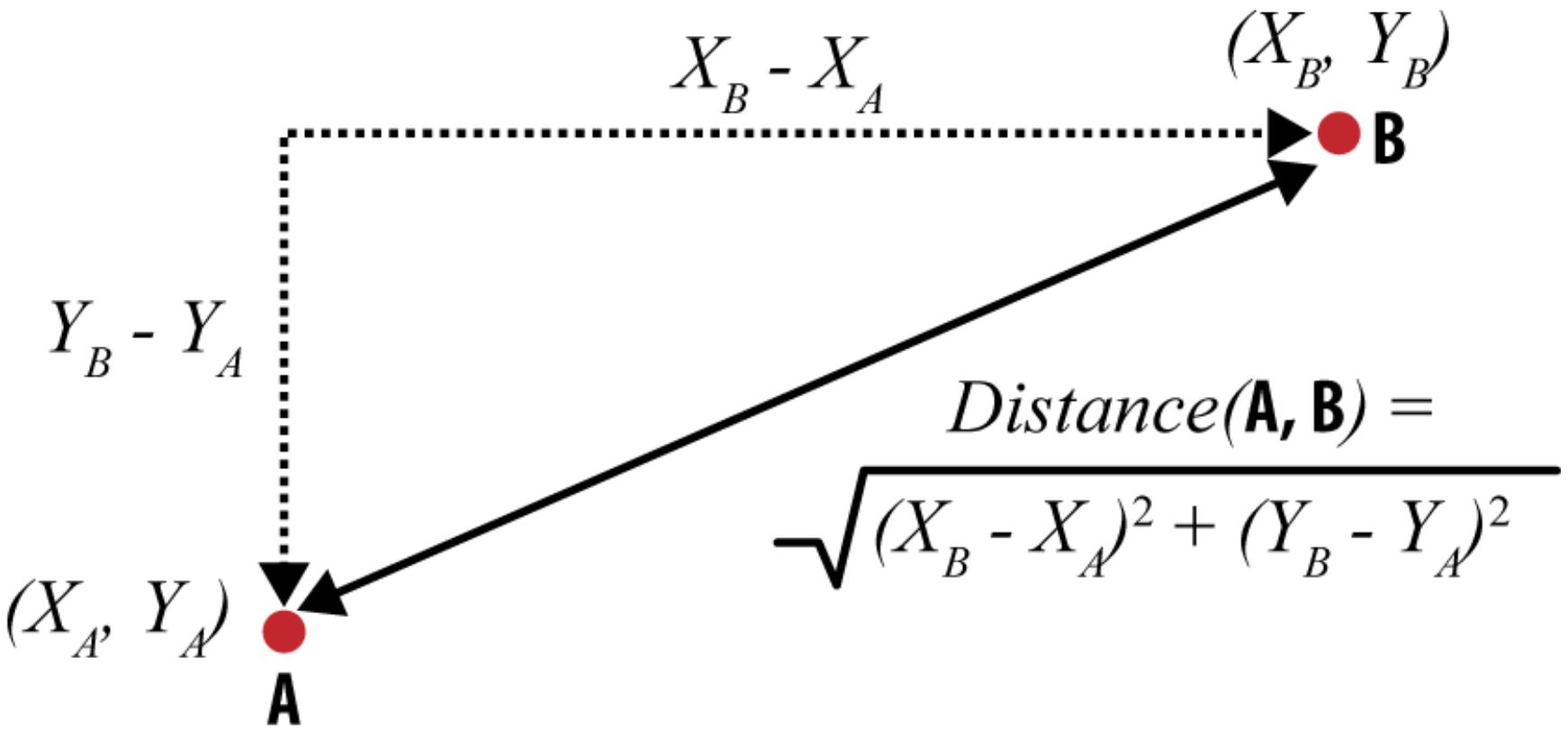
Similarity and Distance

- If two objects can be represented as feature vectors, then we can compute the distance between them

Attribute	Person A	Person B
Age	23	40
Years at current address	2	10
Residential status (1=Owner, 2=Renter, 3=Other)	2	1

Euclidean Distance



Euclidean Distance

$$\sqrt{\left(d_{1,A} - d_{1,B}\right)^2 + \left(d_{2,A} - d_{2,B}\right)^2 + \cdots + \left(d_{n,A} - d_{n,B}\right)^2}$$

$$d(A,B) = \sqrt{(23-40)^2 + (2-10)^2 + (2-1)^2} = 18.8$$

Other Distance Functions

$$d_{Manhattan}(\boldsymbol{X}, \boldsymbol{Y}) = \|\boldsymbol{X} - \boldsymbol{Y}\|_1 = |x_1 - y_1| + |x_2 - y_2| + \cdots$$

(L1-norm, taxicab-distance)

$$d_{Jaccard}(X, Y) = 1 - \frac{|X \cap Y|}{|X \cup Y|}$$

$$d_{Cosine}(\boldsymbol{X}, \boldsymbol{Y}) = 1 - \frac{\boldsymbol{X} \cdot \boldsymbol{Y}}{\|\boldsymbol{X}\|_2 \cdot \|\boldsymbol{Y}\|_2}$$

where $||\cdot||_2$ represents the L2 norm, or Euclidean length, of each feature vector (for a vector this is simply the distance from the origin).

Example: "Whiskey Analytics"

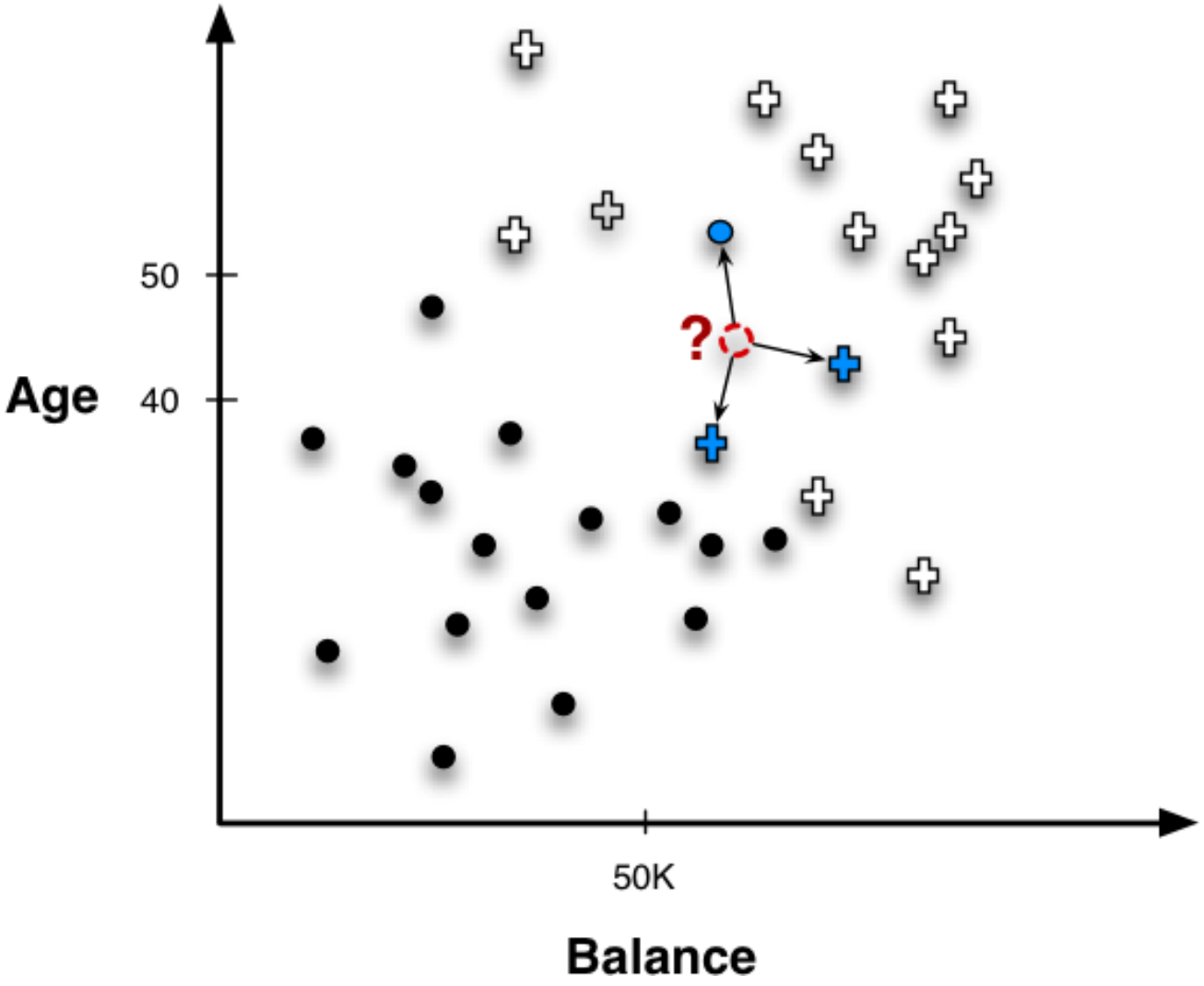
1.	**Color:** *yellow, very pale, pale, pale gold, gold, old gold, full gold, amber,* etc.	(14 values)
2.	**Nose:** *aromatic, peaty, sweet, light, fresh, dry, grassy,* etc.	(12 values)
3.	**Body:** *soft, medium, full, round, smooth, light, firm, oily.*	(8 values)
4.	**Palate:** *full, dry, sherry, big, fruity, grassy, smoky, salty,* etc.	(15 values)
5.	**Finish:** *full, dry, warm, light, smooth, clean, fruity, grassy, smoky,* etc.	(19 values)

Consequently there are 68 binary features of each whiskey.

Whiskey	Distance	Descriptors
Bunnahabhain	—	*gold; firm,med,light; sweet,fruit,clean; fresh,sea; full*
Glenglassaugh	0.643	gold; firm,light,smooth; sweet,grass; fresh,grass
Tullibardine	0.647	gold; firm,med,smooth; sweet,fruit,full,grass,clean; sweet; big,arome,sweet
Ardbeg	0.667	sherry; firm,med,full,light; sweet; dry,peat,sea;salt
Bruichladdich	0.667	pale; firm,light,smooth; dry,sweet,smoke,clean; light; full
Glenmorangie	0.667	p.gold; med,oily,light; sweet,grass,spice; sweet,spicy,grass,sea,fresh; full,long

Nearest Neighbors

Nearest Neighbors for Predictive Modeling



Nearest Neighbors for Predictive Modeling

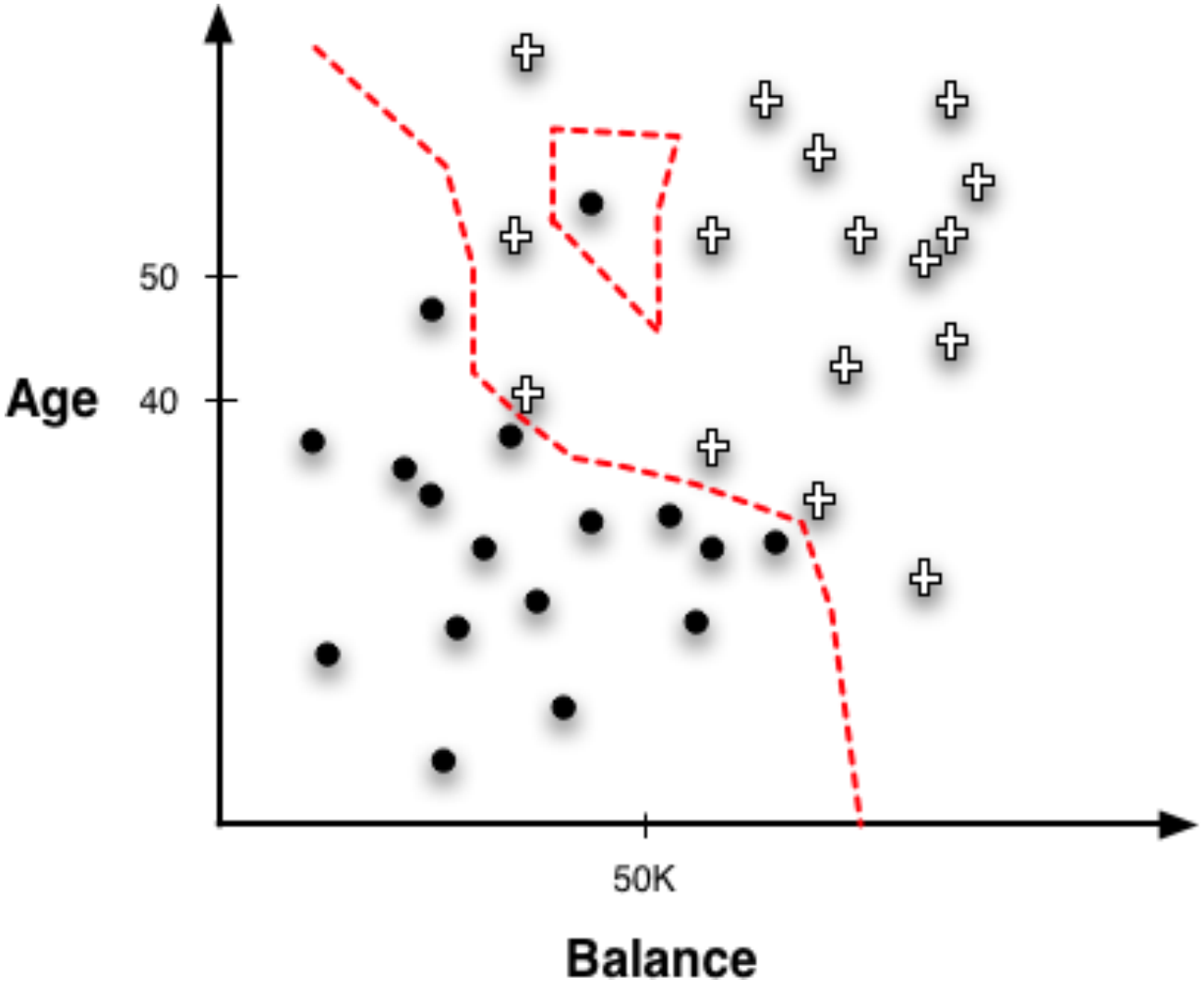
Customer	Age	Income (1000s)	Cards	Response (target)	Distance from David
David	*37*	*50*	*2*	*?*	0
John	35	35	3	Yes	$\sqrt{(35-37)^2+(35-50)^2+(3-2)^2} = 15.16$
Rachael	22	50	2	No	$\sqrt{(22-37)^2+(50-50)^2+(2-2)^2} = 15$
Ruth	63	200	1	No	$\sqrt{(63-37)^2+(200-50)^2+(1-2)^2} = 152.23$
Jefferson	59	170	1	No	$\sqrt{(59-37)^2+(170-50)^2+(1-2)^2} = 122$
Norah	25	40	4	Yes	$\sqrt{(25-37)^2+(40-50)^2+(4-2)^2} = 15.74$

How Many Neighbors and How Much Influence?

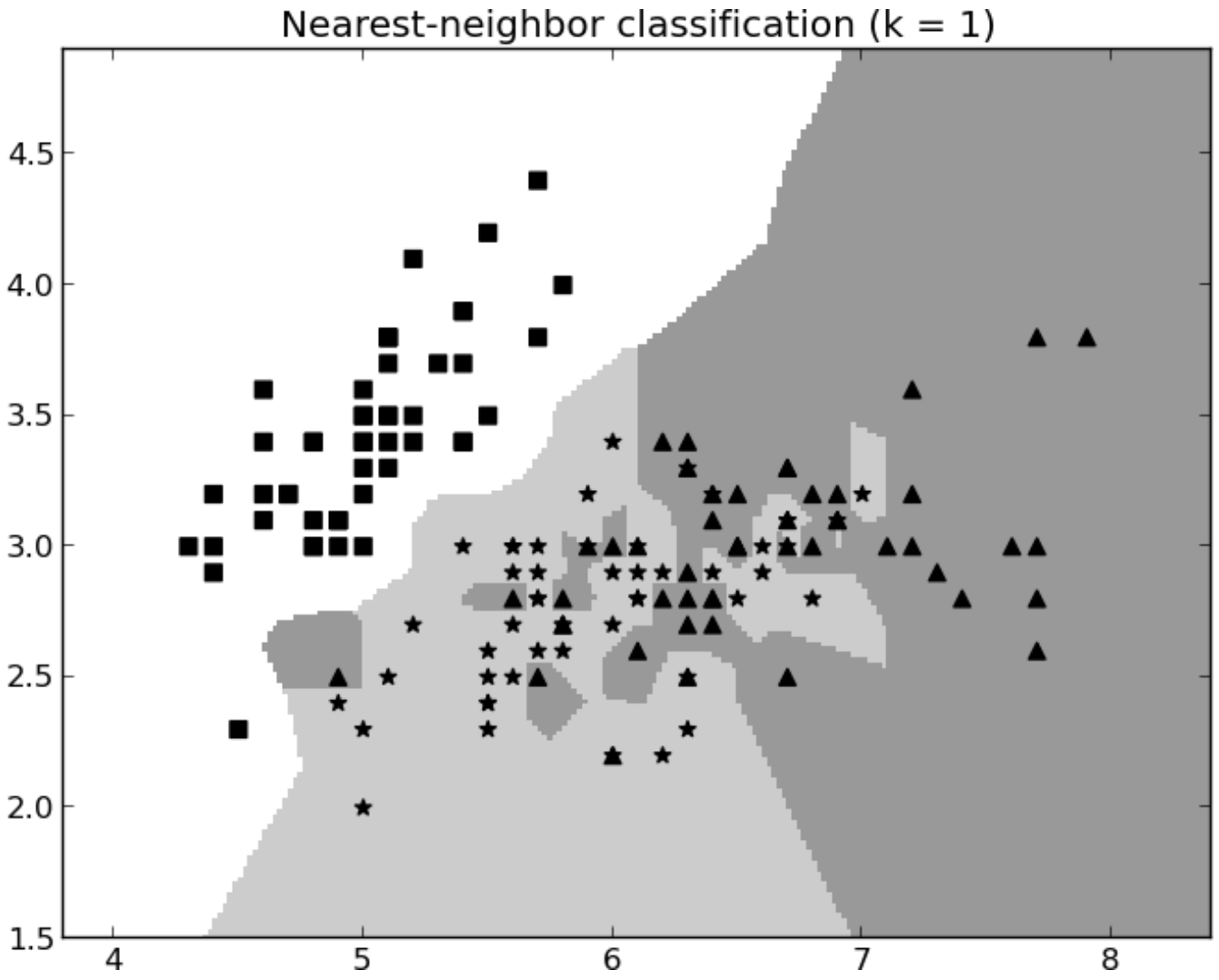
k Nearest Neighbors

- $k = ?$
- $k = 1\,?$
- $k = n\,?$

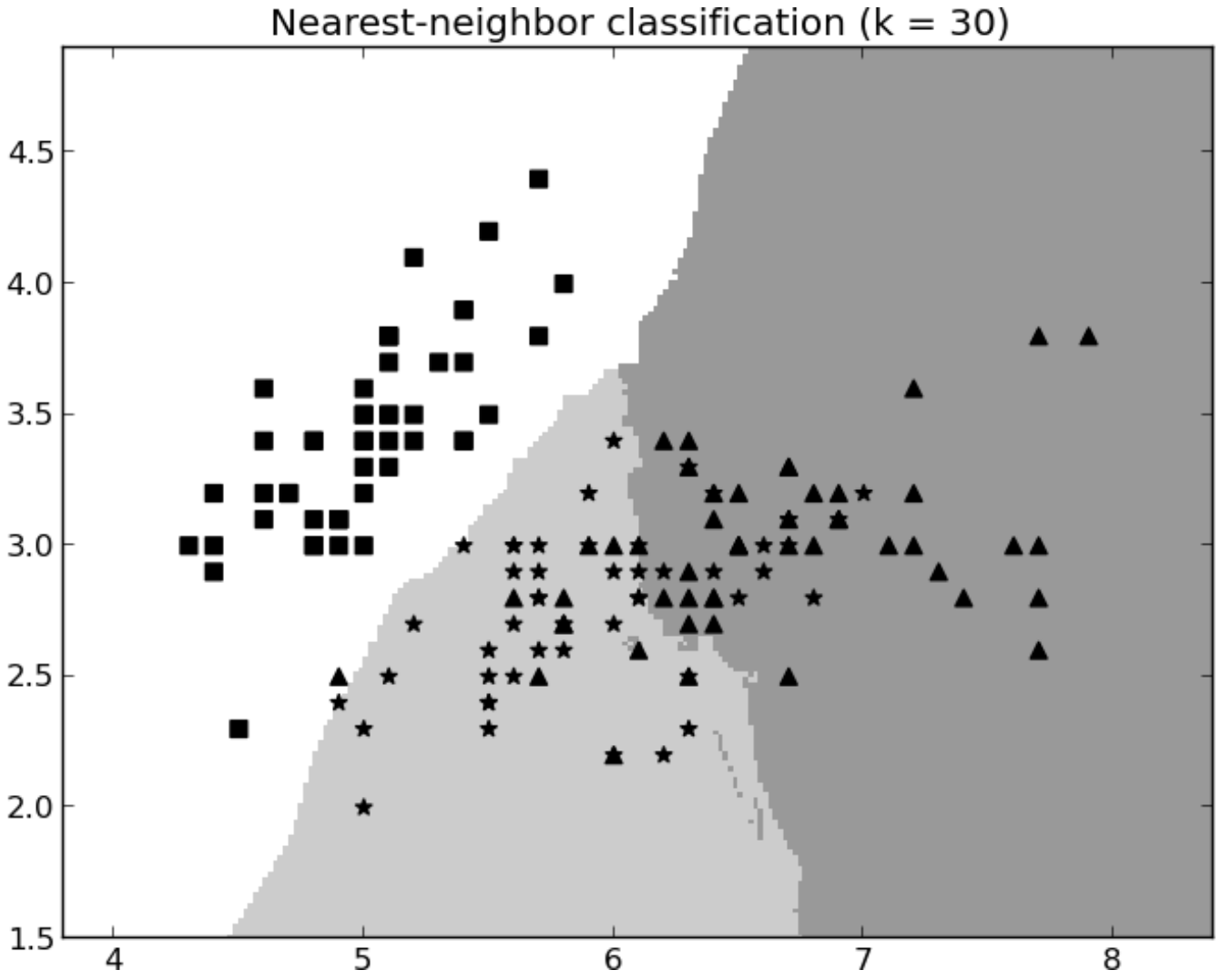
Geometric Interpretation, Over-fitting, and Complexity



1-Nearest Neighbor



30-Nearest Neighbors



Issues with Nearest-Neighbor Models

- Dimensionality and domain knowledge
  - There might be too many features (and some are irrelevant)
  - The distance function need to consider the scale and importance of the features.
- Computational efficiency
  - Not suitable for online advertisement, whose decisions have to be made in a few tens of milliseconds.