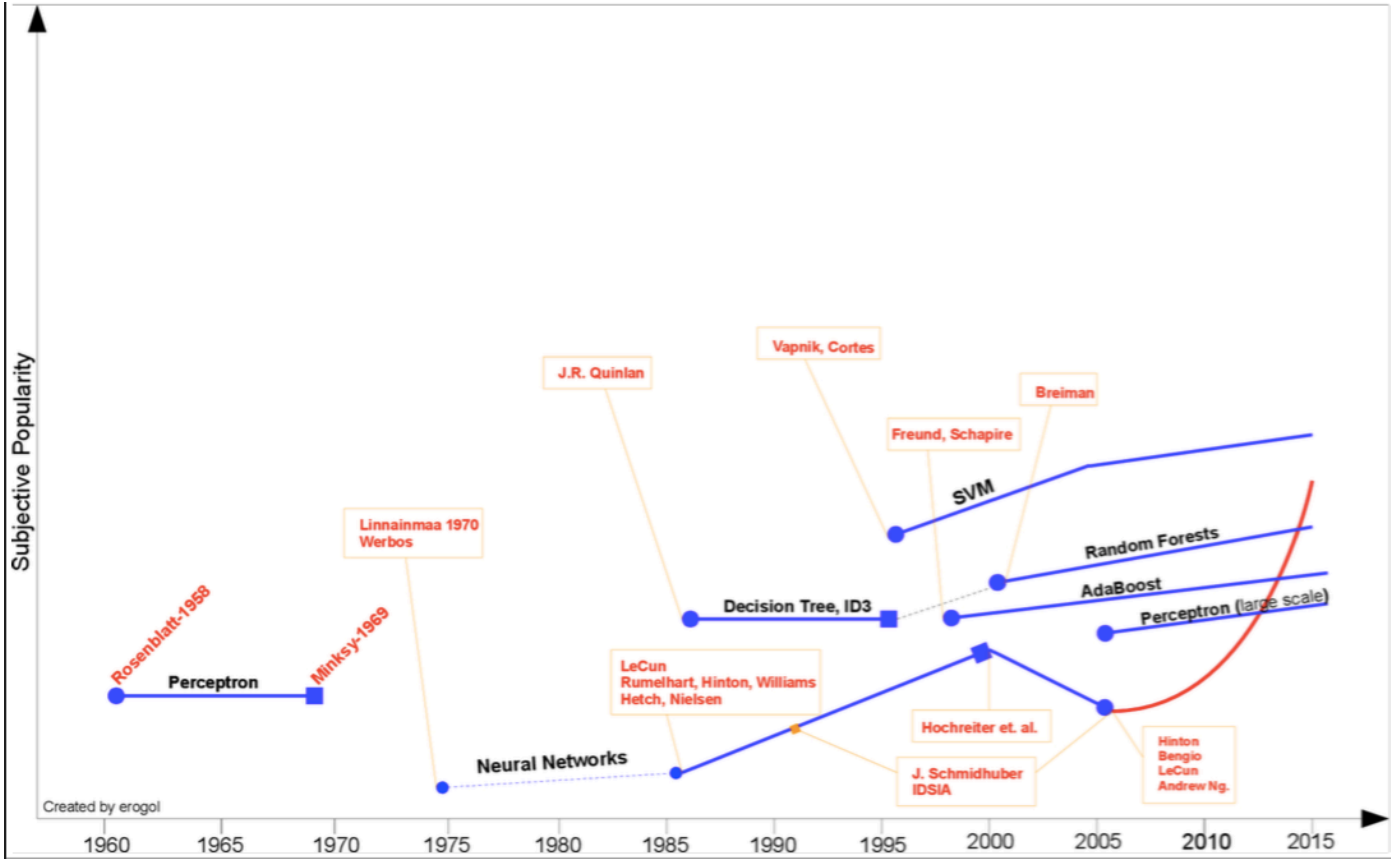
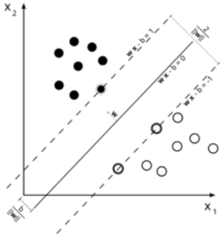

Subjective Popularity
Rosenblatt-1958
Perceptron
Minksy-1969
Linnainmaa 1970
Werbos
Neural Networks
J.R. Quinlan
Decision Tree, ID3
LeCun
Rumelhart, Hinton, Williams
Hetch, Nielsen
Vapnik, Cortes
SVM
Freund, Schapire
Breiman
Random Forests
AdaBoost
Perceptron (large scale)
Hochreiter et. al.
J. Schmidhuber
IDSIA
Hinton
Bengio
LeCun
Andrew Ng.
Created by erogol
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015



- The present form of support vector machine (SVM) was largely developed at AT&T Bell Laboratories by Vapnik and co-workers.
- Known as a maximum margin classifier.
- Originally proposed for classification and soon applied to regression and time series prediction.
- One of the most efficient supervised learning methods.

Problem

- Given a set of training samples

$$(x_1, y_1), (x_2, y_2), \cdots, (x_N, y_N), x_i \in R^n, y_i \in \{-1, 1\},$$

find a function $f(x, \alpha)$ to classify the samples, such that

$$f(x_i, \alpha) \begin{cases} > 0, & \forall y_i = +1; \\ < 0, & \forall y_i = -1, \end{cases}$$

where α denotes the parameters.

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- For a testing sample x, we can predict its label by $\text{sign}[f(x, \alpha)]$.

Problem

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where α denotes the parameters.

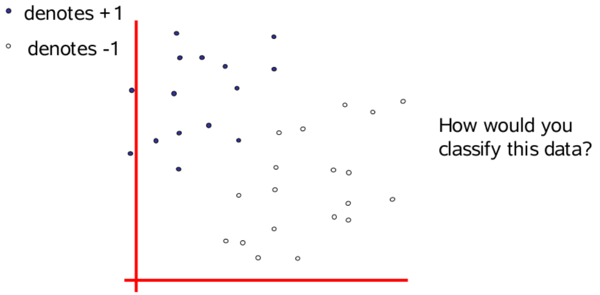
- For a testing sample x, we can predict its label by $\text{sign}[f(x, \alpha)]$.
- $f(x, \alpha) = 0$ is called the separation hyperplane.

Linear classifiers

Linear hyperplane

$$f(x, w, b) = \langle x, w \rangle + b = 0$$

Consider the linearly separable case, there are infinite number of hyperplanes that can do the job.



Linear classifiers

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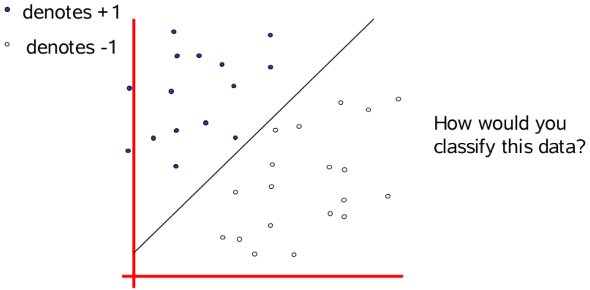

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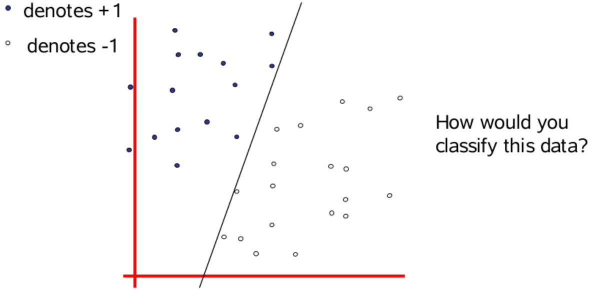

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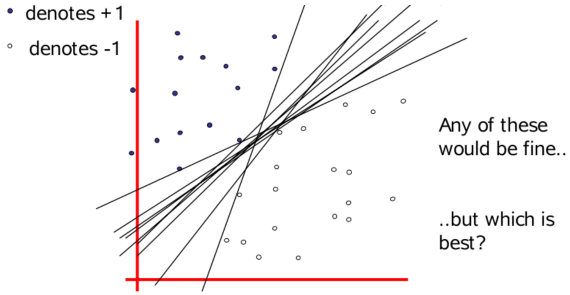
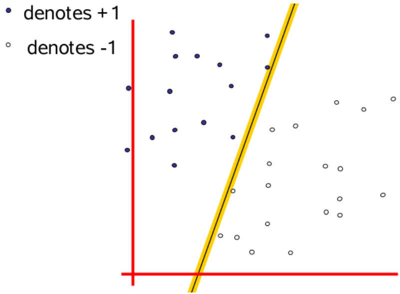
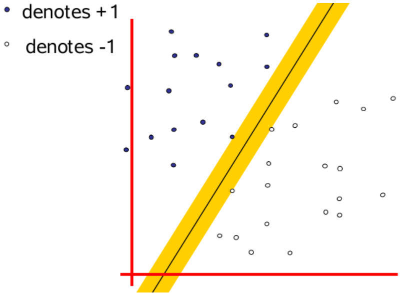


Margin of a linear classifier



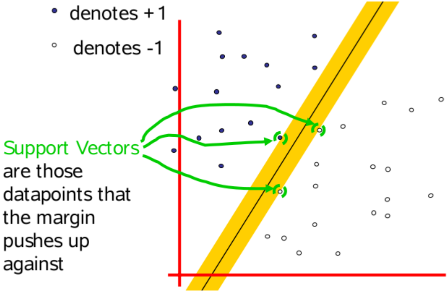
Definition: the width that the boundary could be increased by before hitting a data point.

Maximum margin linear classifier



Definition: the linear classifier with the maximum margin.

Support vectors



Problem formulation

To formulate the margin, we further requires that for all samples

$$f(x_i, \alpha) = \langle x_i, w \rangle + b \begin{cases} \geq +1, & \forall y_i = +1; \\ \leq -1, & \forall y_i = -1. \end{cases}$$

or

$$y_i(\langle x_i, w \rangle + b) \geq 1, \quad i = 1, \ldots, N.$$

Problem formulation

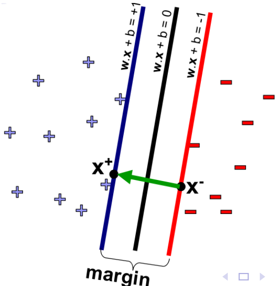
To formulate the margin, we further requires that for all samples

$$f(x_i, \alpha) = \langle x_i, w \rangle + b \begin{cases} \geq +1, & \forall y_i = +1; \\ \leq -1, & \forall y_i = -1. \end{cases}$$

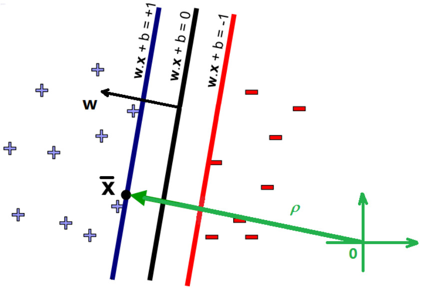
or

$$y_i(\langle x_i, w \rangle + b) \geq 1, \quad i = 1, \ldots, N.$$

- We have introduced two additional hyperplanes $\langle x, w \rangle + b = \pm 1$ parallel to the separation hyperplane $\langle x, w \rangle + b = 0$



What is the margin? The distance between the two new hyperplanes.



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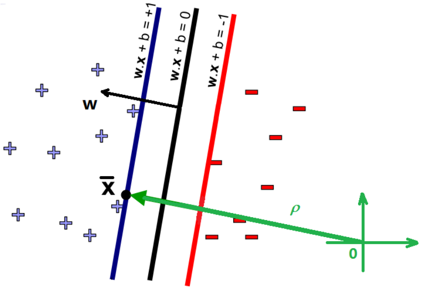
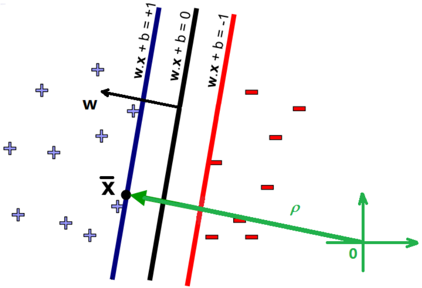


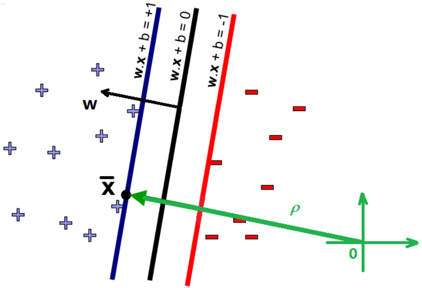
- The minimum distance between the hyperplane $\langle x, w \rangle + b = 1$ and the origin is $\rho_1 = \frac{1-b}{\|w\|}$. (why?)

What is the margin? The distance between the two new hyperplanes.



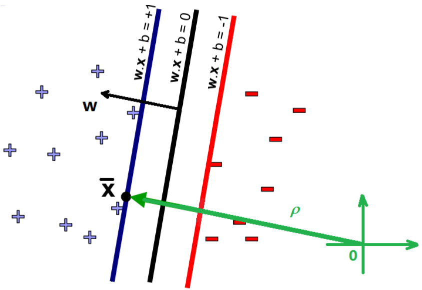
- The minimum distance between the hyperplane $\langle x, w \rangle + b = 1$ and the origin is $\rho_1 = \frac{1-b}{\|w\|}$. (why?)
- The minimum distance between the hyperplane $\langle x, w \rangle + b = -1$ and the origin is $\rho_2 = \frac{-1-b}{\|w\|}$.

What is the margin? The distance between the two new hyperplanes.



- The minimum distance between the hyperplane $\langle x, w \rangle + b = 1$ and the origin is $\rho_1 = \frac{1-b}{\|w\|}$. (why?)
- The minimum distance between the hyperplane $\langle x, w \rangle + b = -1$ and the origin is $\rho_2 = \frac{-1-b}{\|w\|}$.
- The margin is $|\rho_1 - \rho_2| = 2/\|w\|$.

How to calculate ρ_1 and ρ_2?



Note $\bar{x} = \rho_1 w/\|w\|$, where $w/\|w\|$ is the unit vector along the direction w. Since $\bar{x}$ is on the blue hyperplane, then

$$\langle \rho_1 w/\|w\|, w\rangle + b = 1$$

which follows $\rho_1 = \frac{1-b}{\|w\|}$. Similarly, we obtain $\rho_2 = \frac{-1-b}{\|w\|}$.

Maximizing the margin is the same thing as minimizing the norm of $\mathbf{w}$

Our goal is to maximize the margin. Among all possible hyperplanes meeting the constraints, we will choose the hyperplane with the smallest $\|\mathbf{w}\|$ because it is the one which will have the biggest margin.

This give us the following optimization problem:

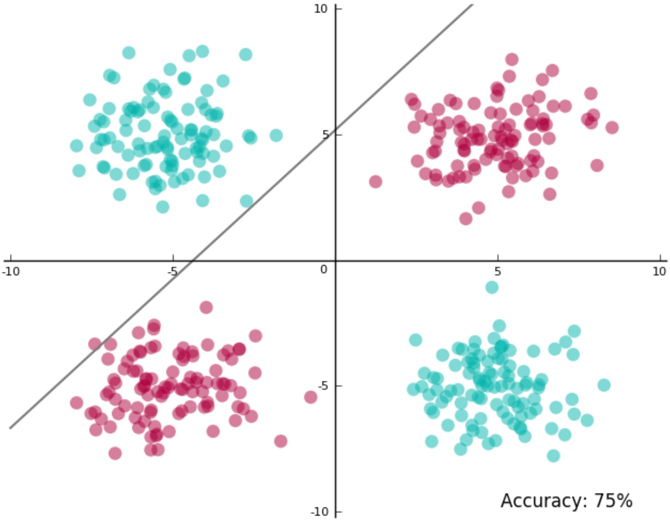
Minimize in $(\mathbf{w}, b)$

$$\|\mathbf{w}\|$$

subject to $y_i(\mathbf{w} \cdot \mathbf{x_i} + b) \geq 1$

(for any $i = 1, \dots, n$)

Solving this problem is like solving and equation. Once we have solved it, we will have found the couple $(\mathbf{w}, b)$ for which $\|\mathbf{w}\|$ is the smallest possible and the constraints we fixed are met. Which means we will have the equation of the optimal hyperplane !


10
5
-10
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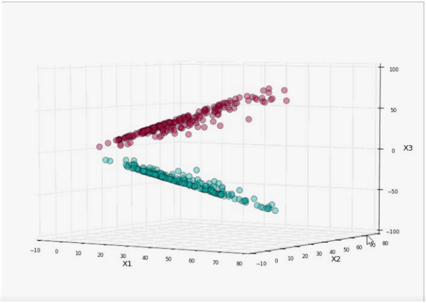
-5
-10
Accuracy: 75%

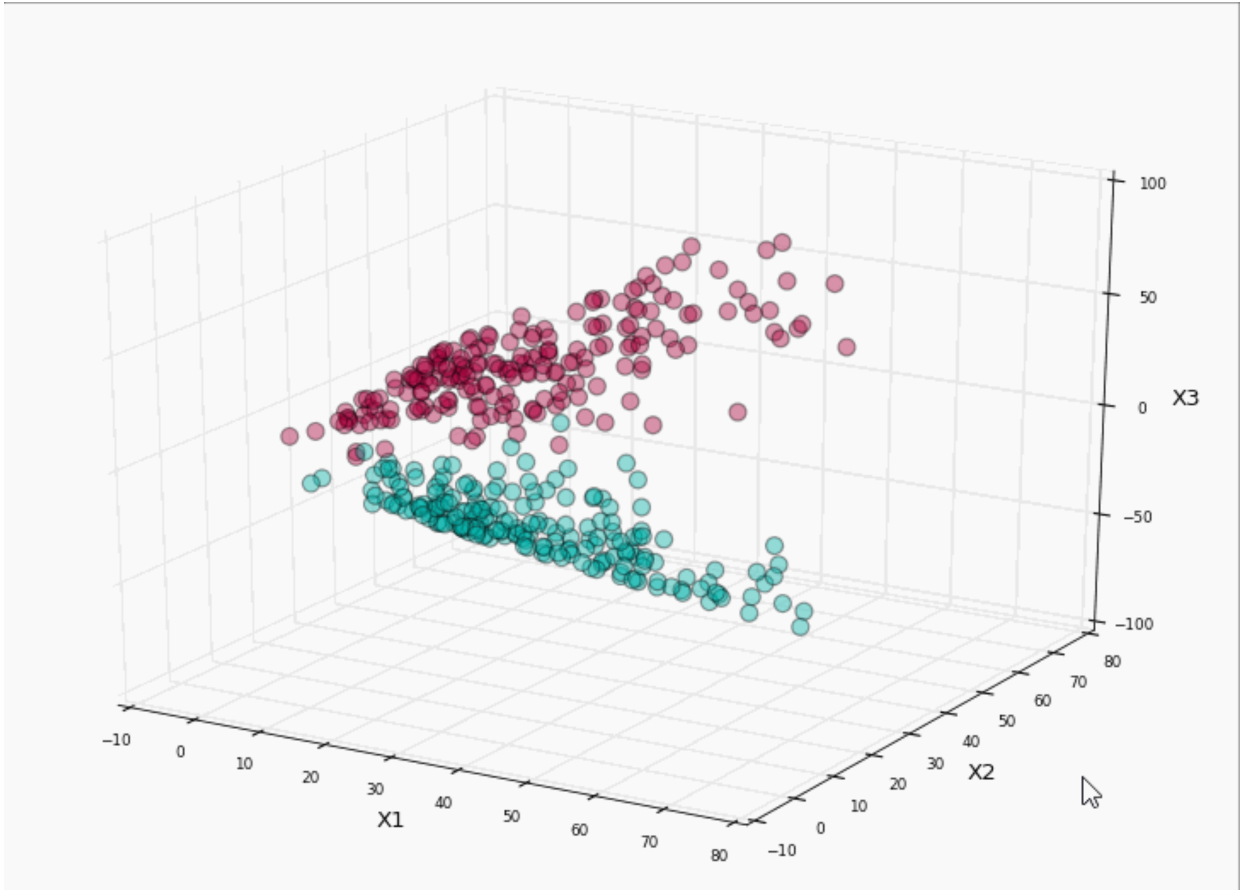
The kernel-based function is exactly equivalent to preprocessing the data by applying similarity function to all inputs, then learning a linear model in the new transformed space.

We start with the dataset in the above figure, and project it into a three-dimensional space where the new coordinates are:

$$X_1 = x_1^2$$
$$X_2 = x_2^2$$
$$X_3 = \sqrt{2}x_1x_2$$

This is what the projected data looks like. Do you see a place where we just might be able to slip in a plane?




X1
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80
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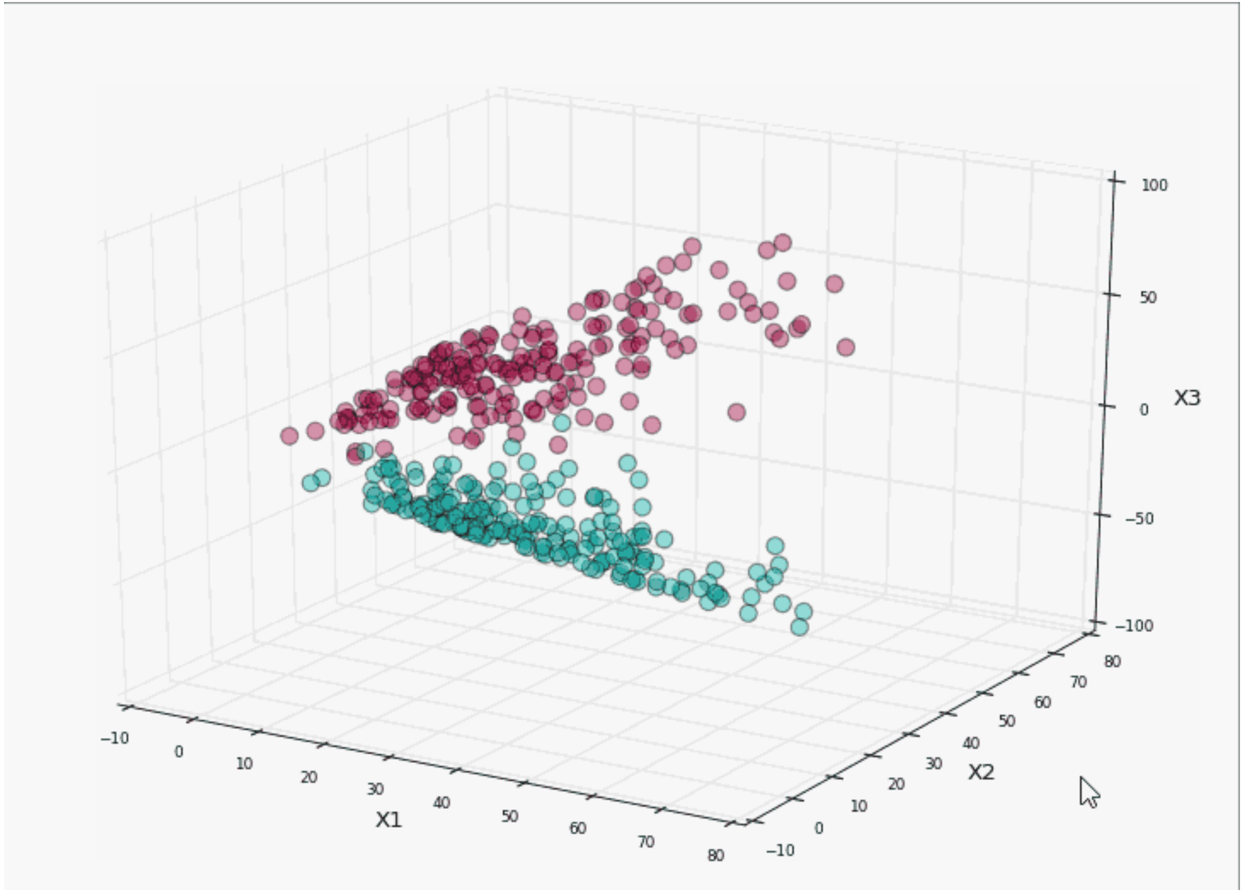
−50
0
50
100



X1
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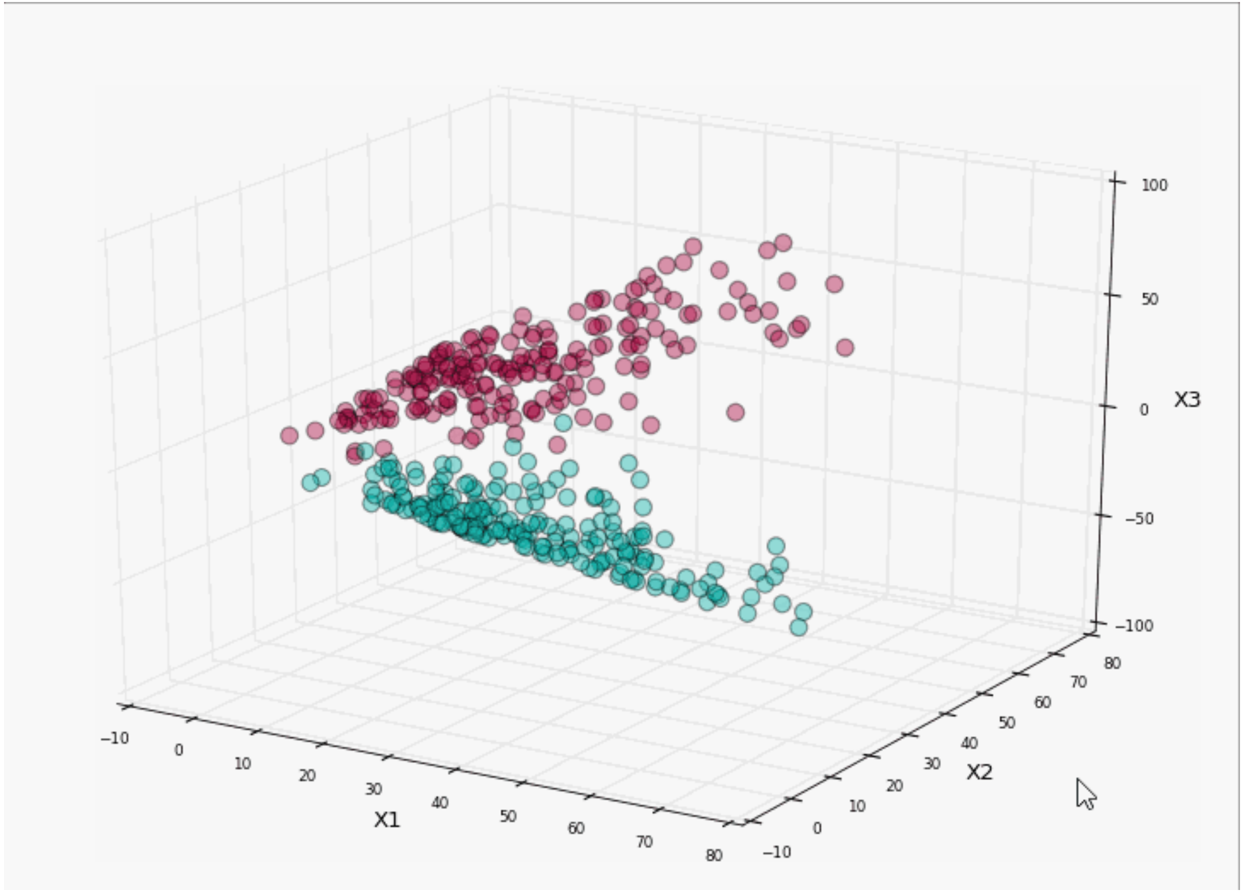

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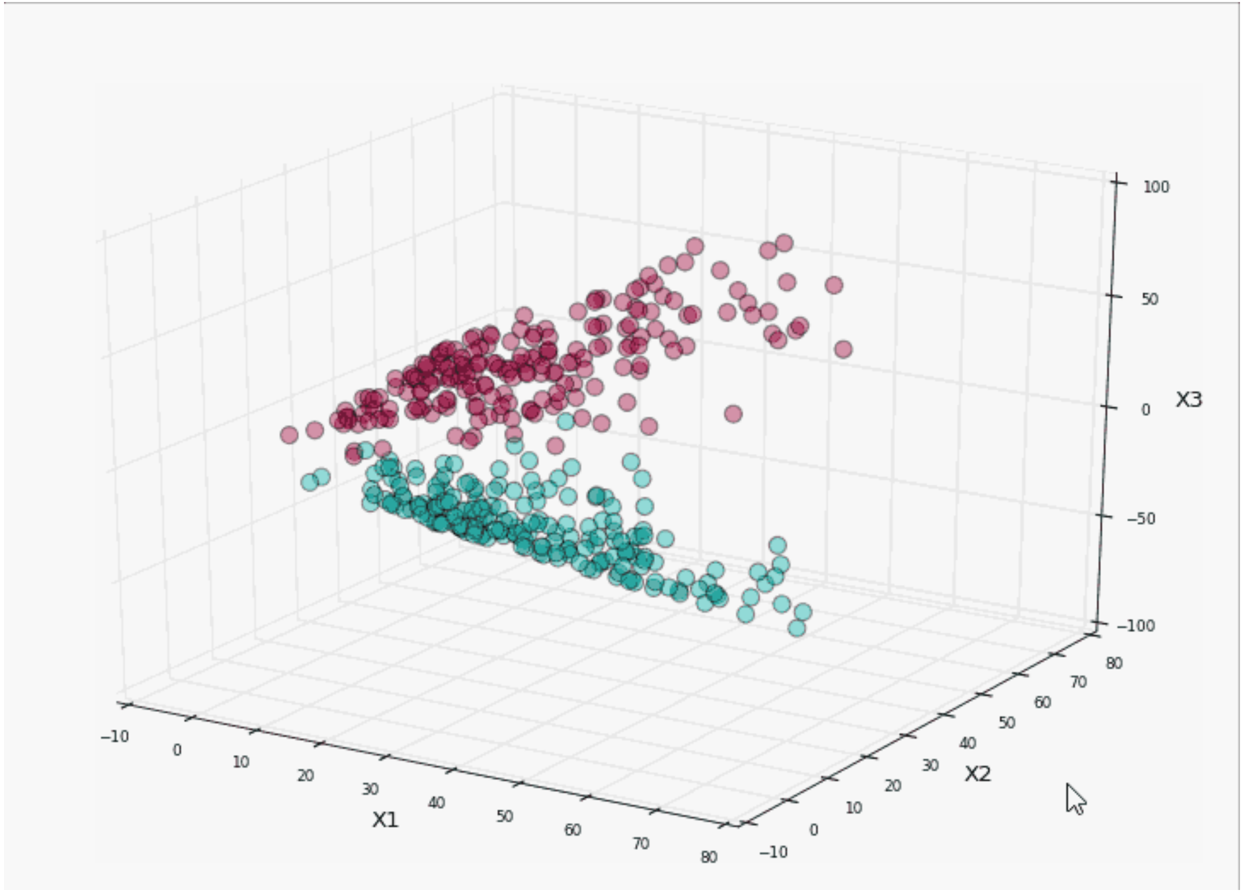

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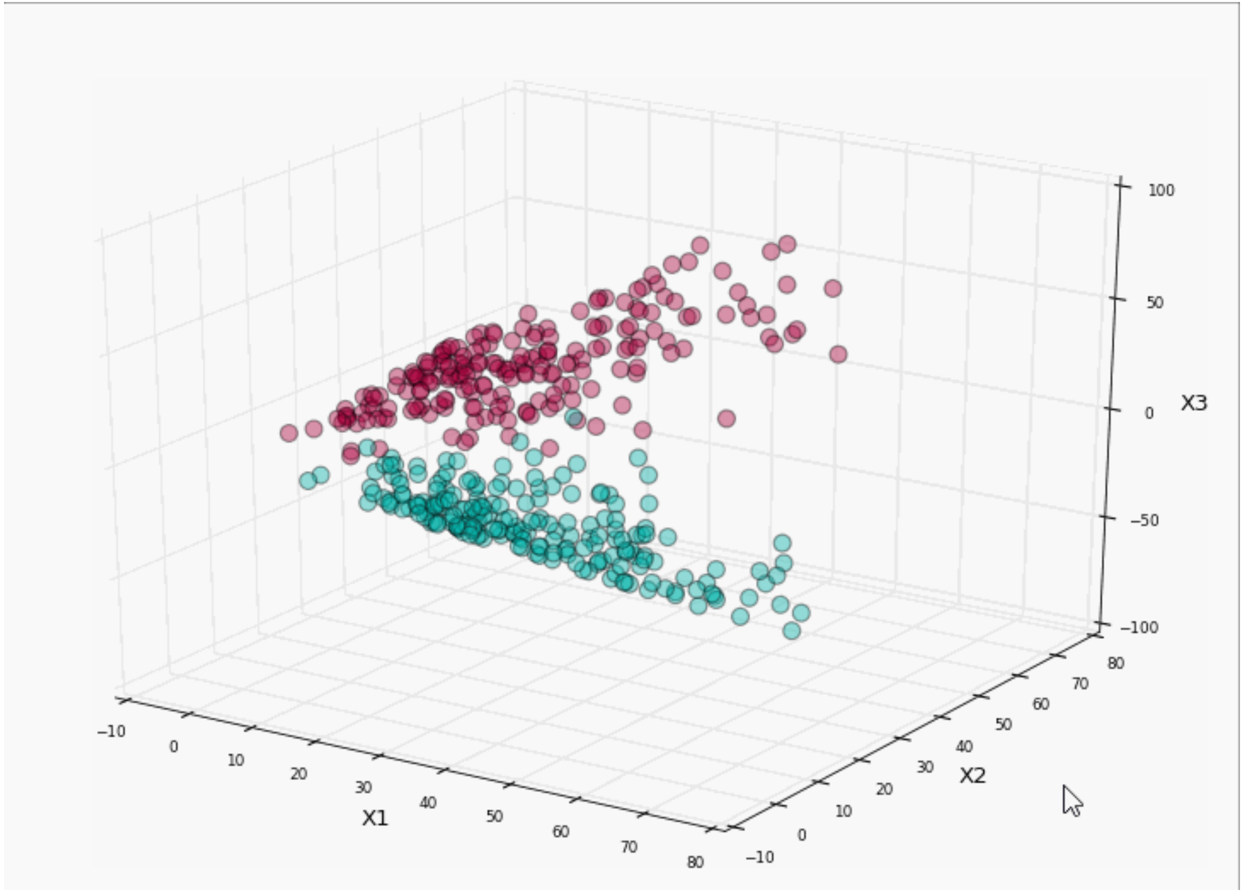

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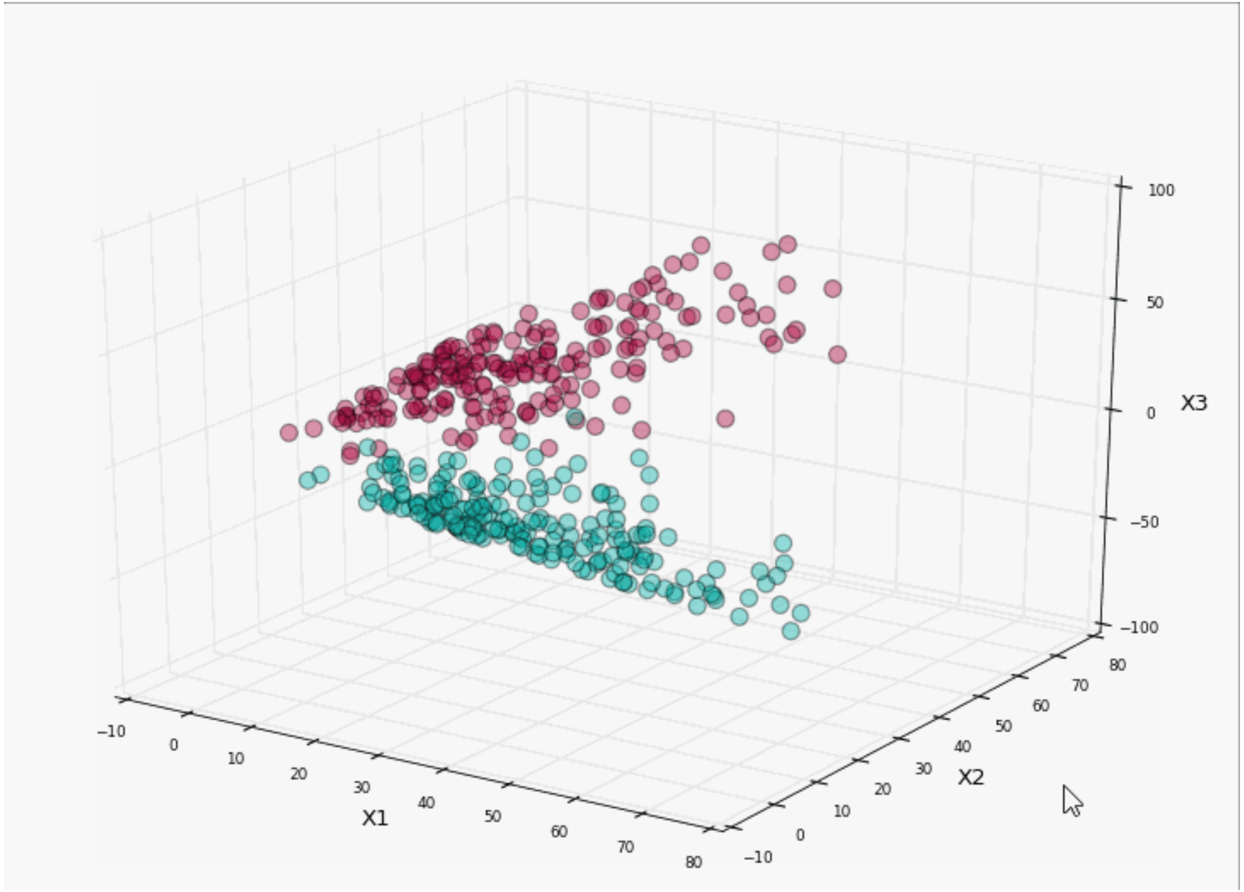

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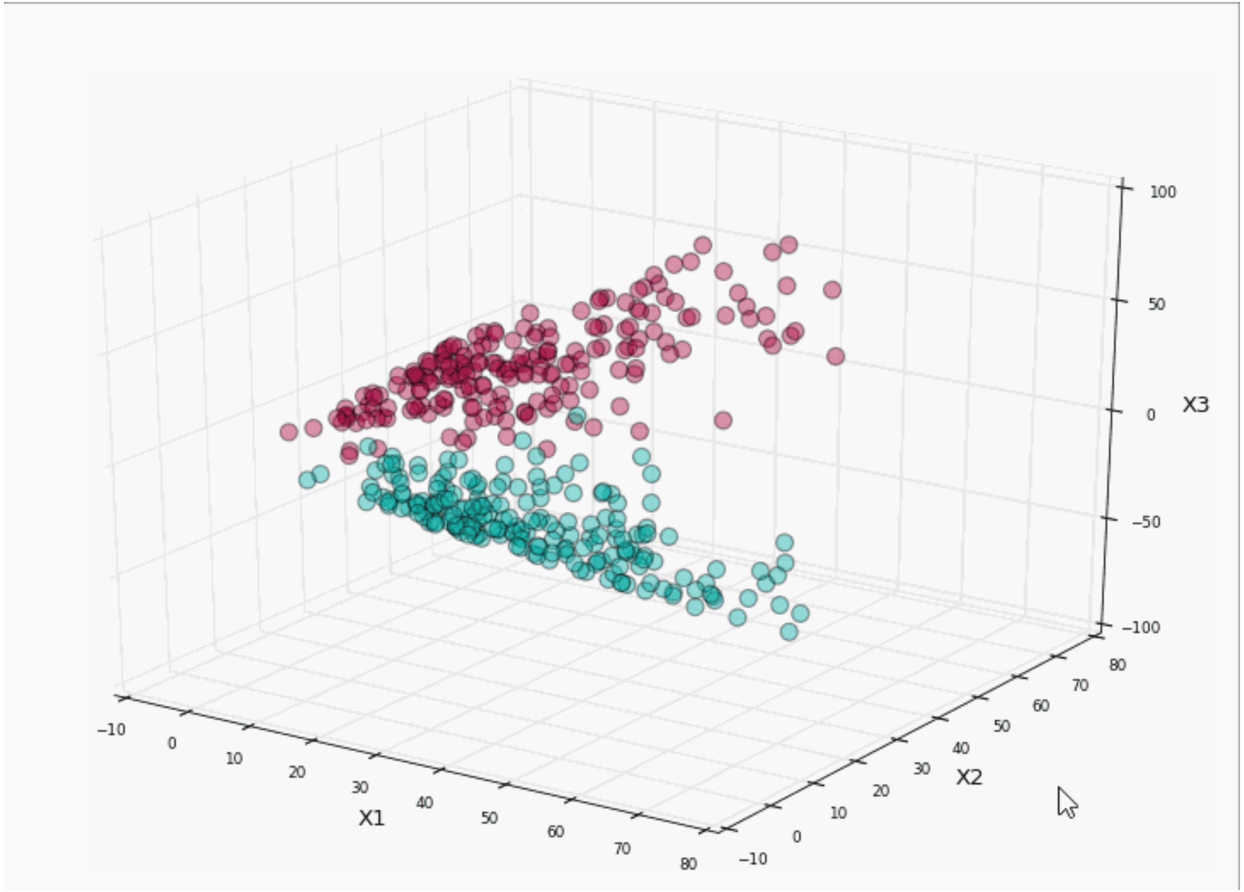

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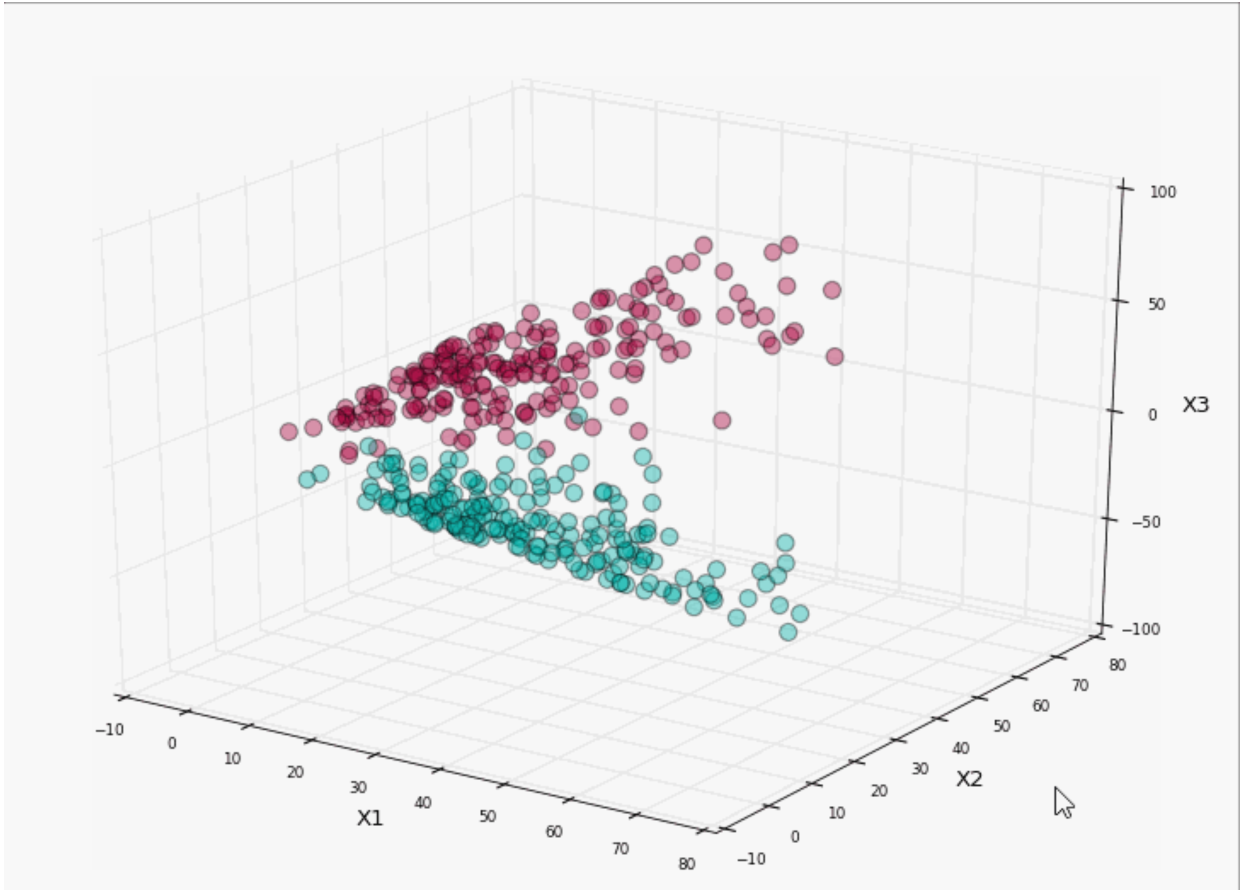

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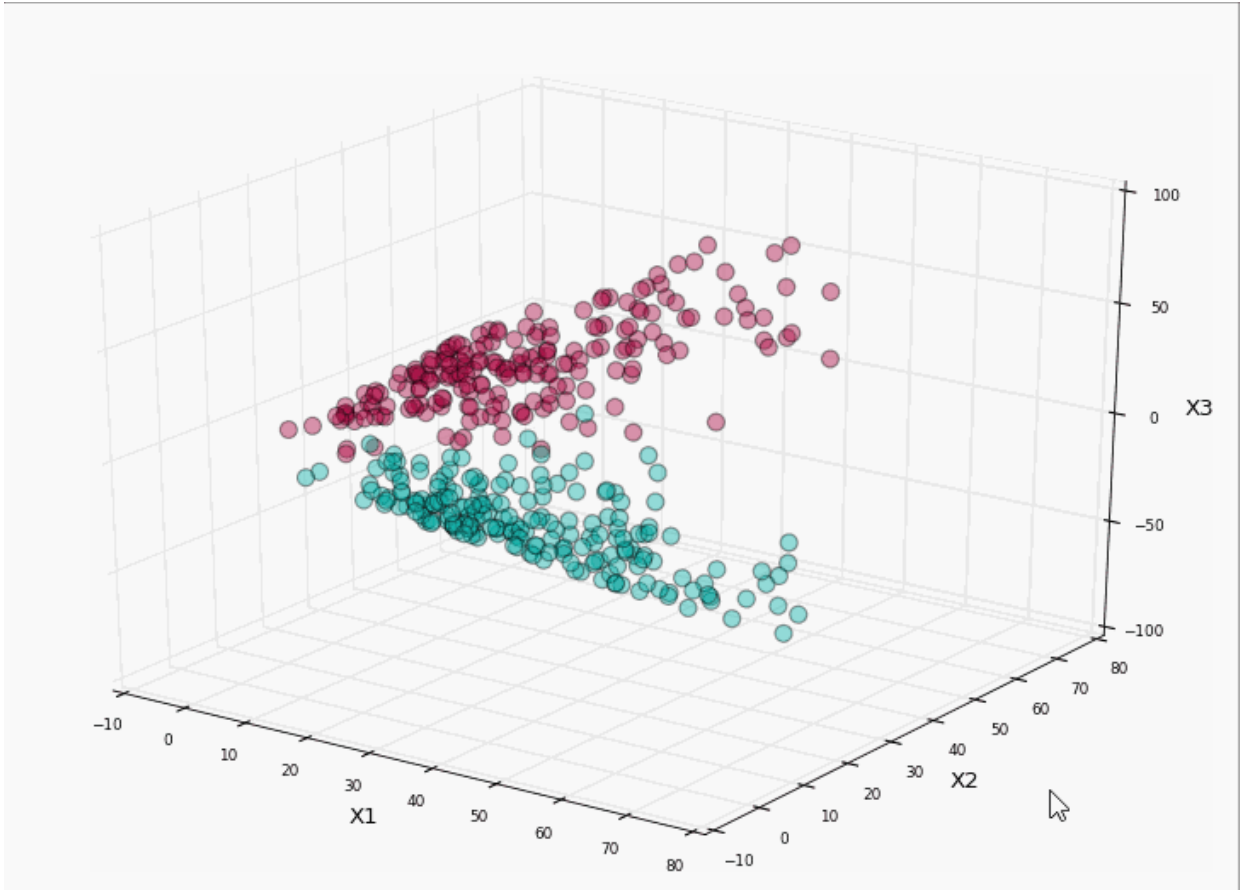
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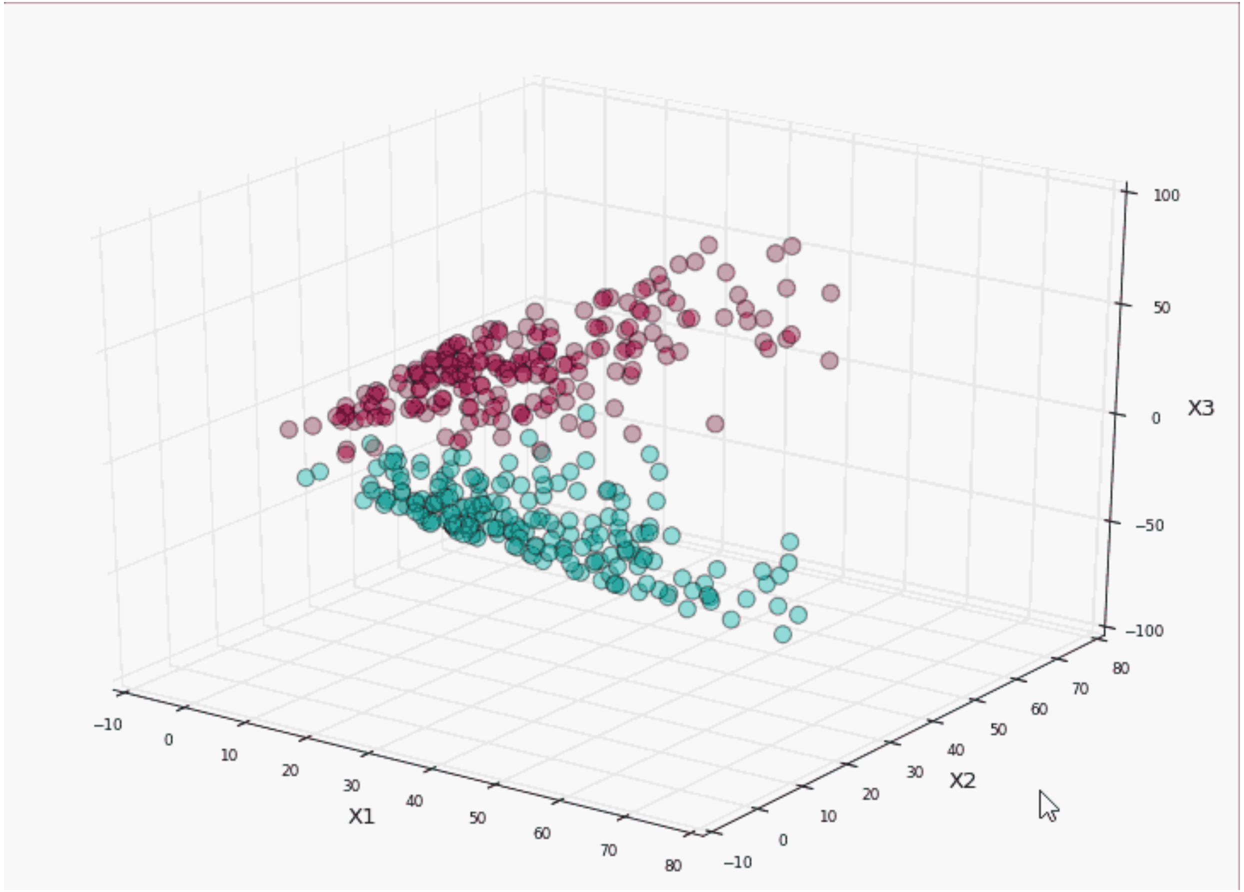

X1
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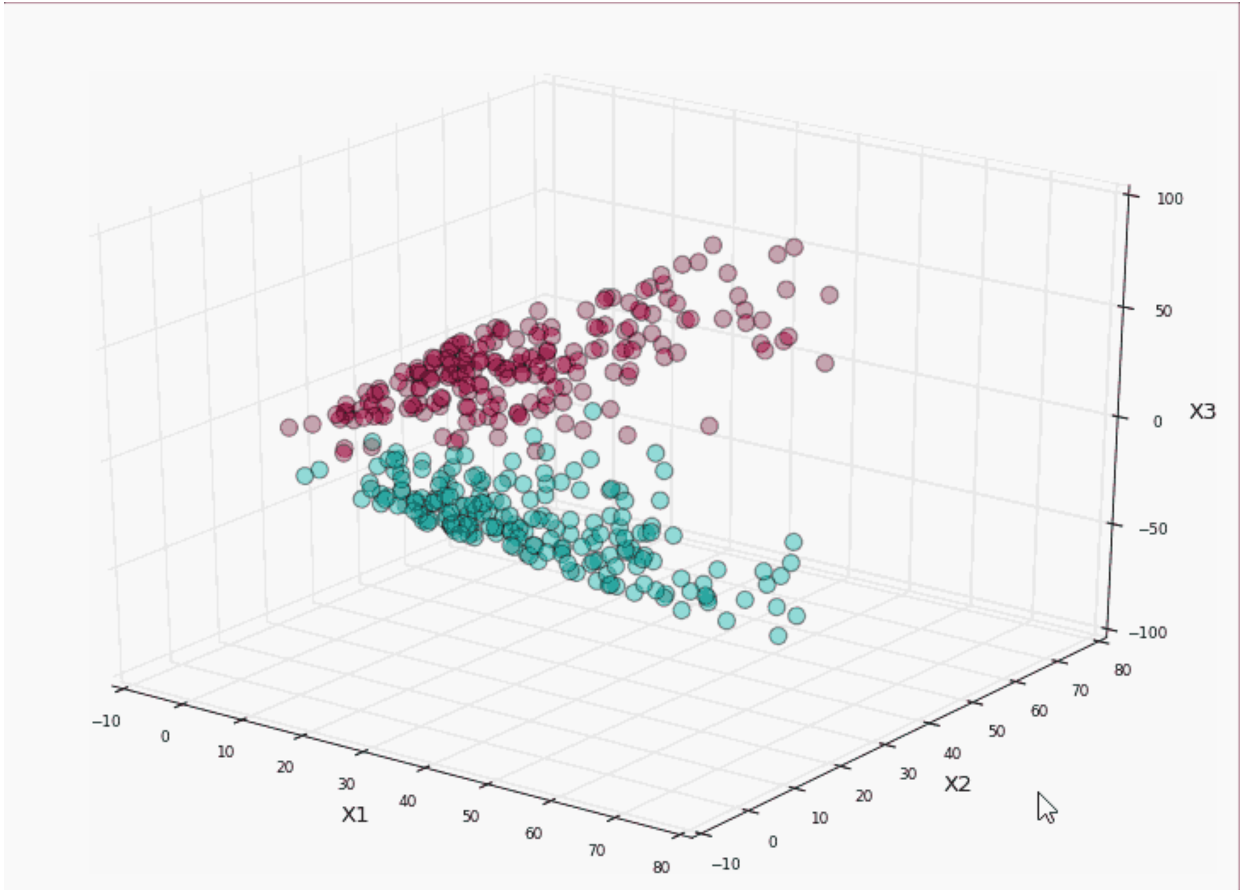
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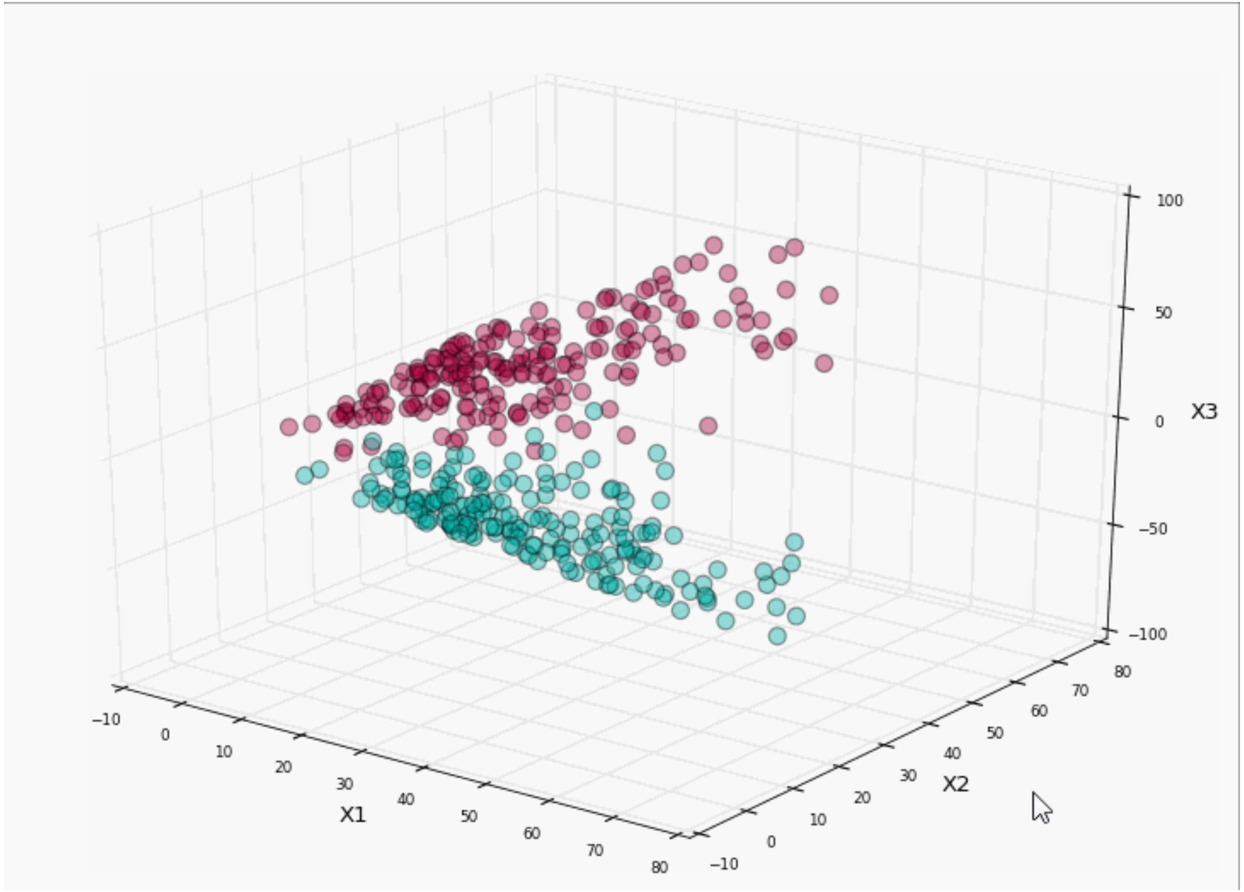
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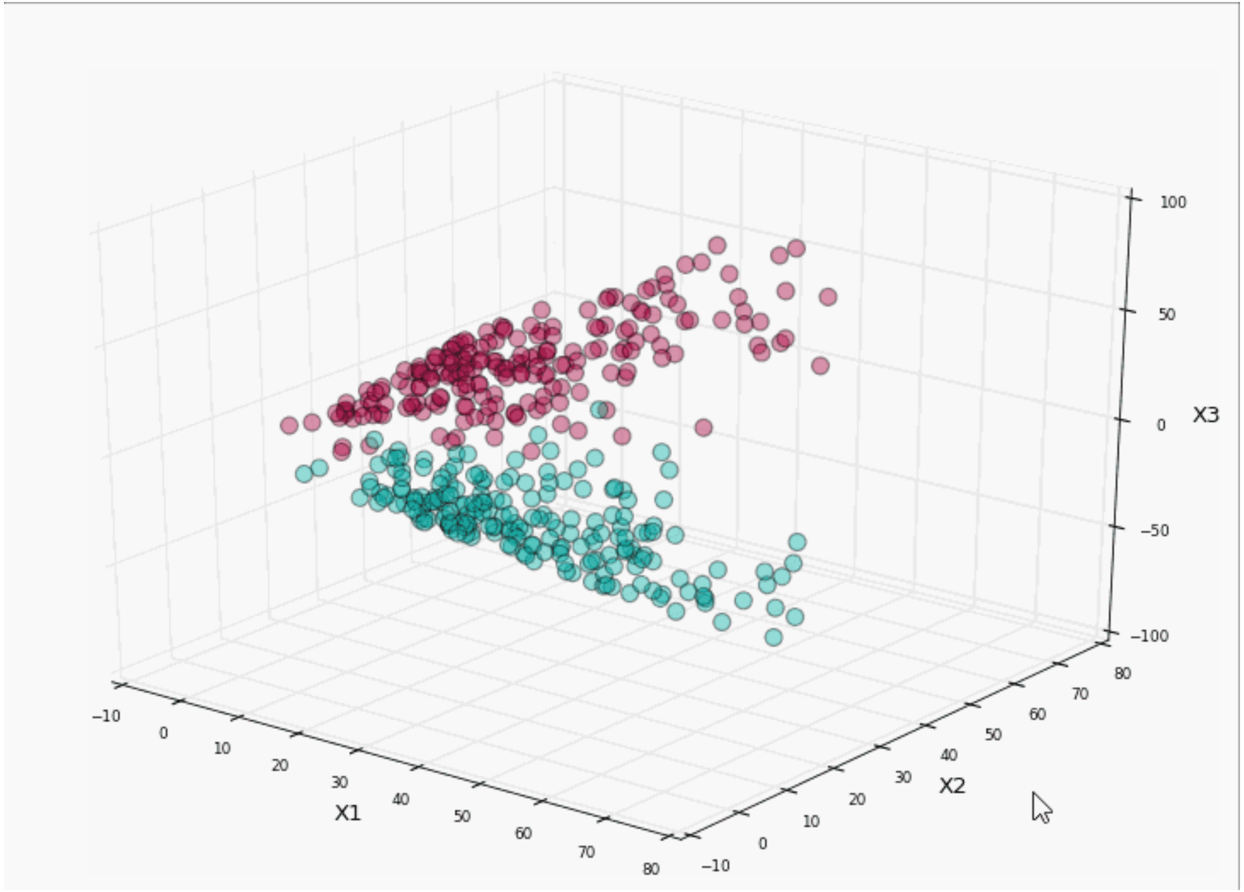
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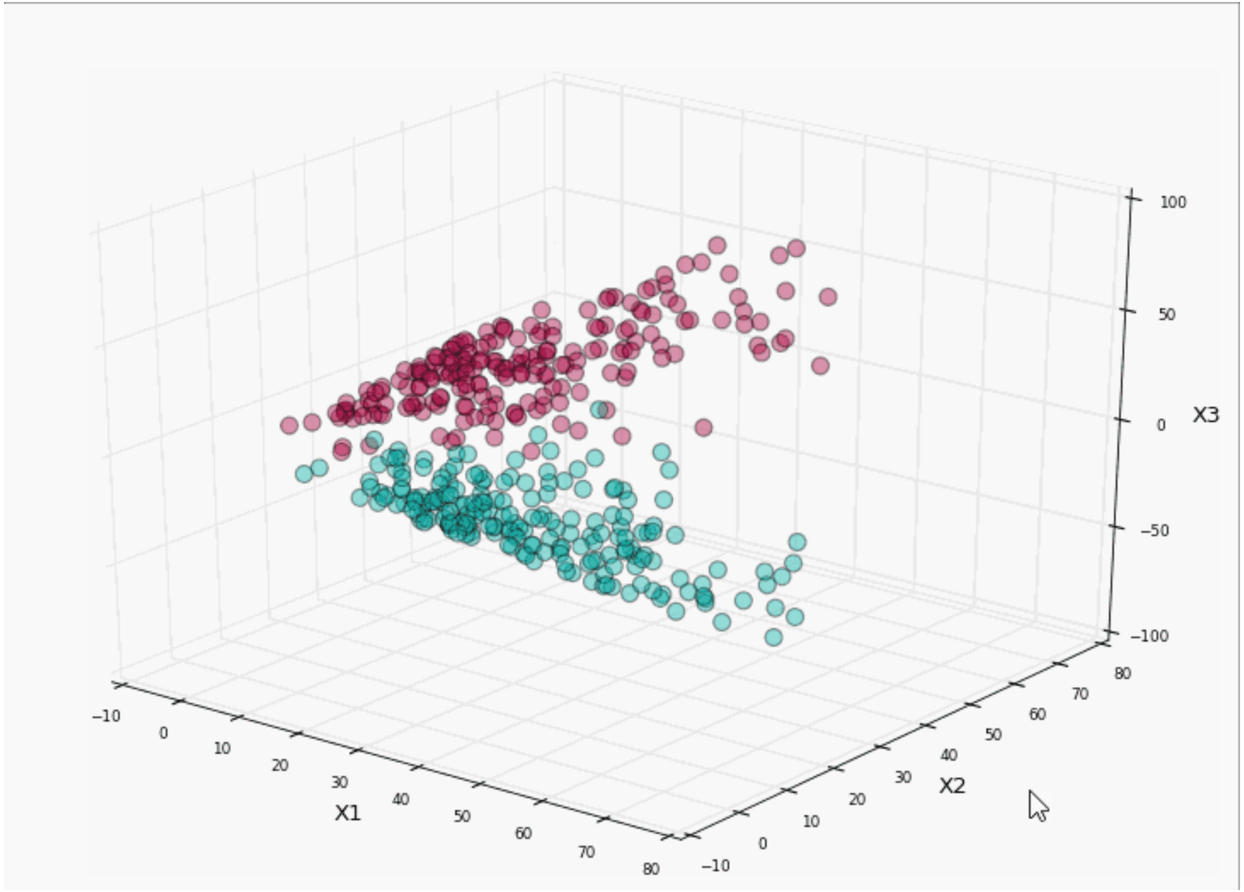
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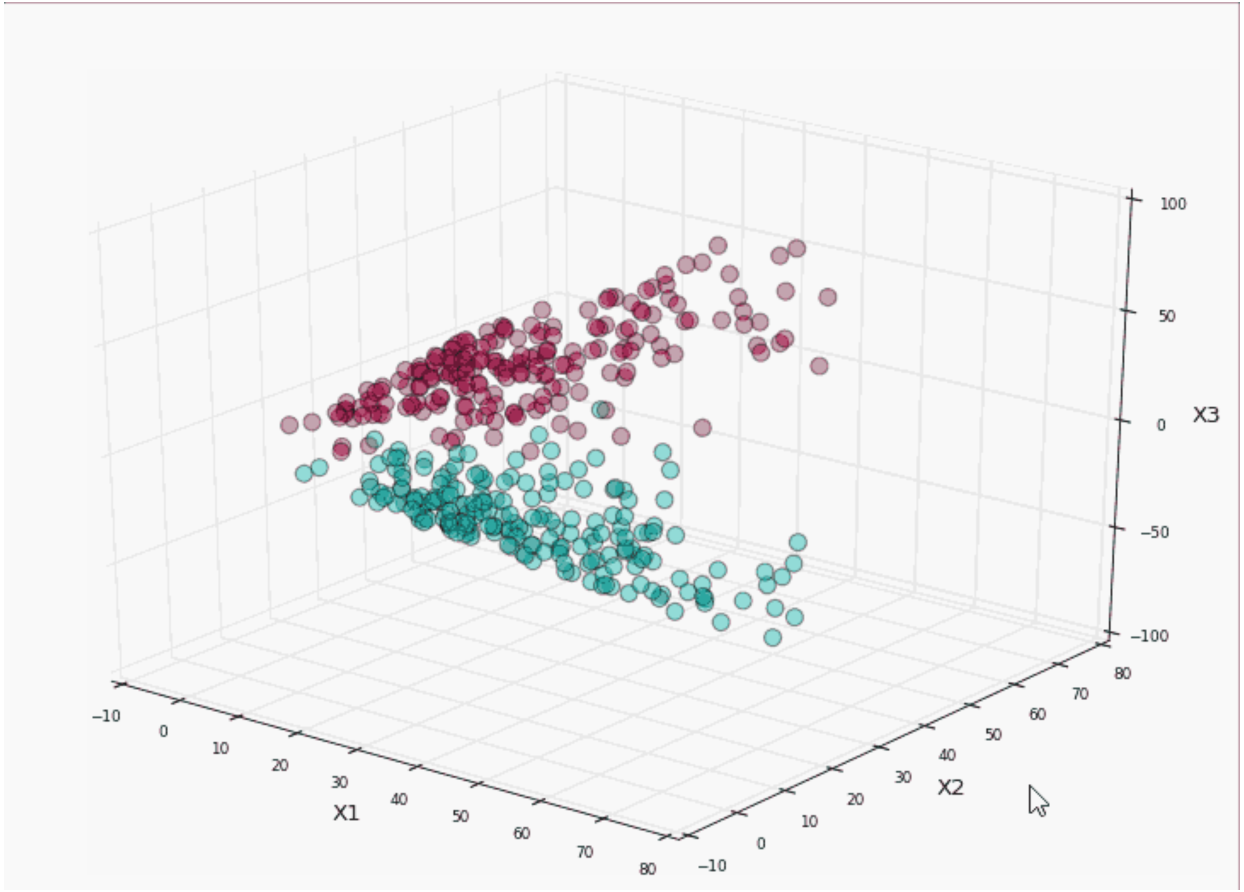
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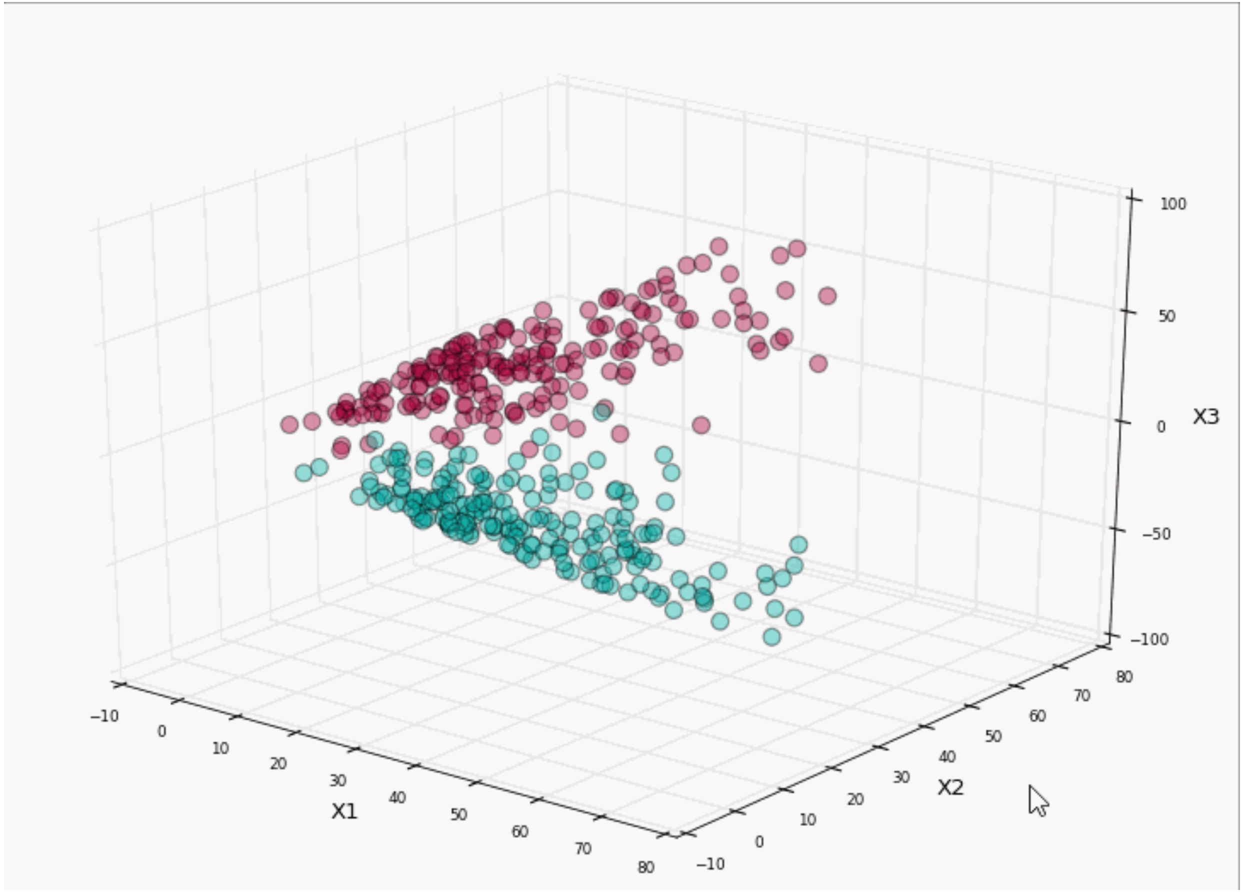
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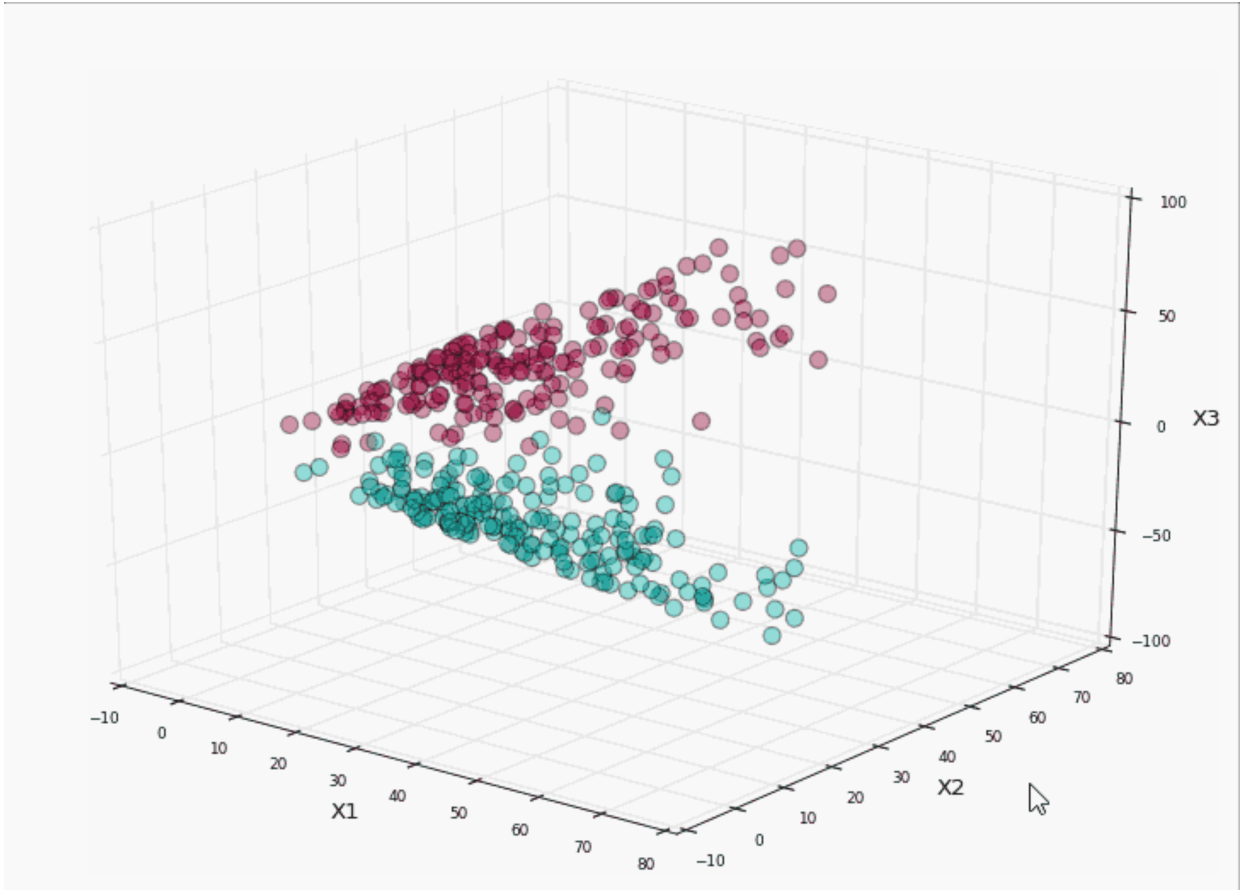

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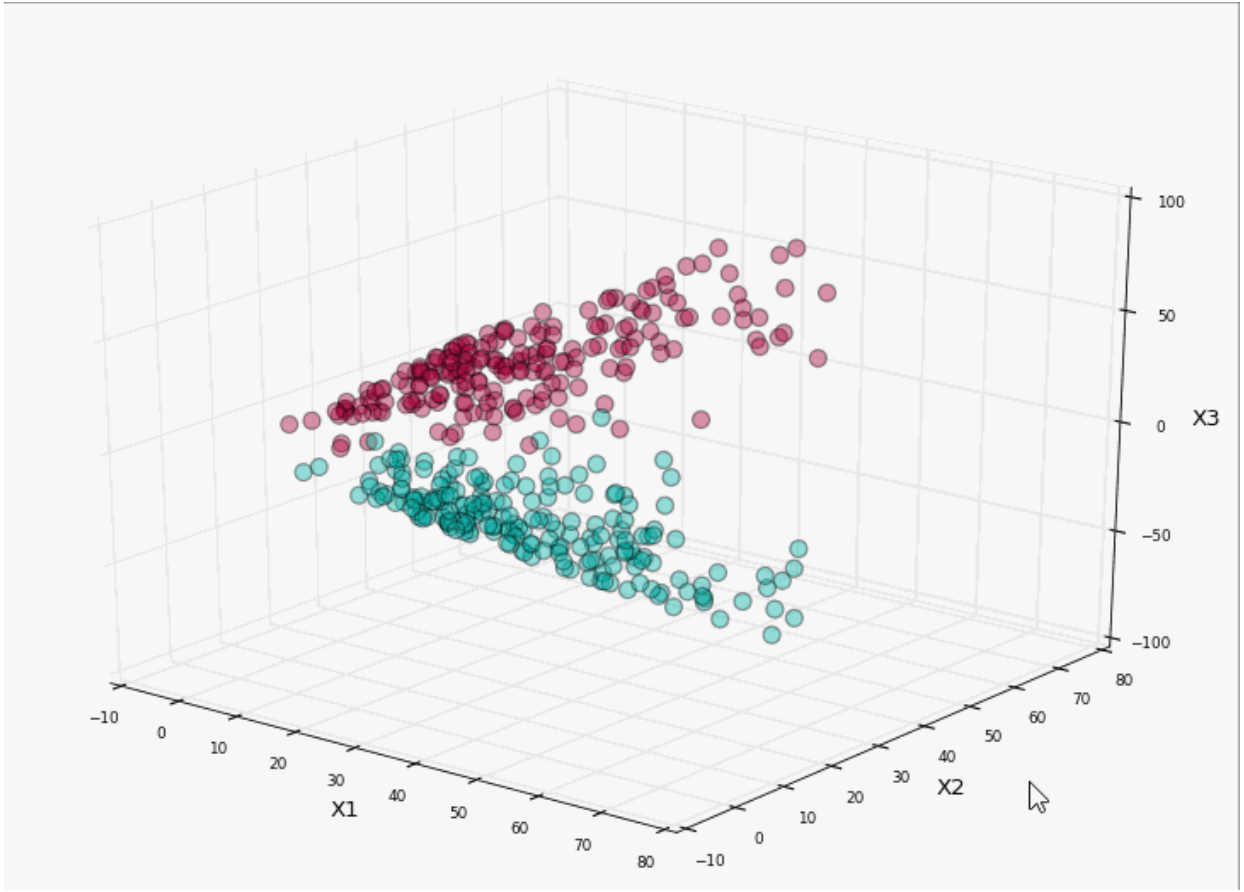
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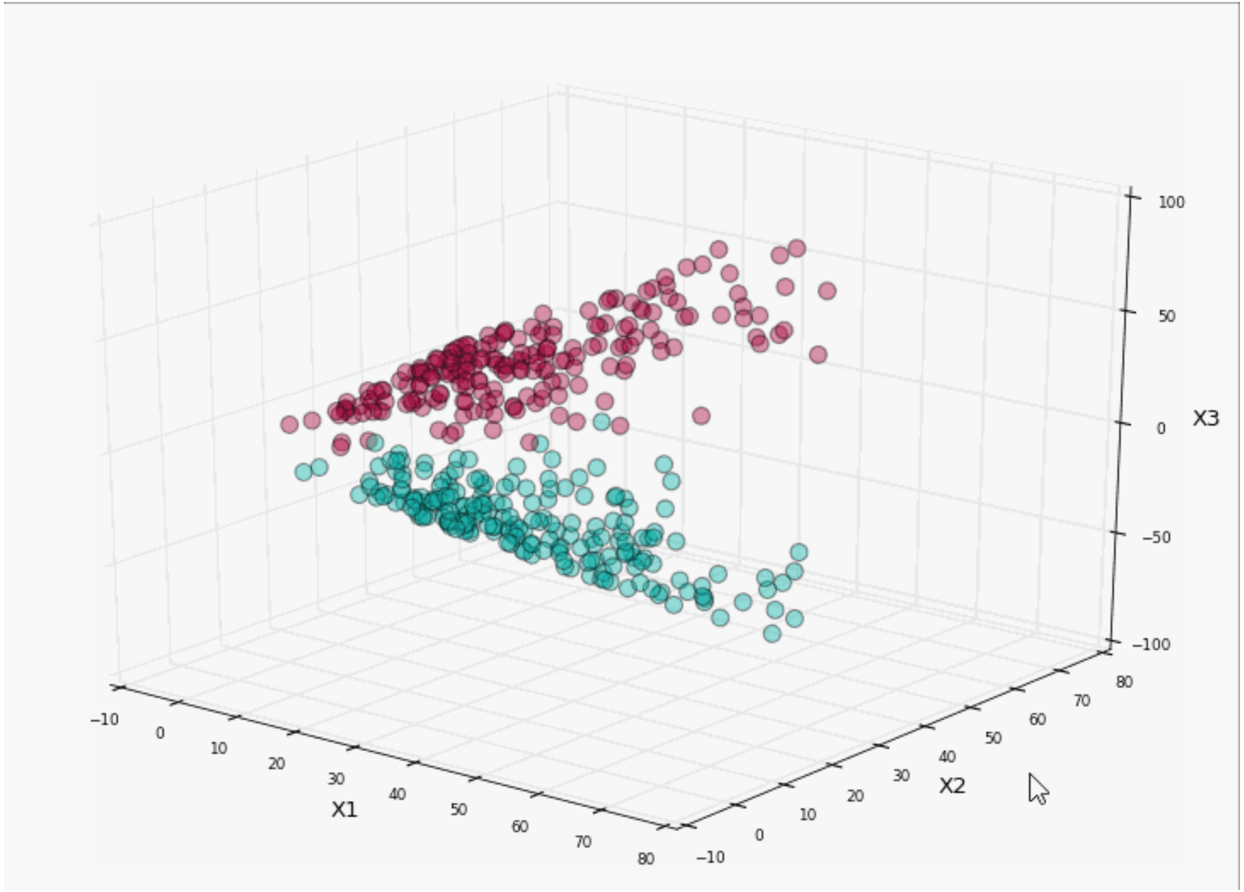

X1
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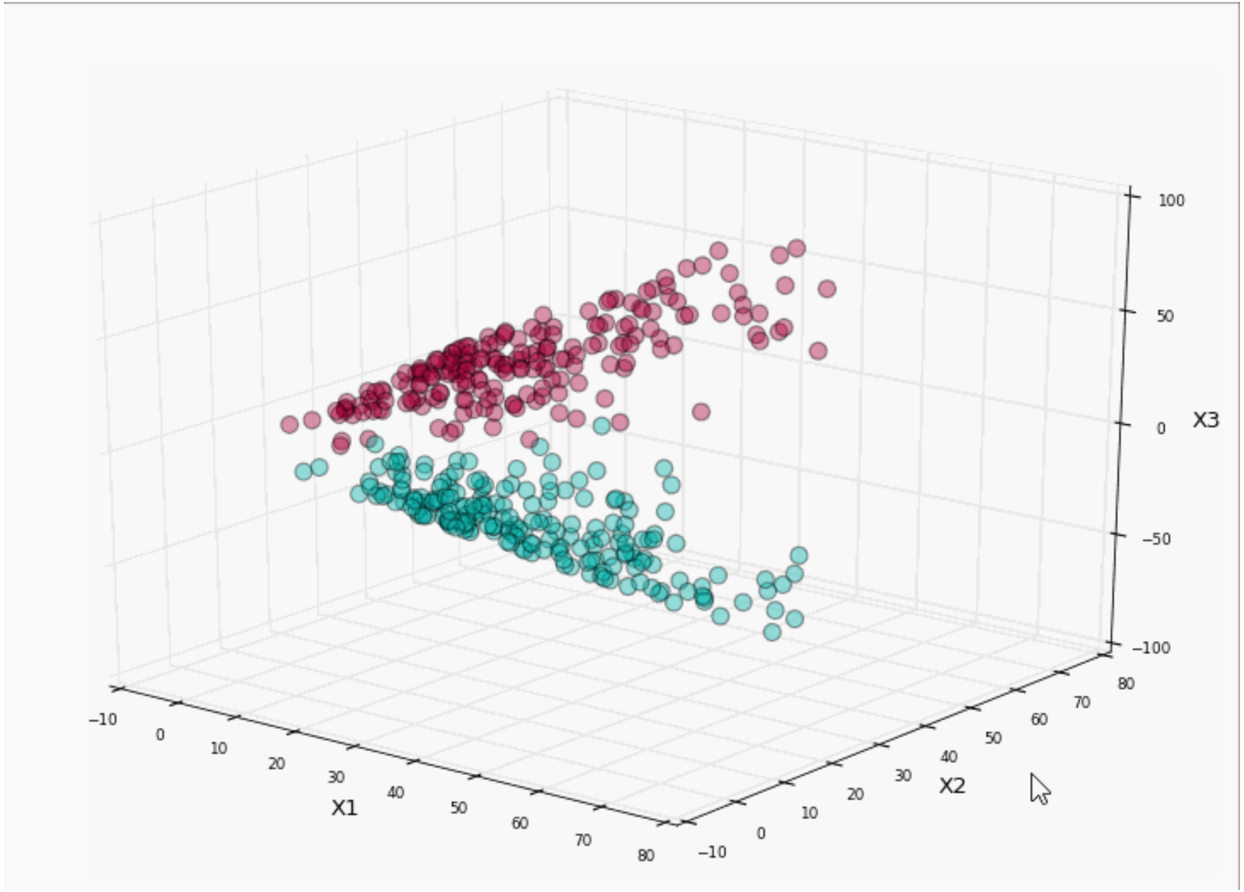

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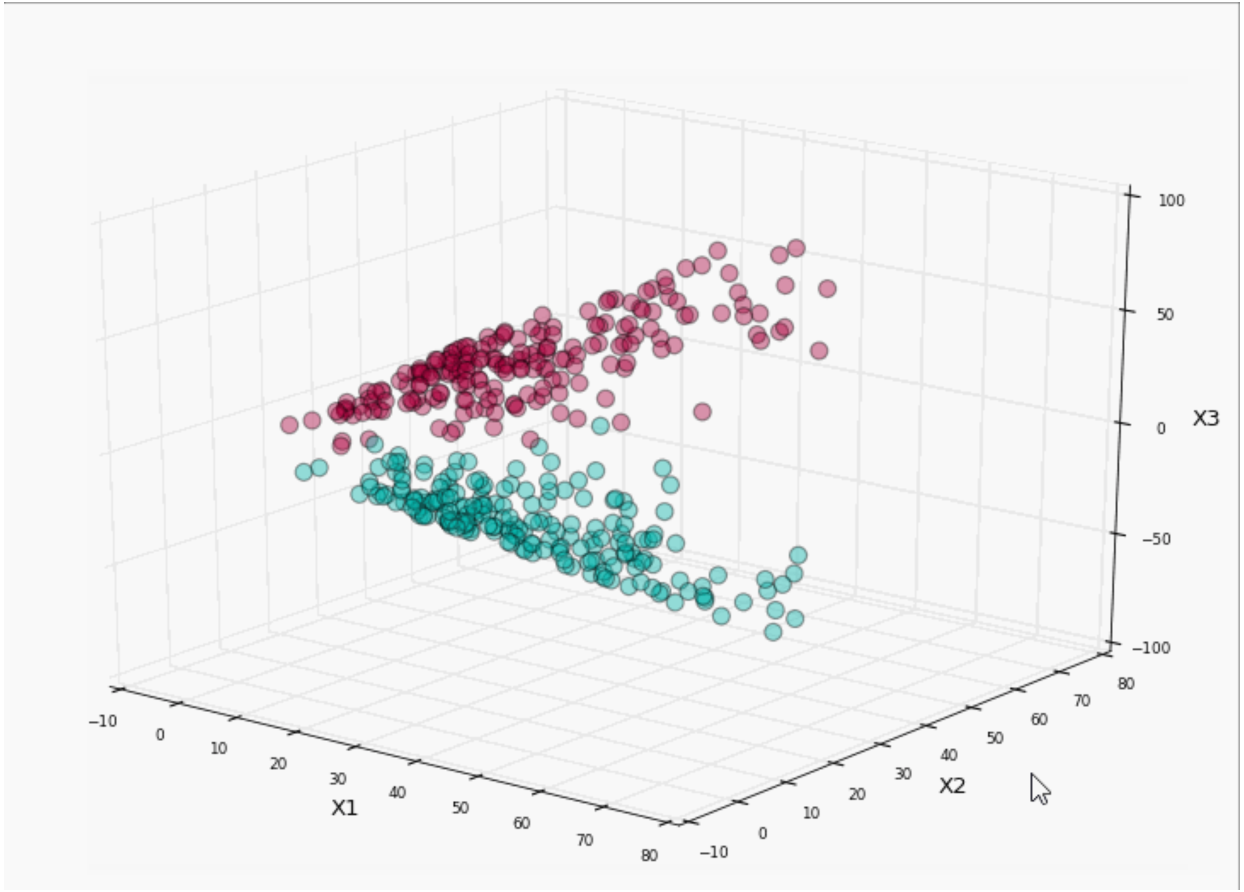

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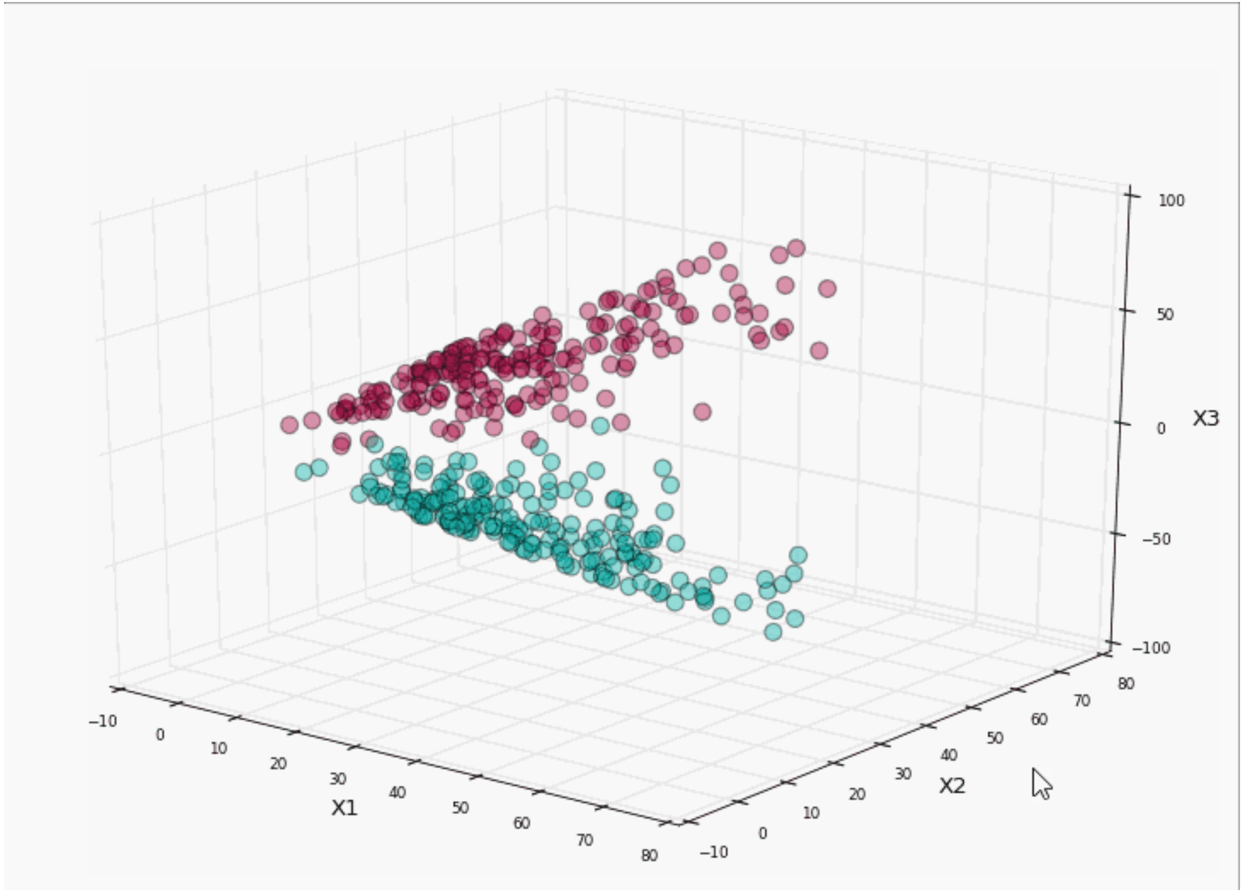
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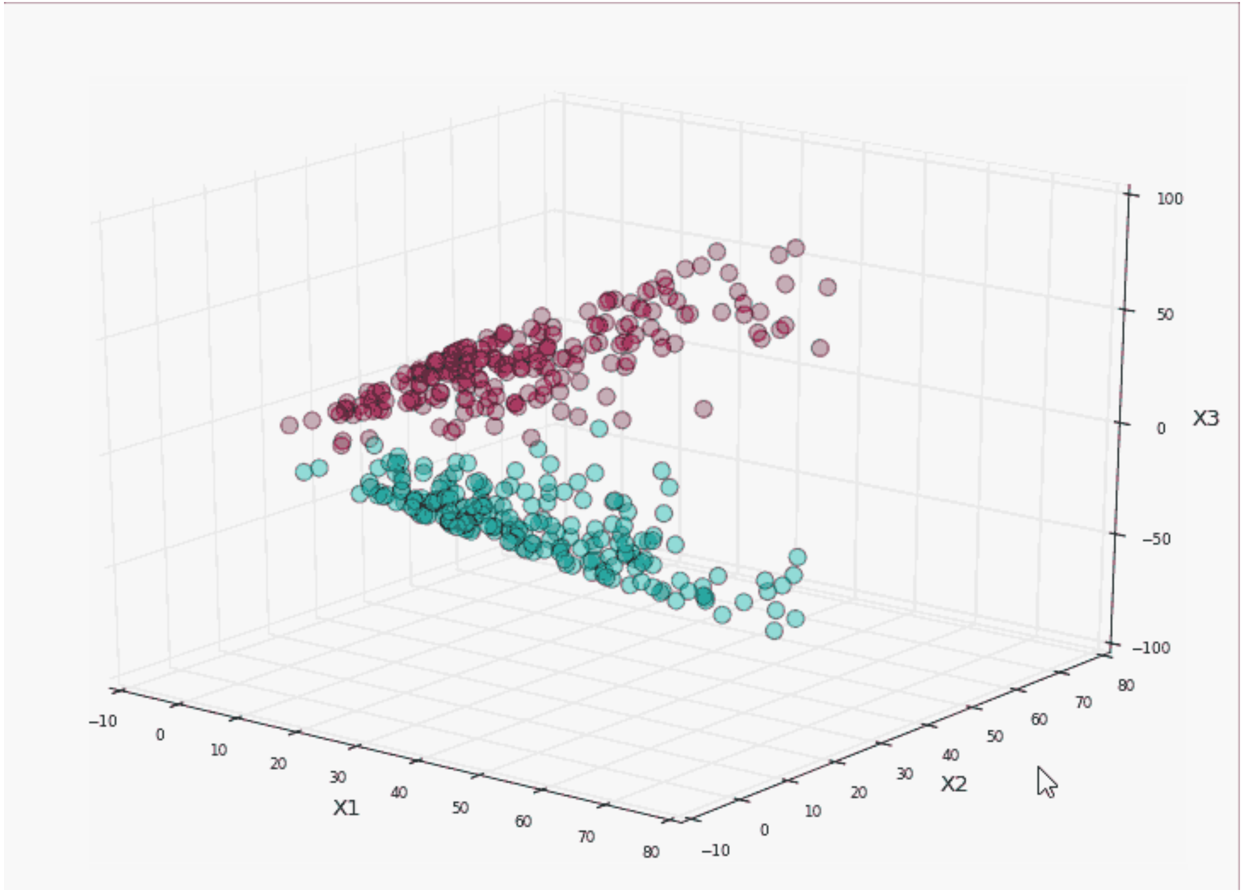

X1
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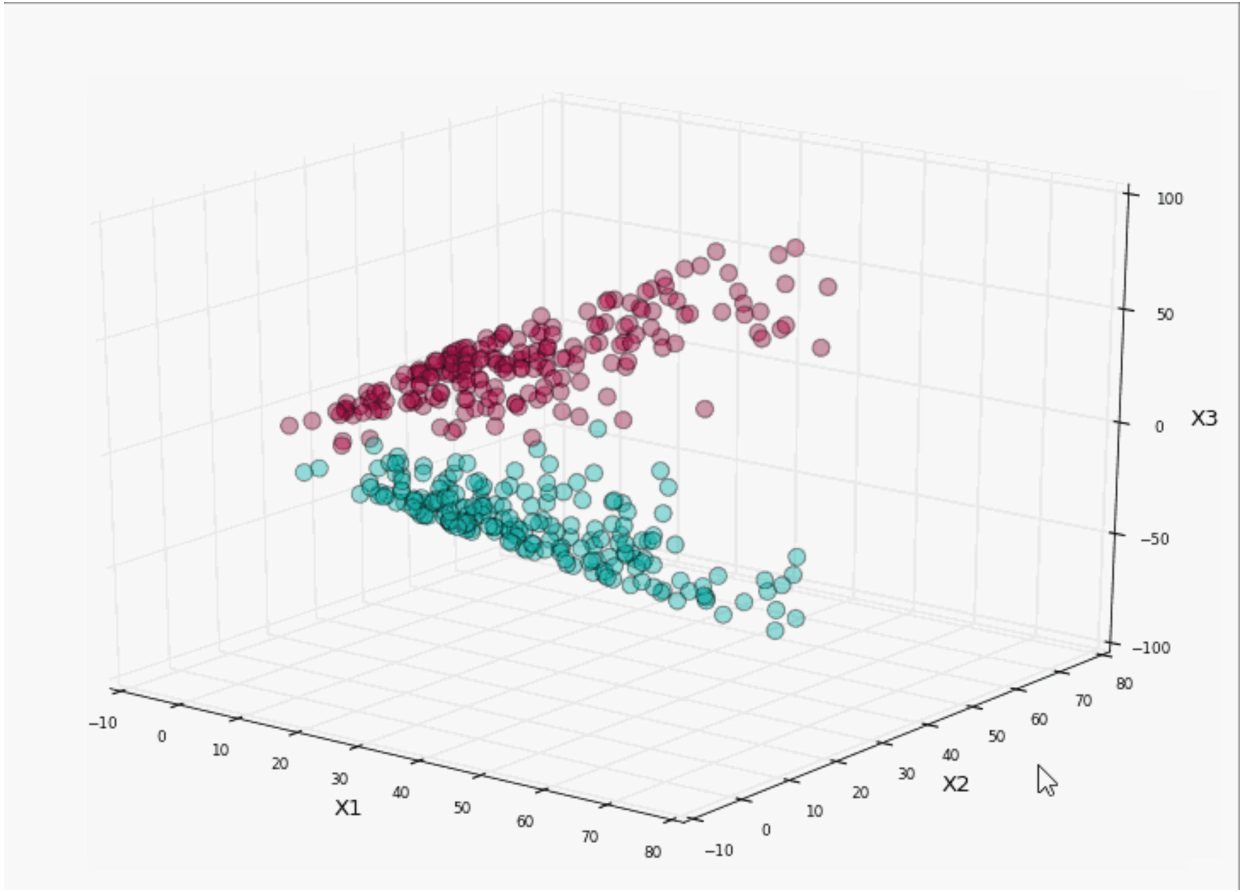
X3



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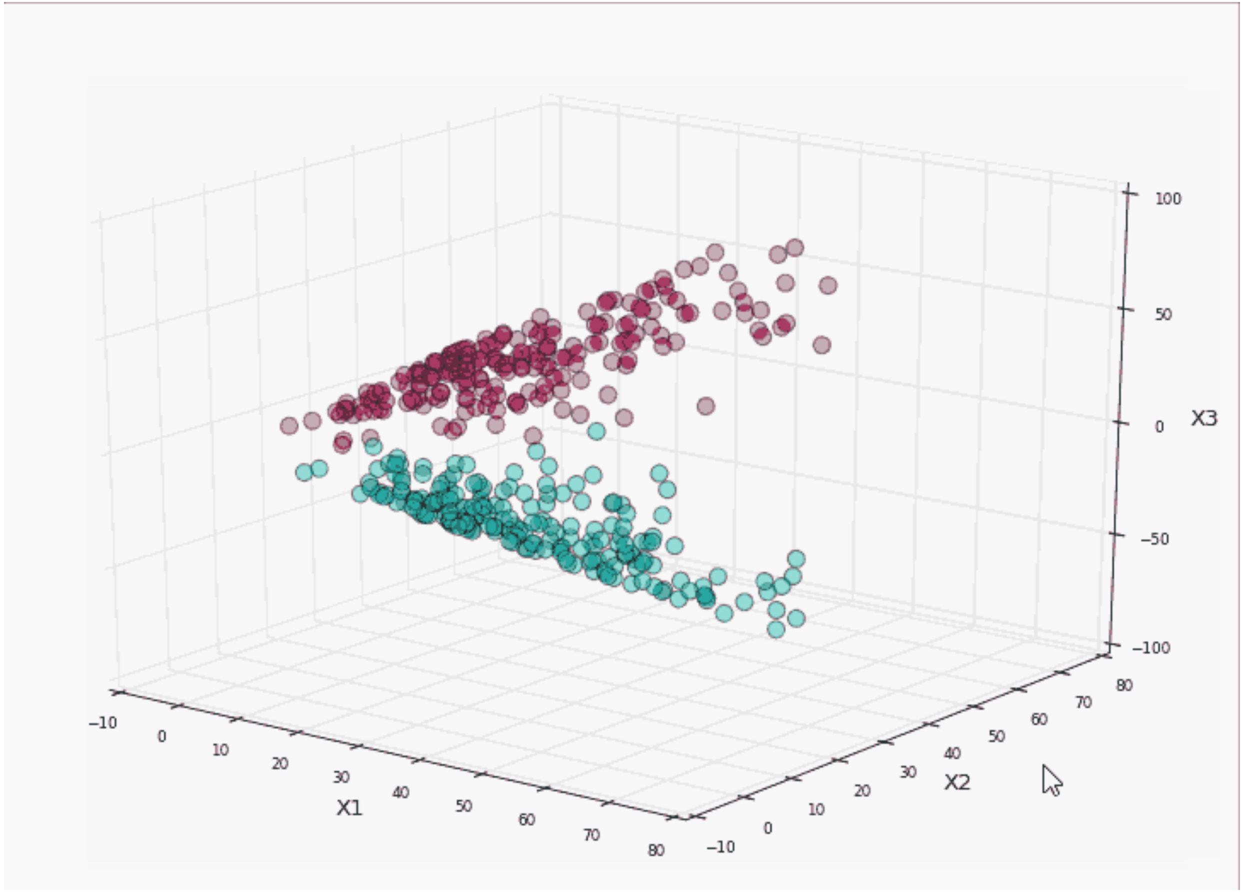

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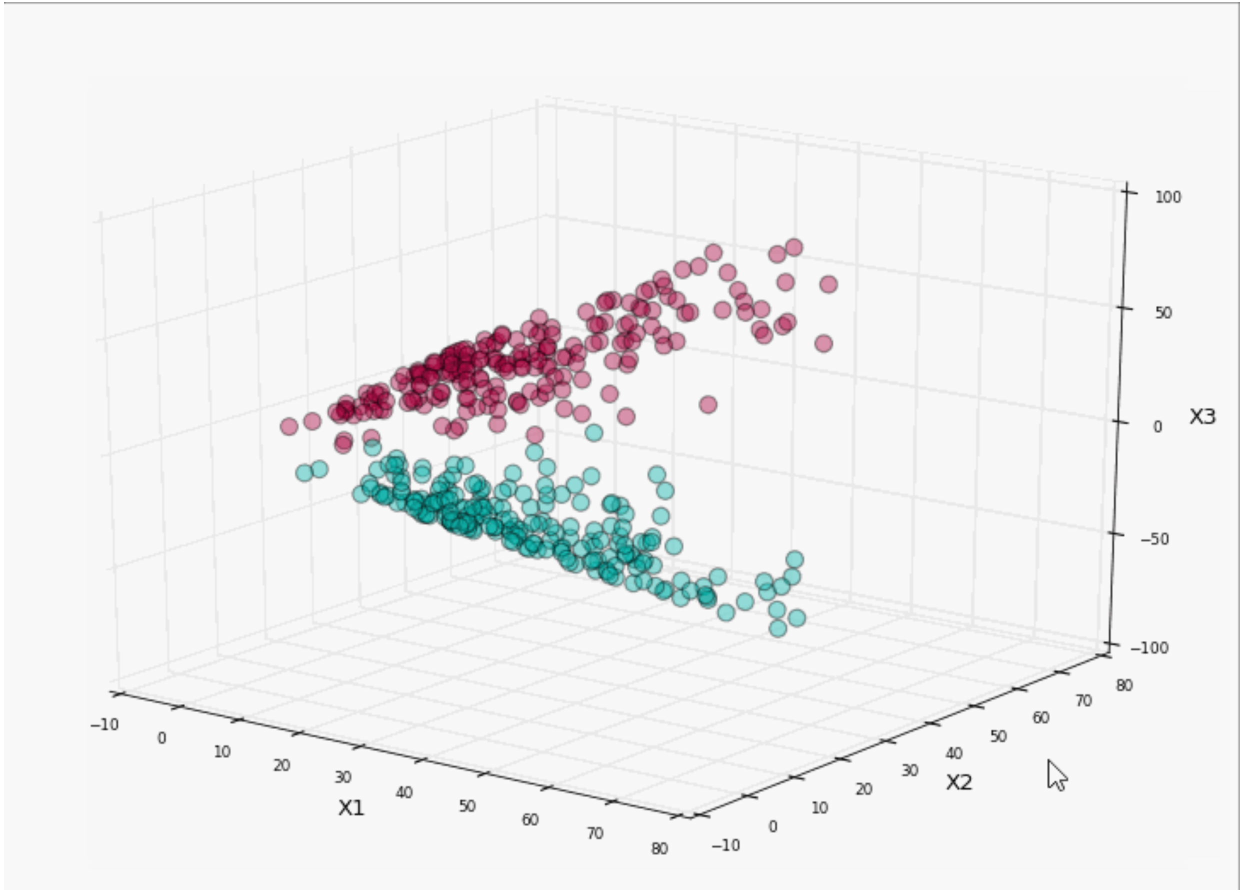

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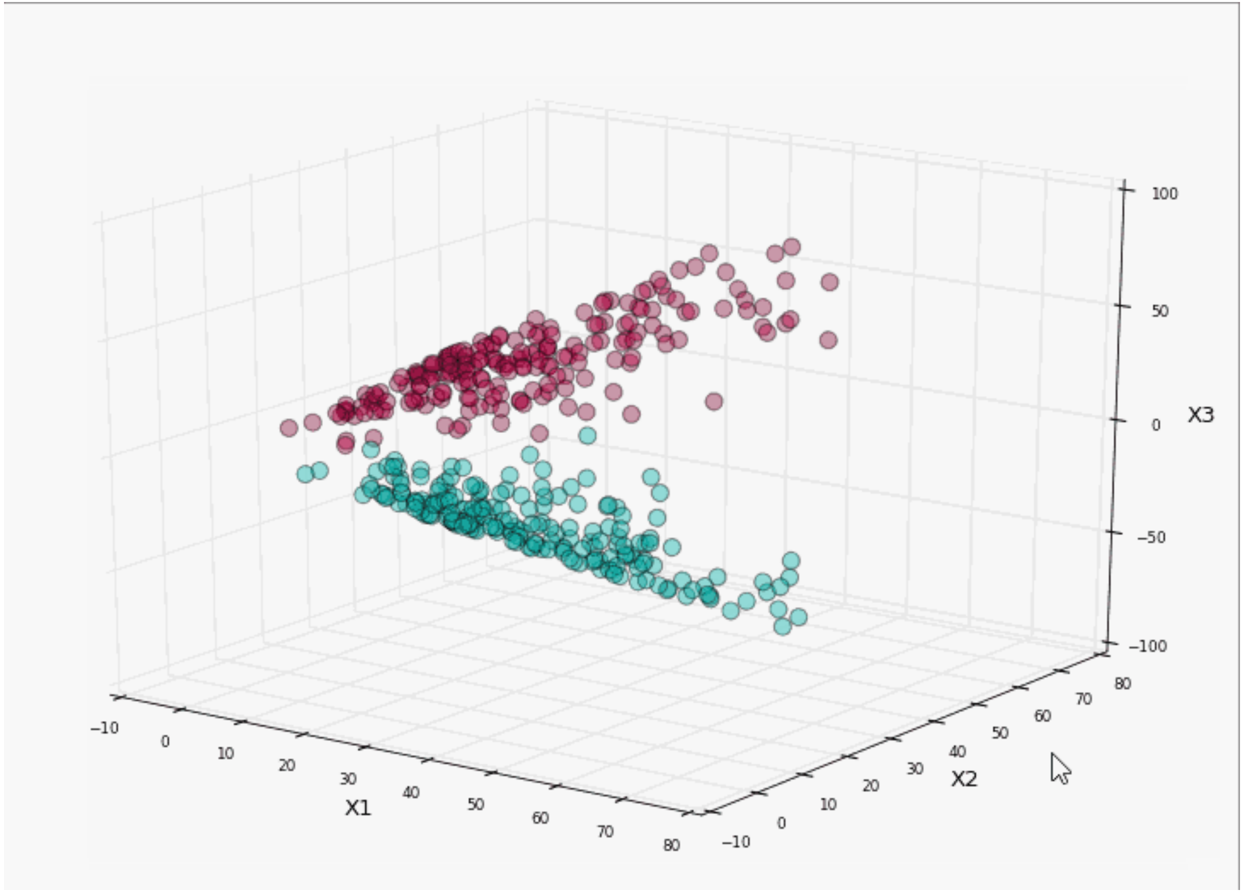
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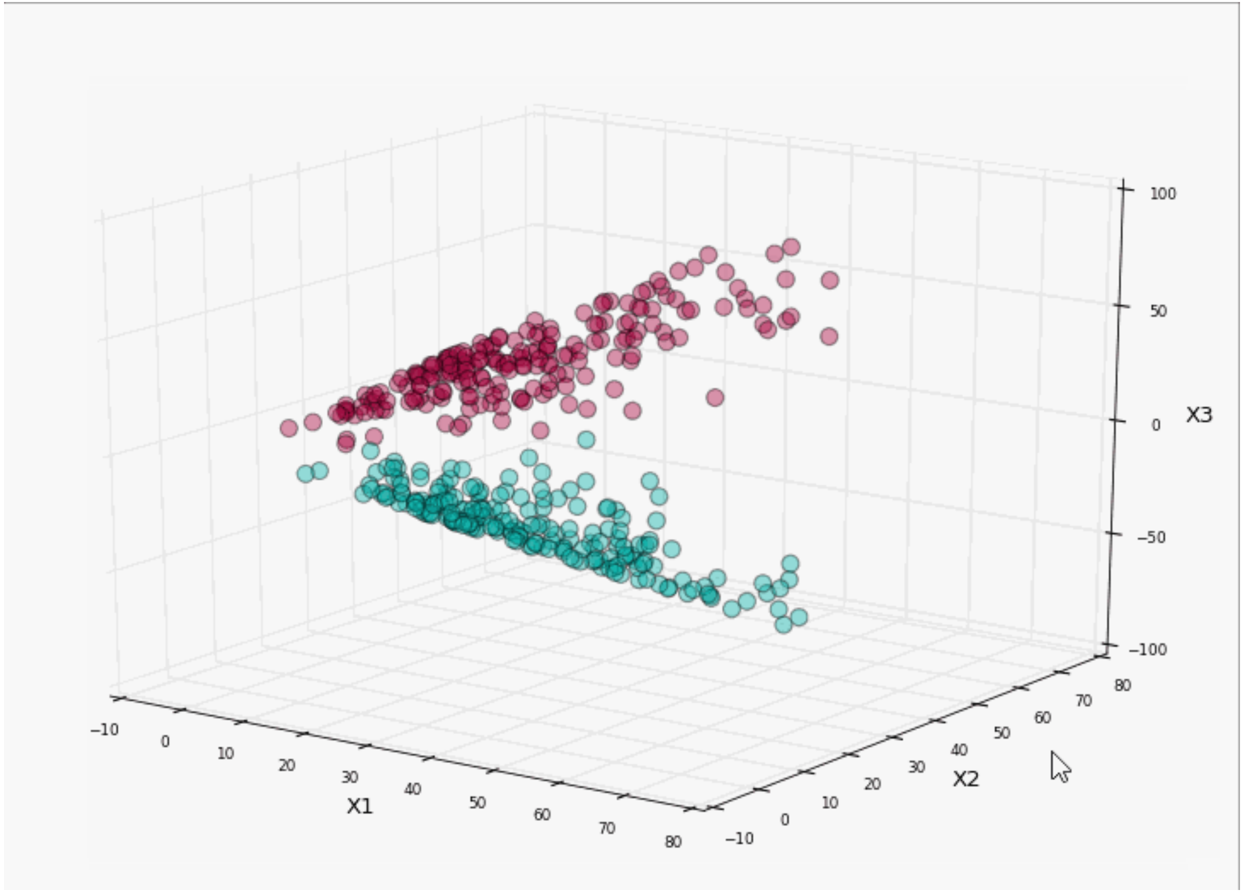
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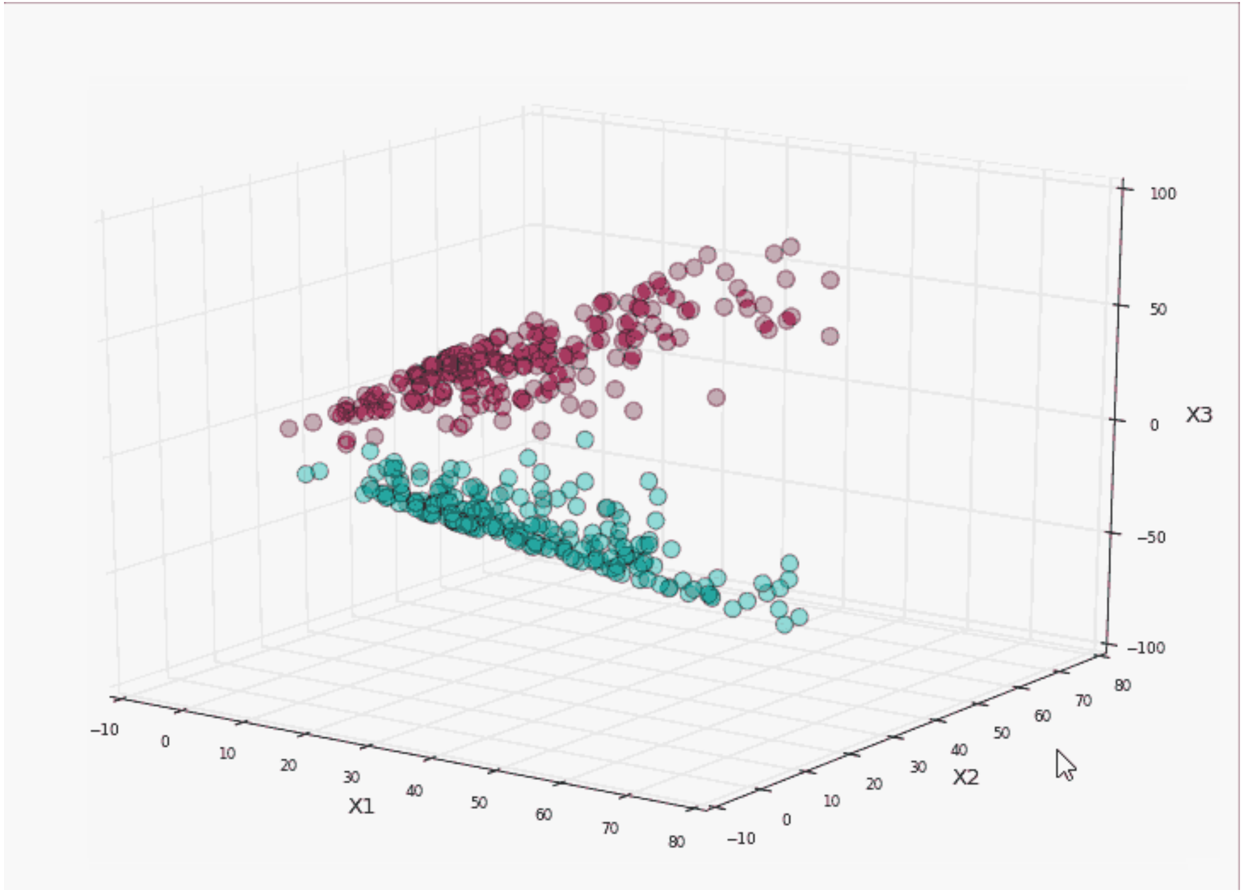
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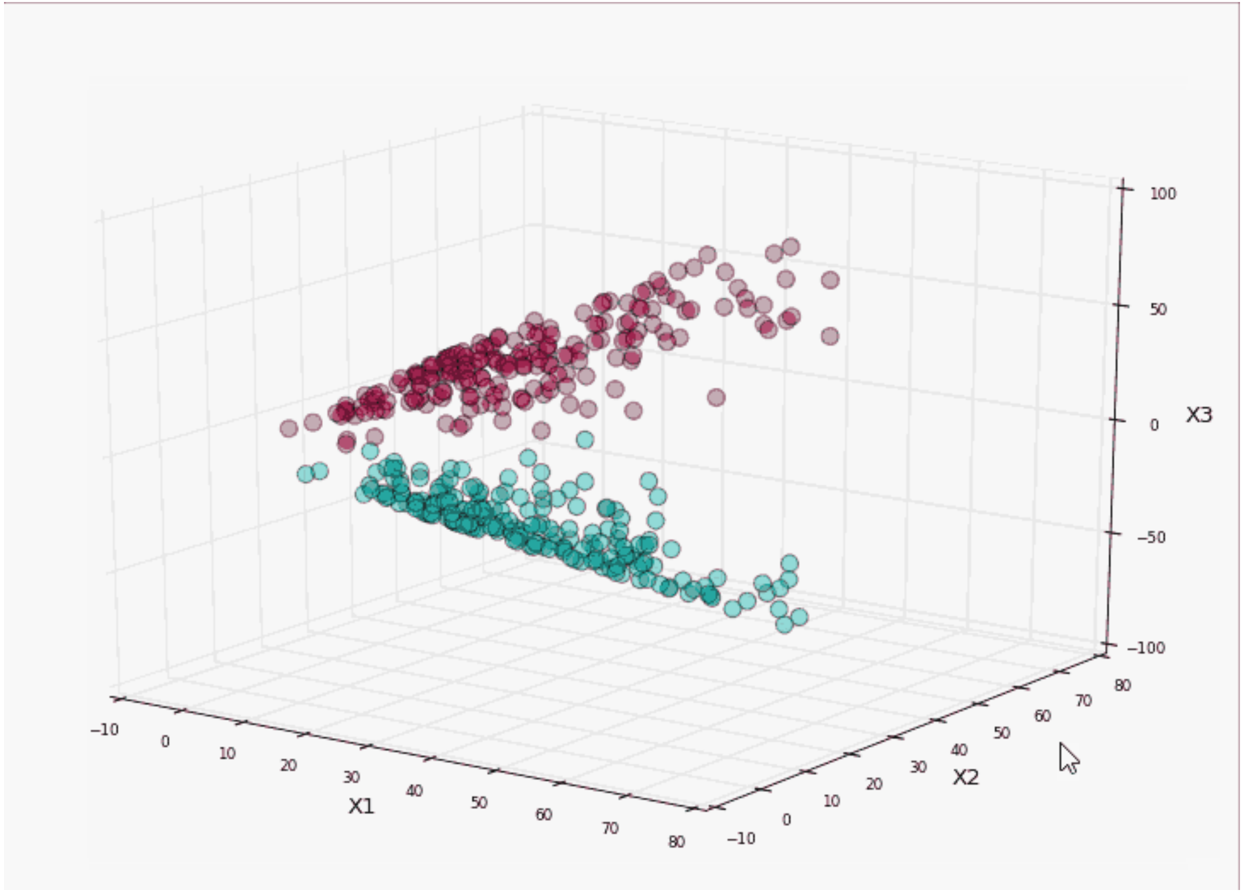
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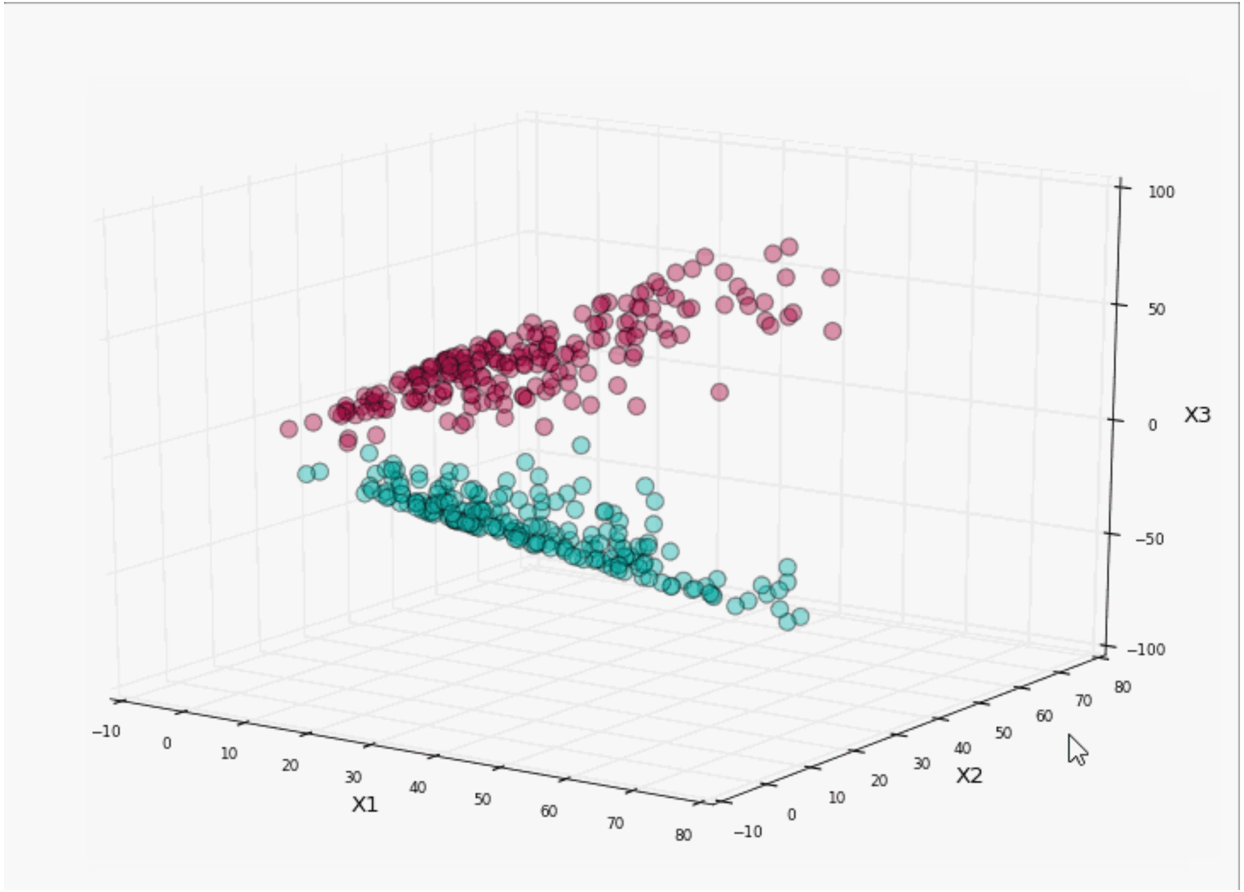

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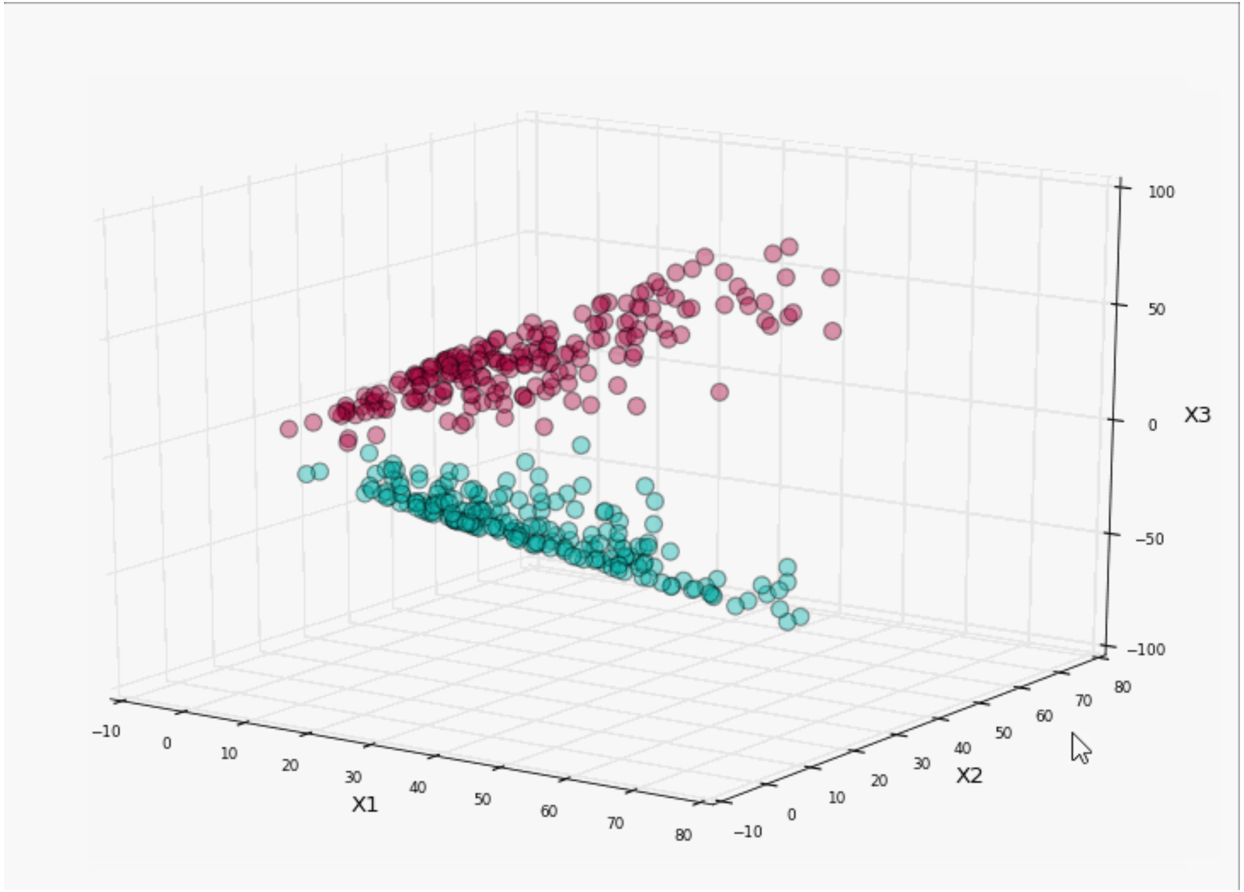
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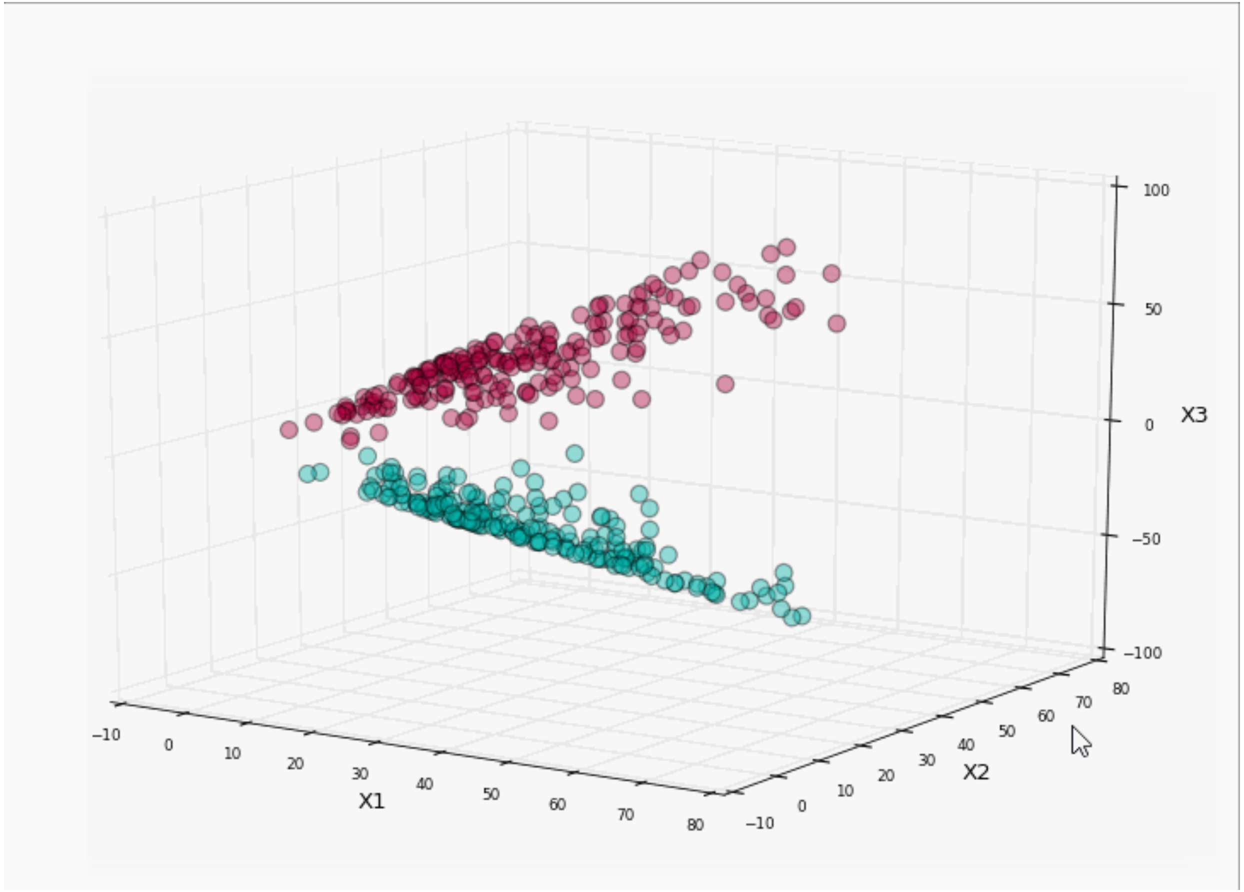

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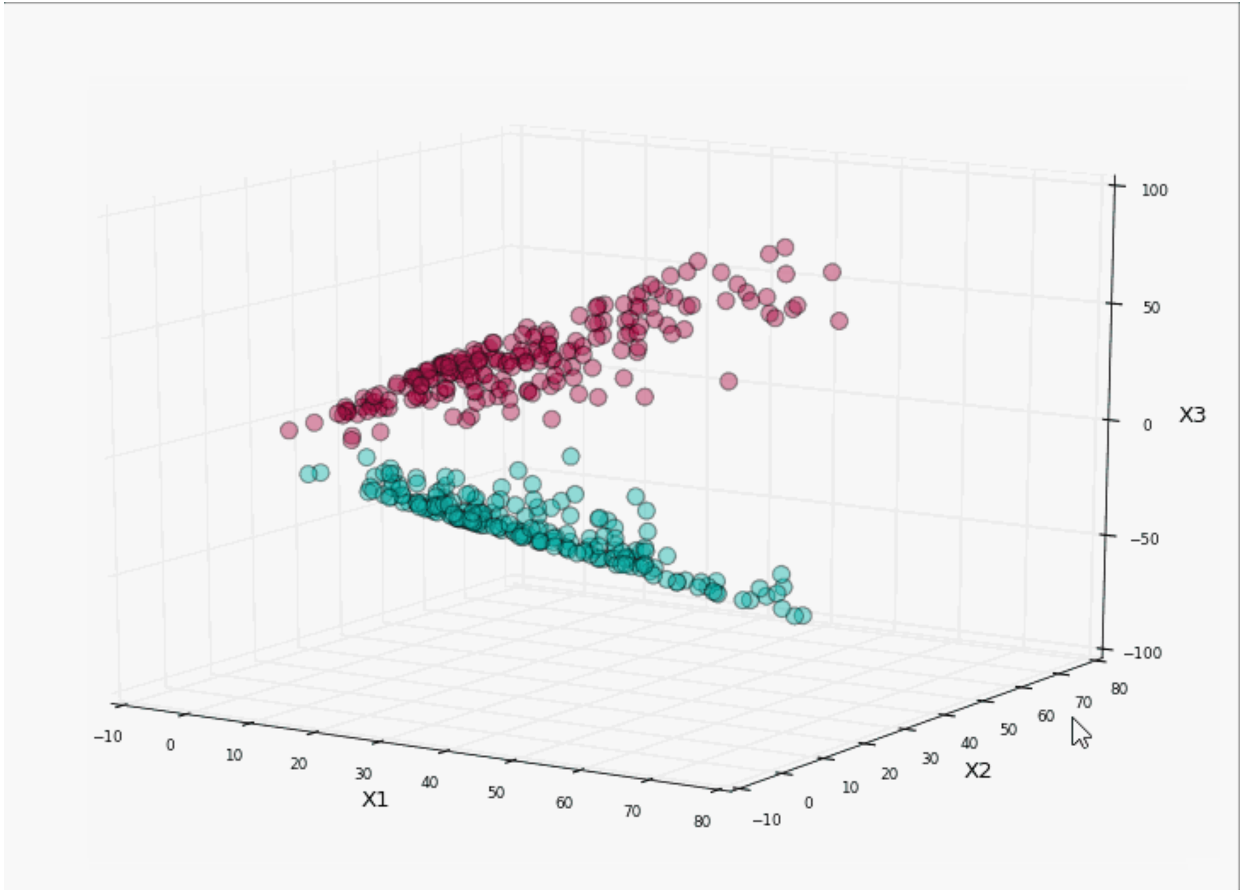
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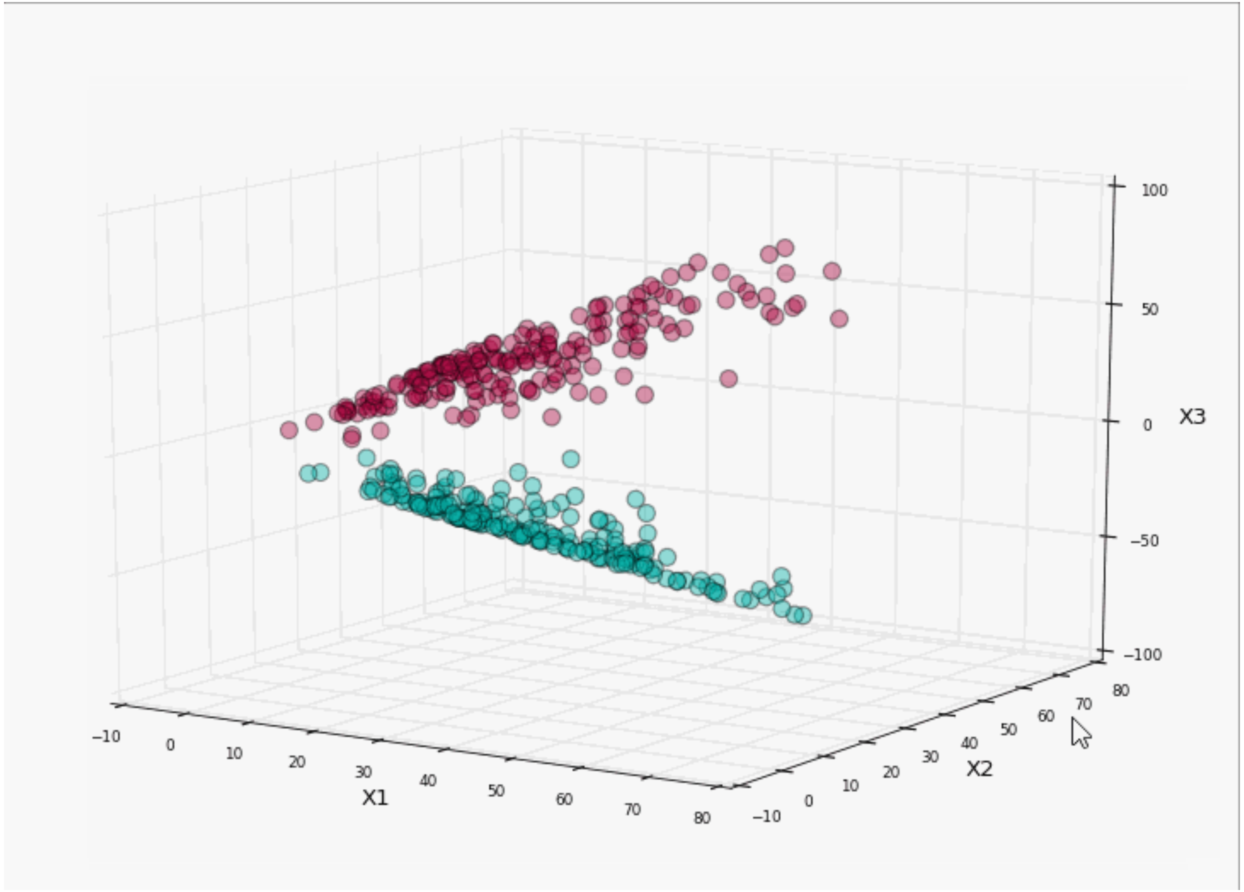

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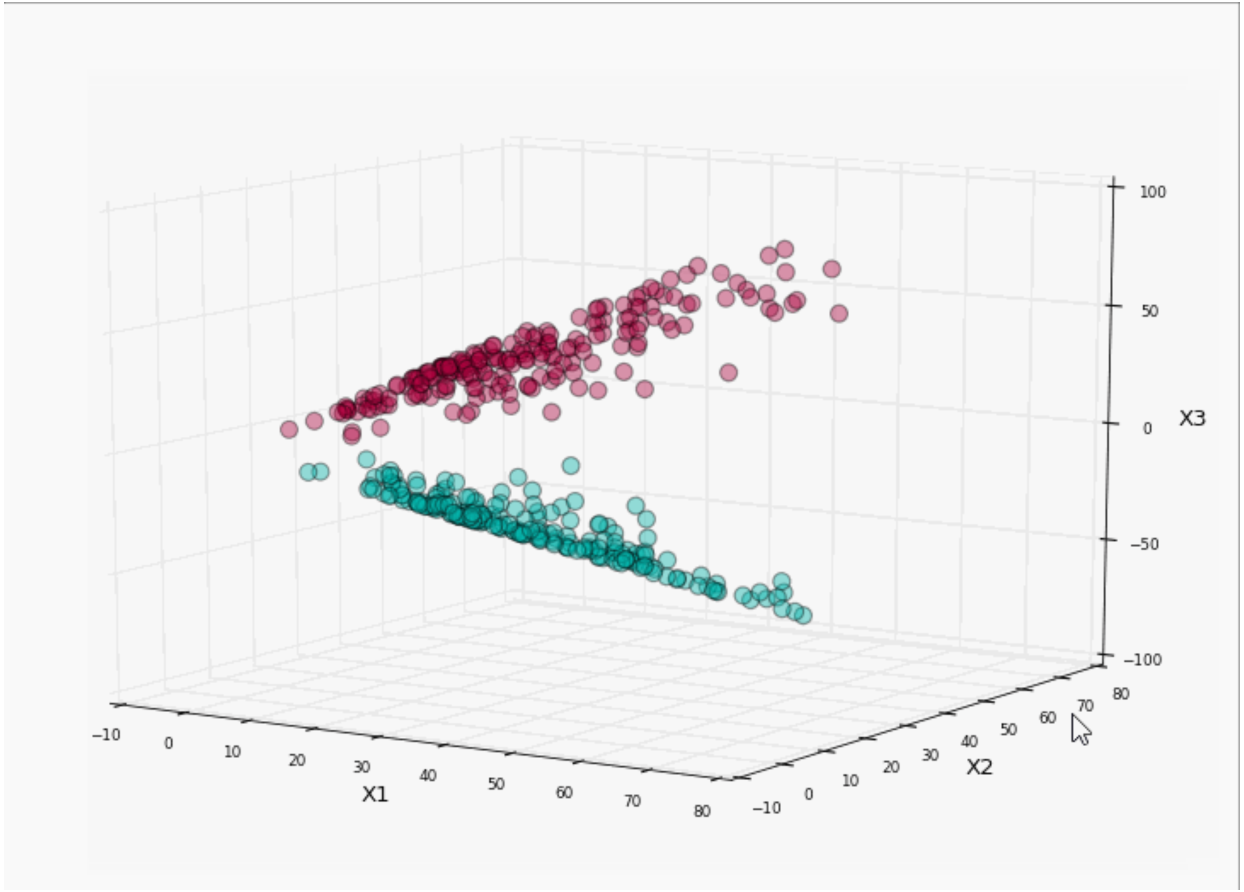

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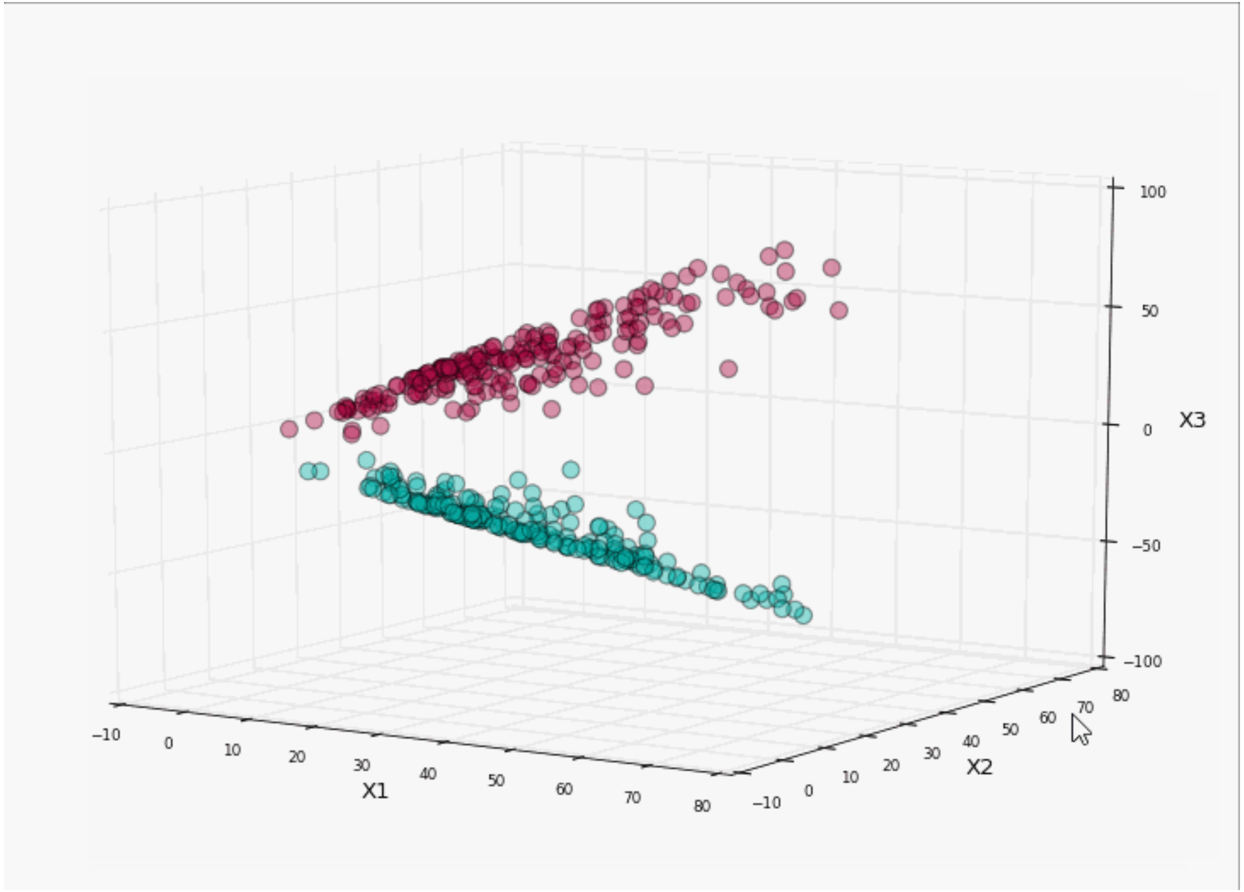

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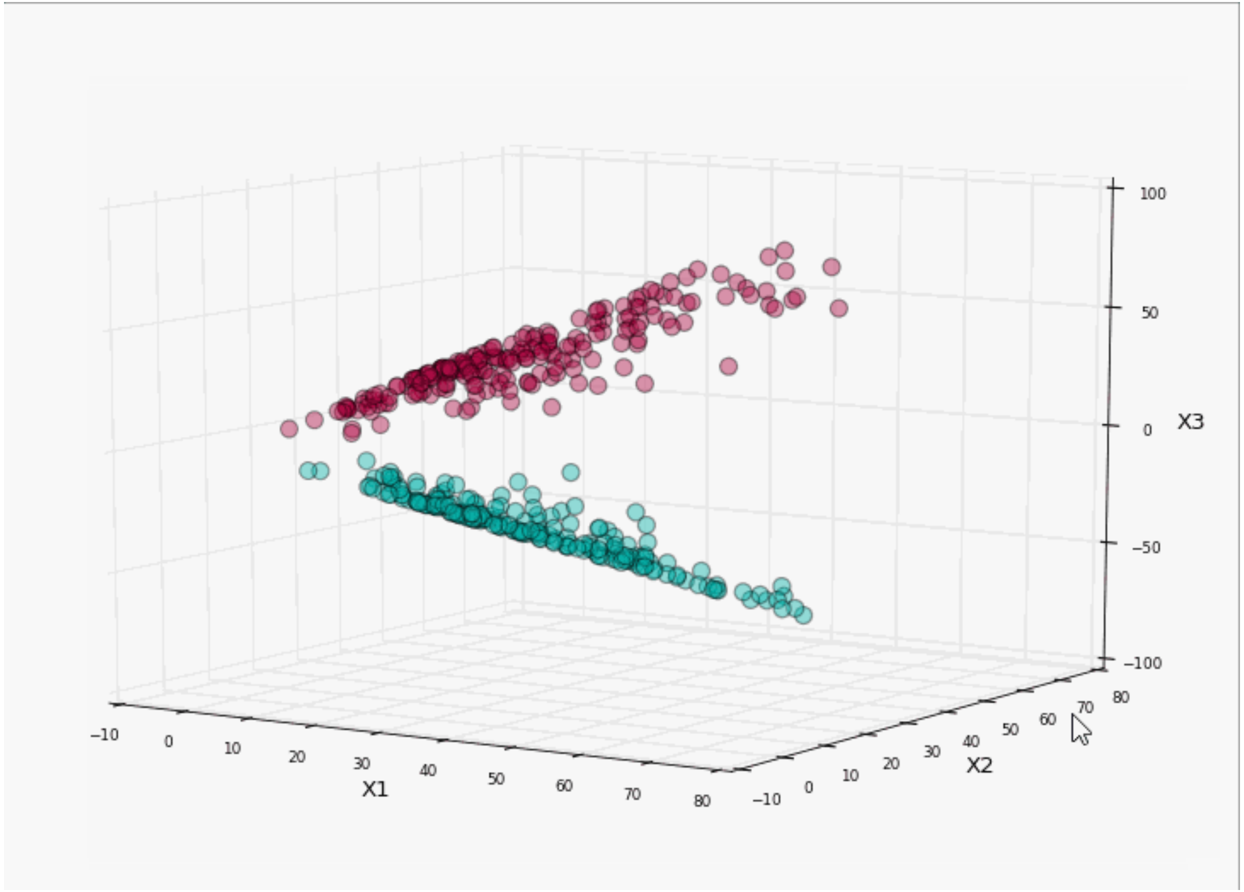

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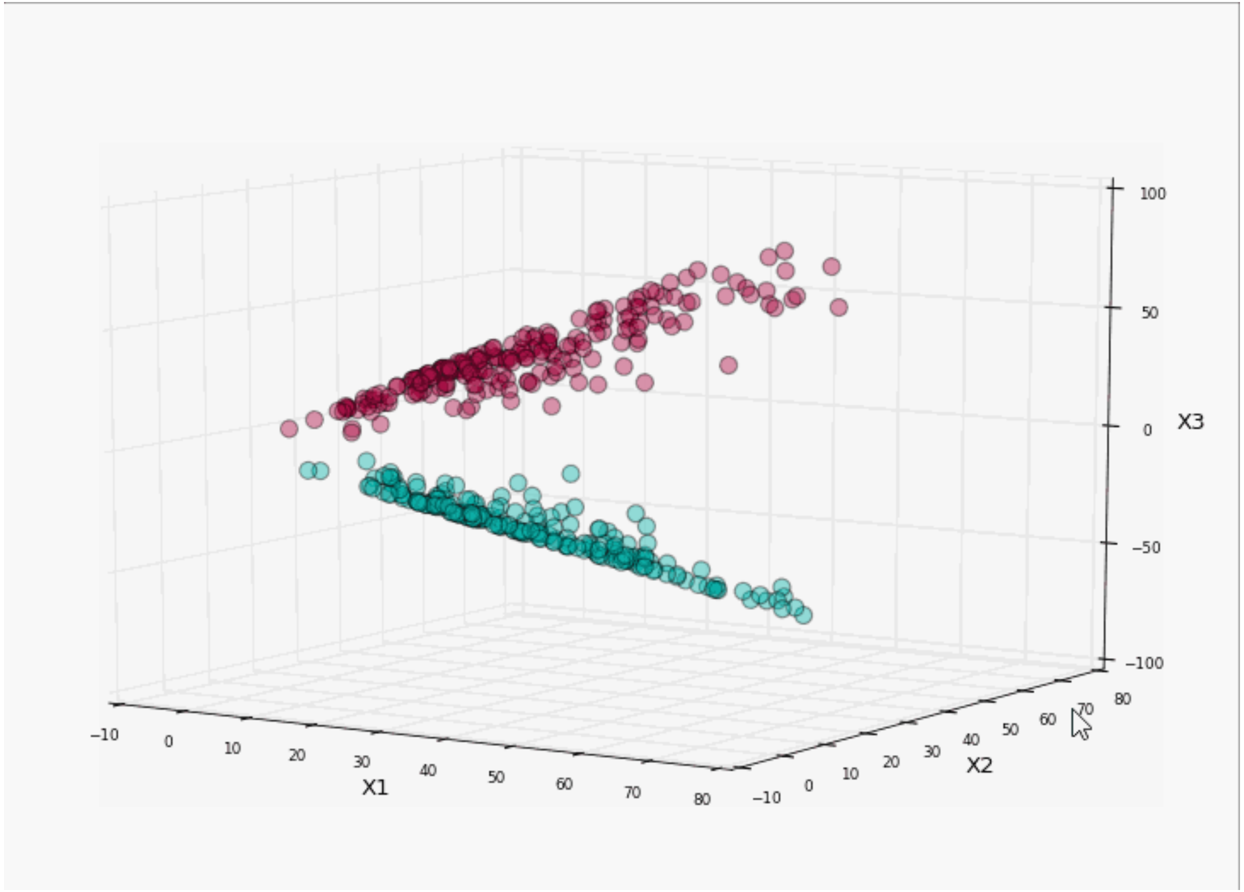

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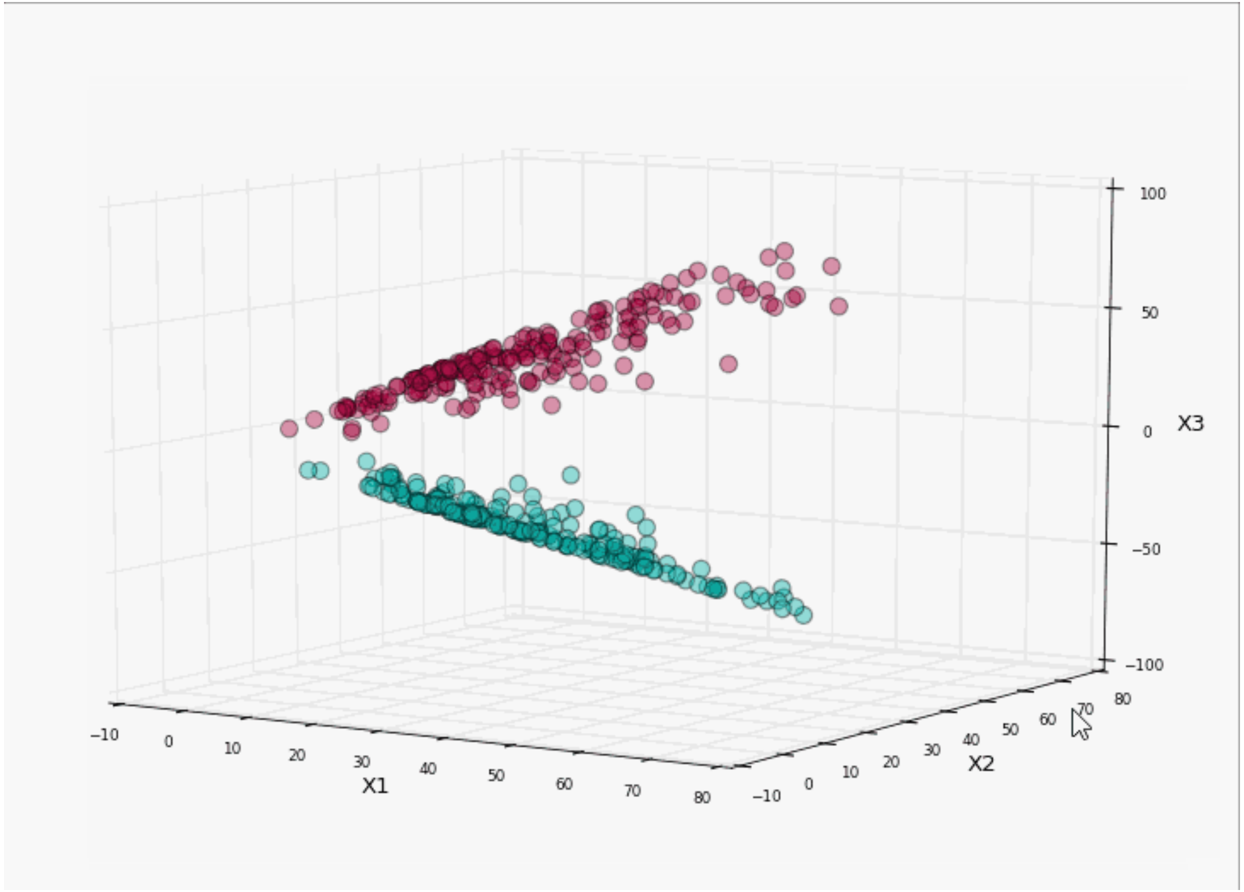
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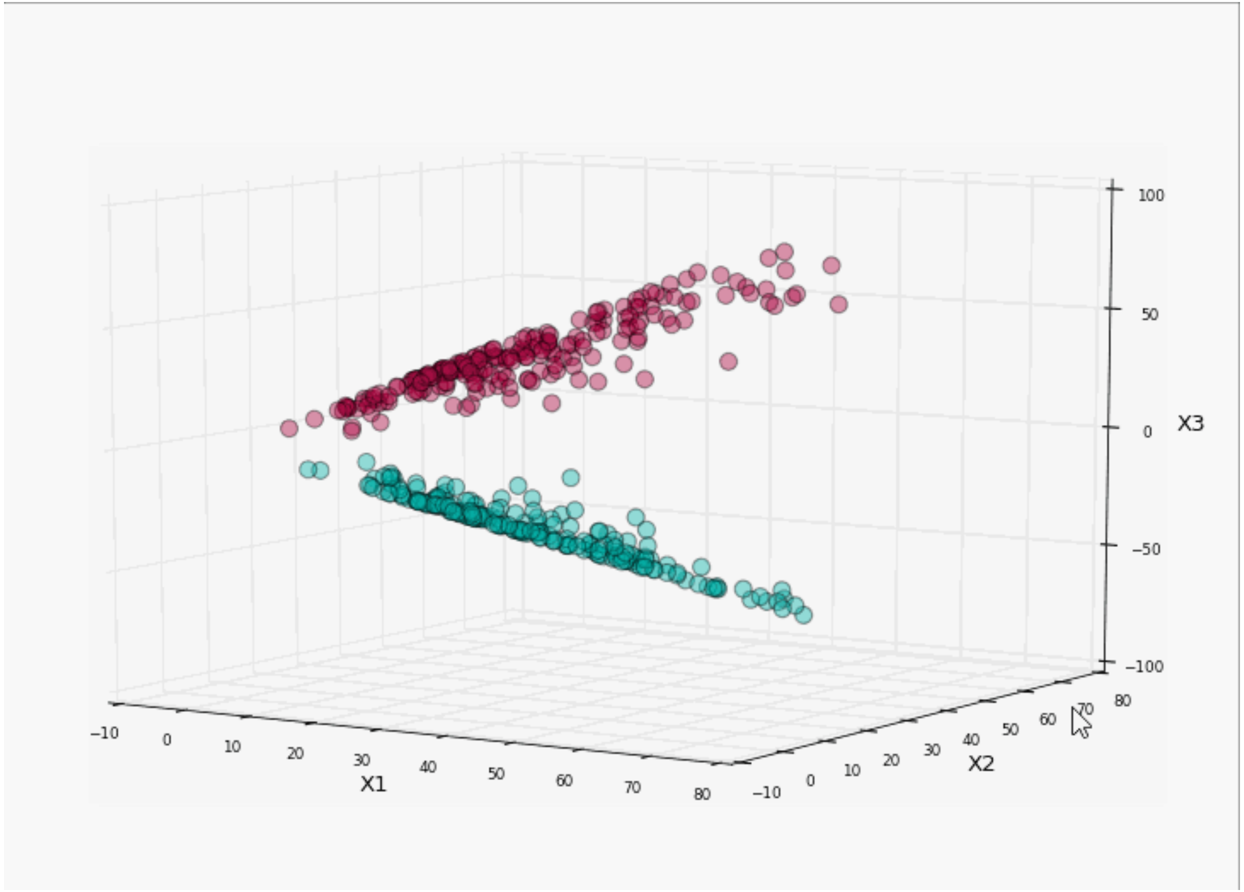

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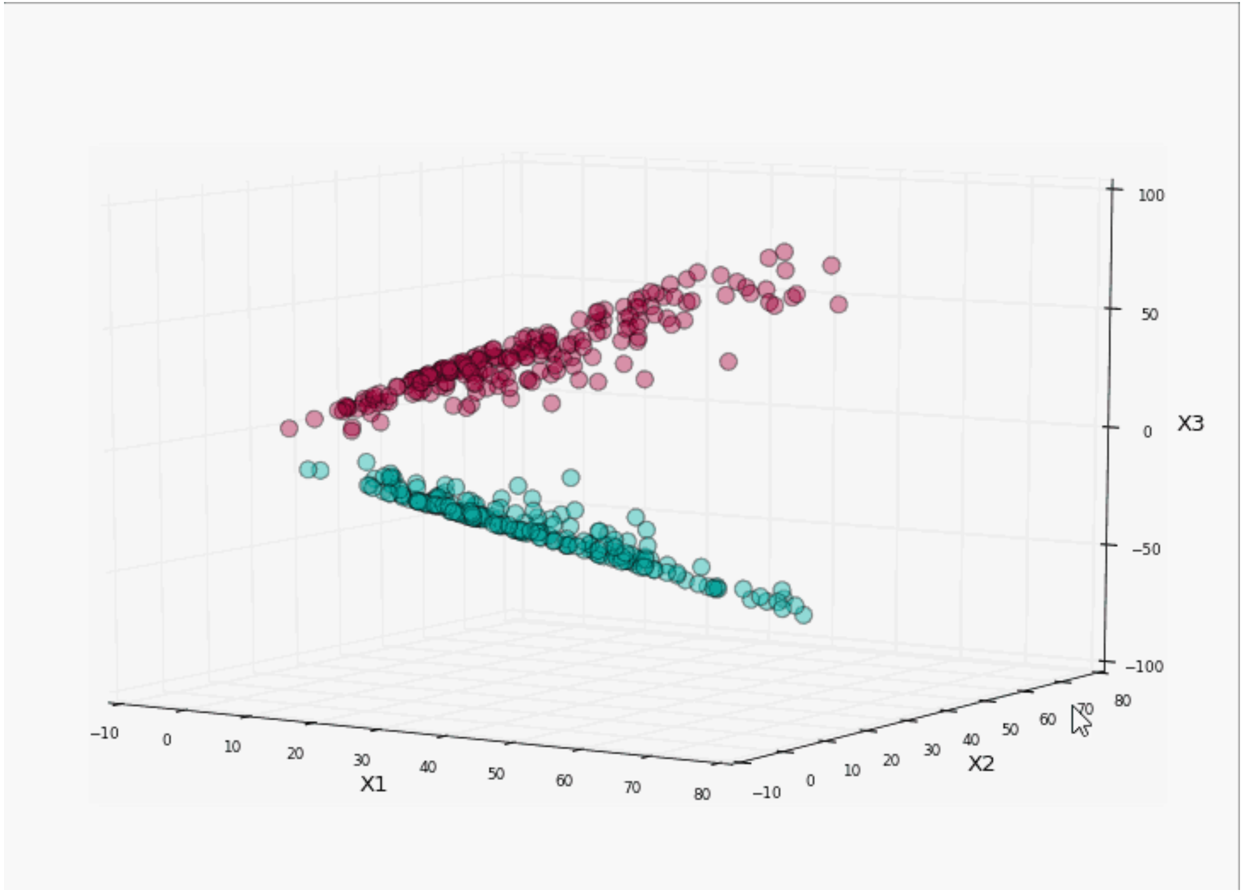

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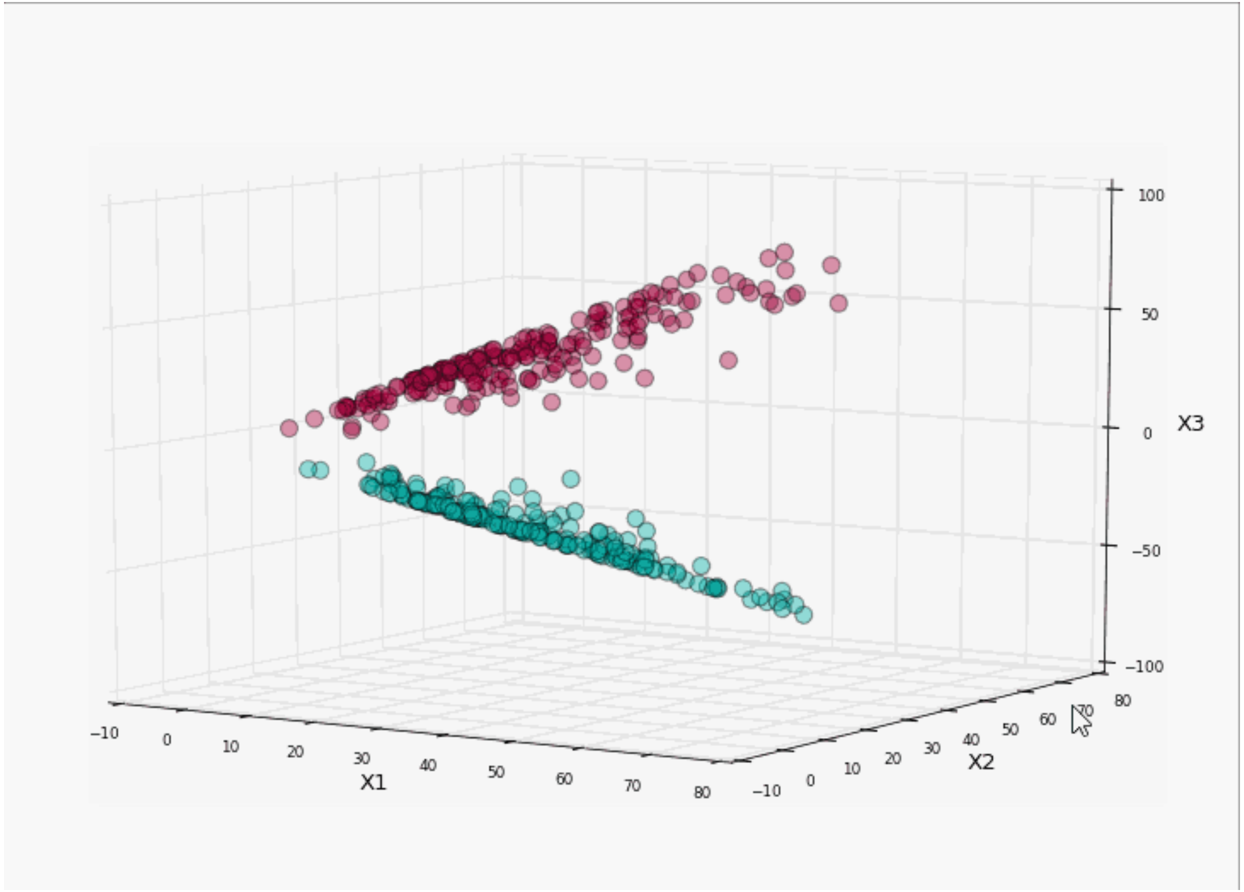

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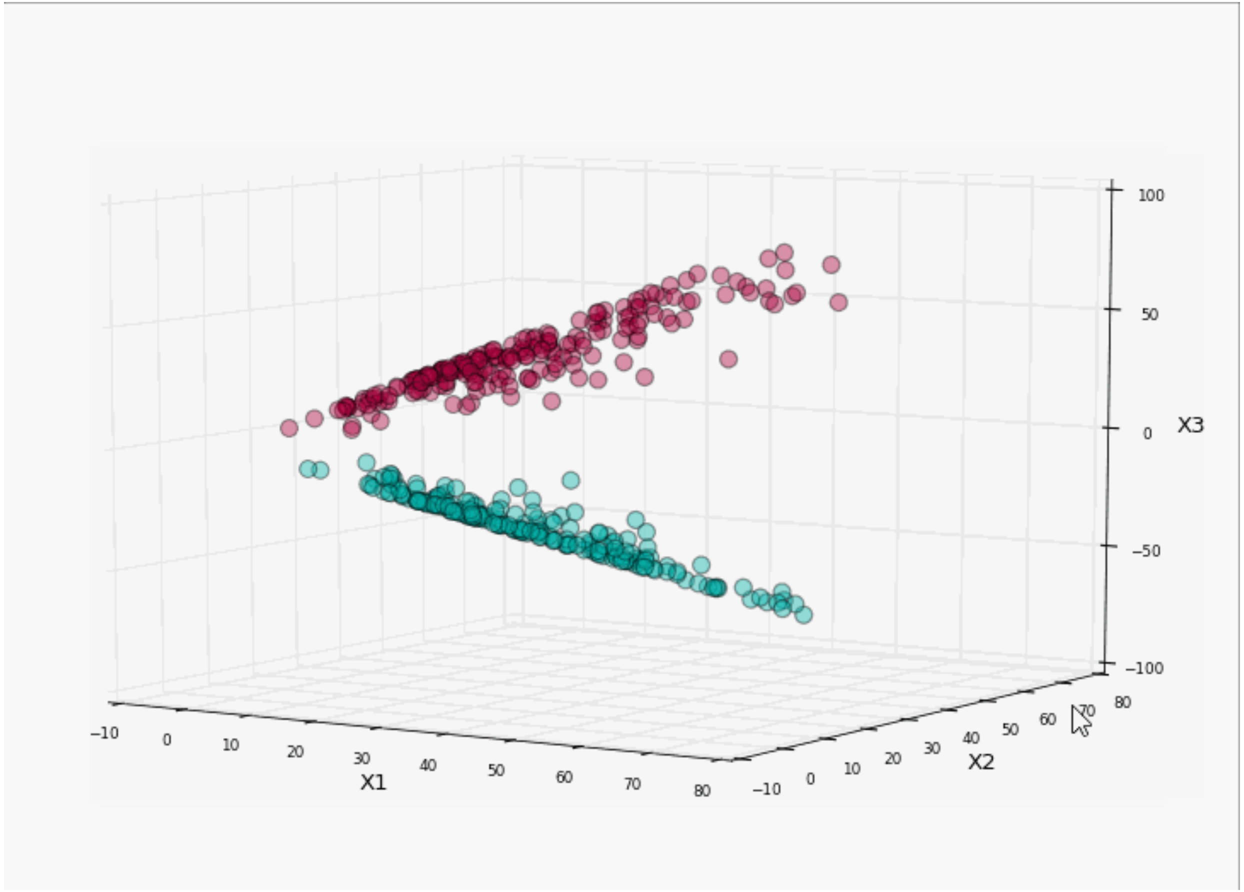
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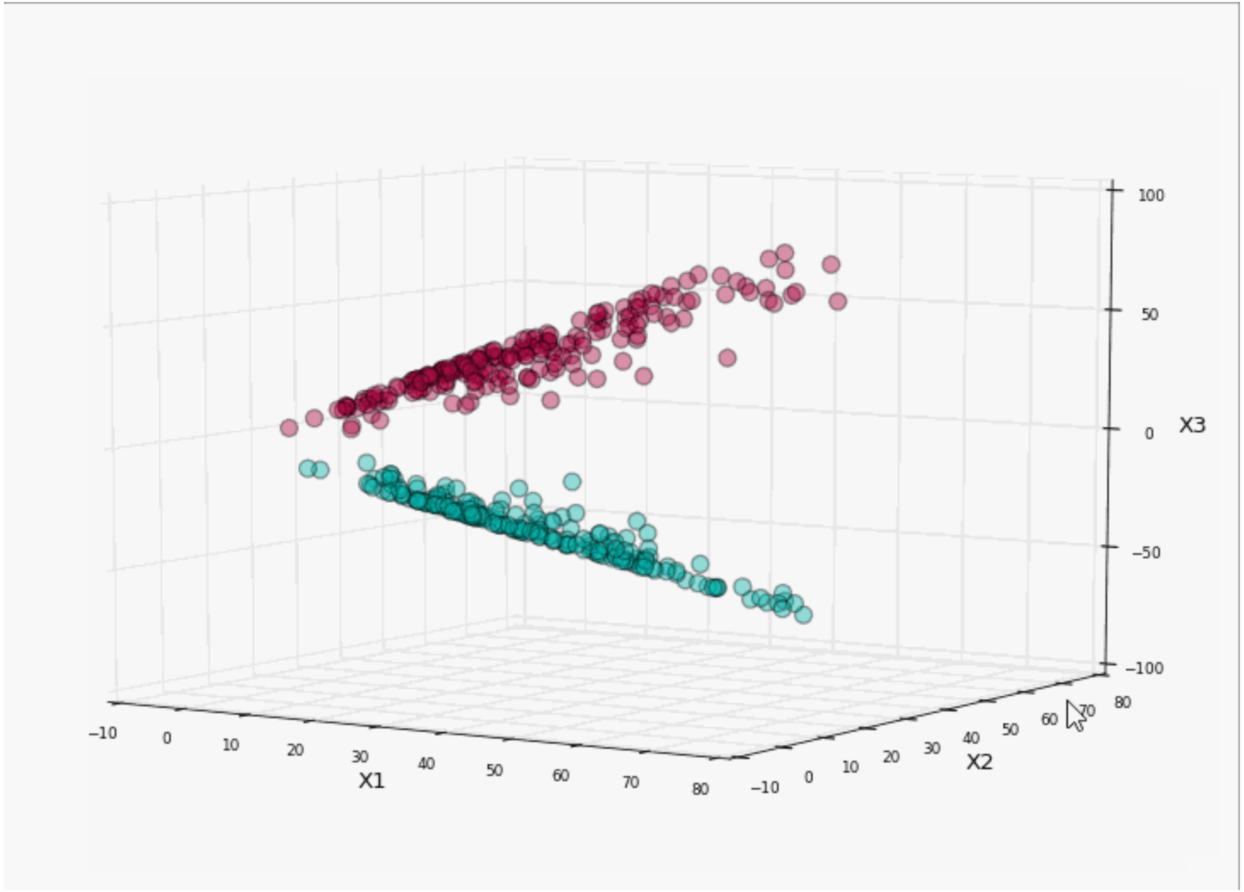

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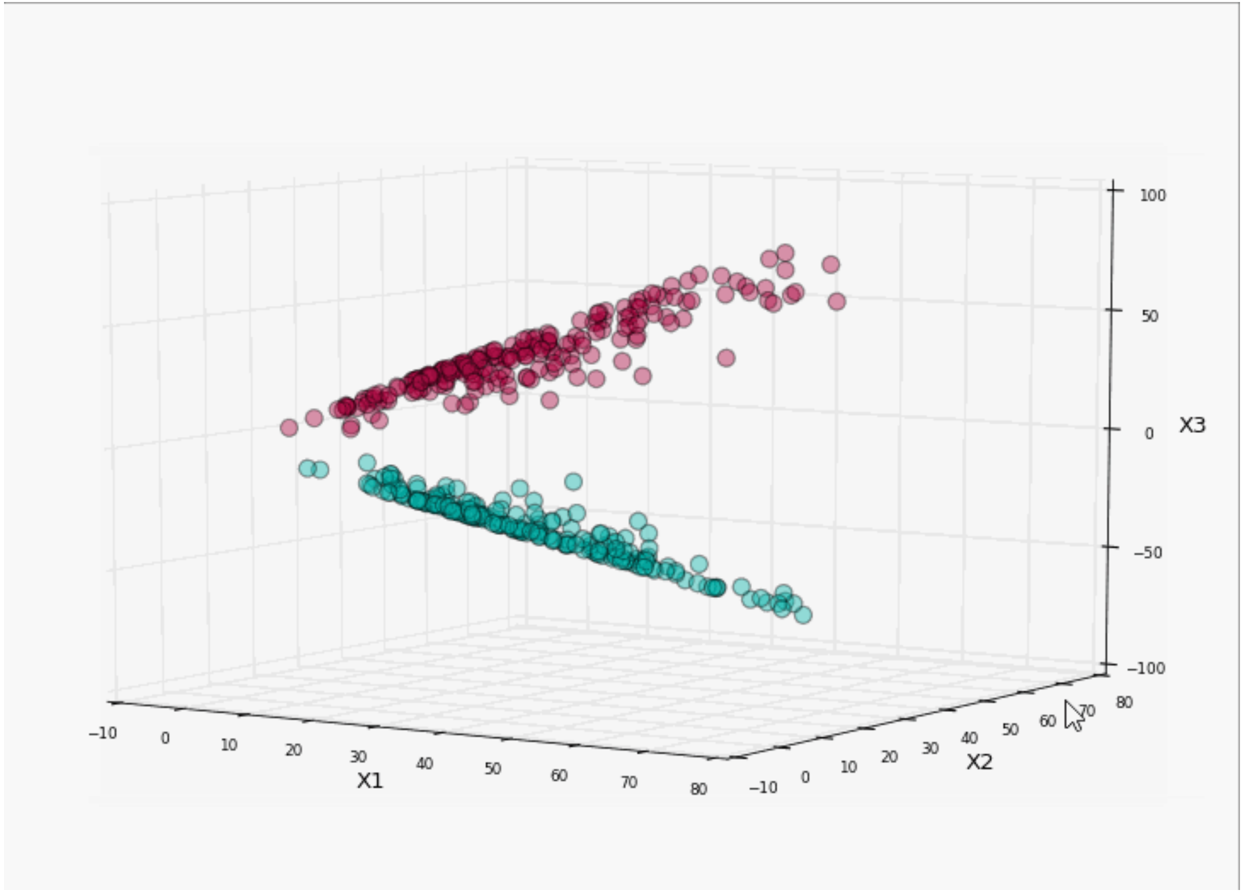
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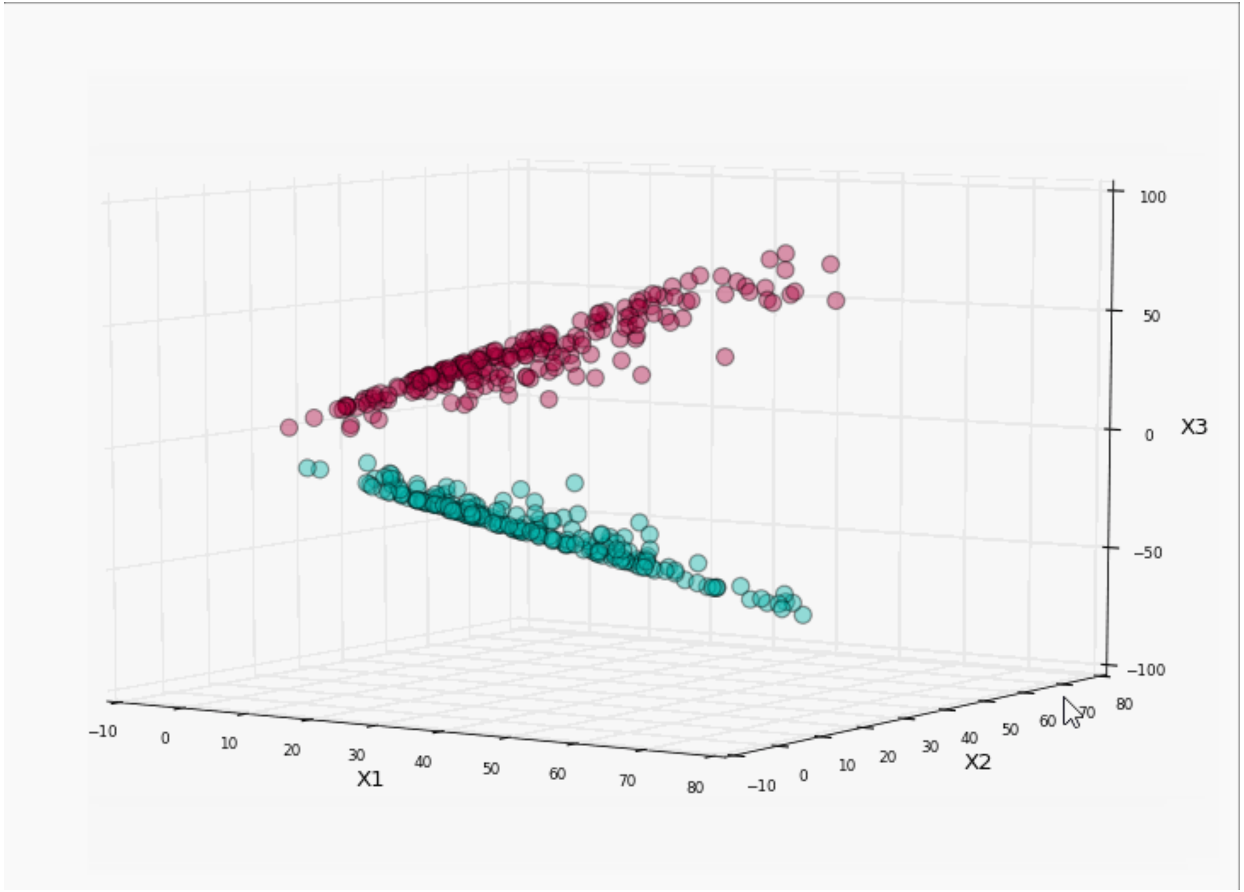

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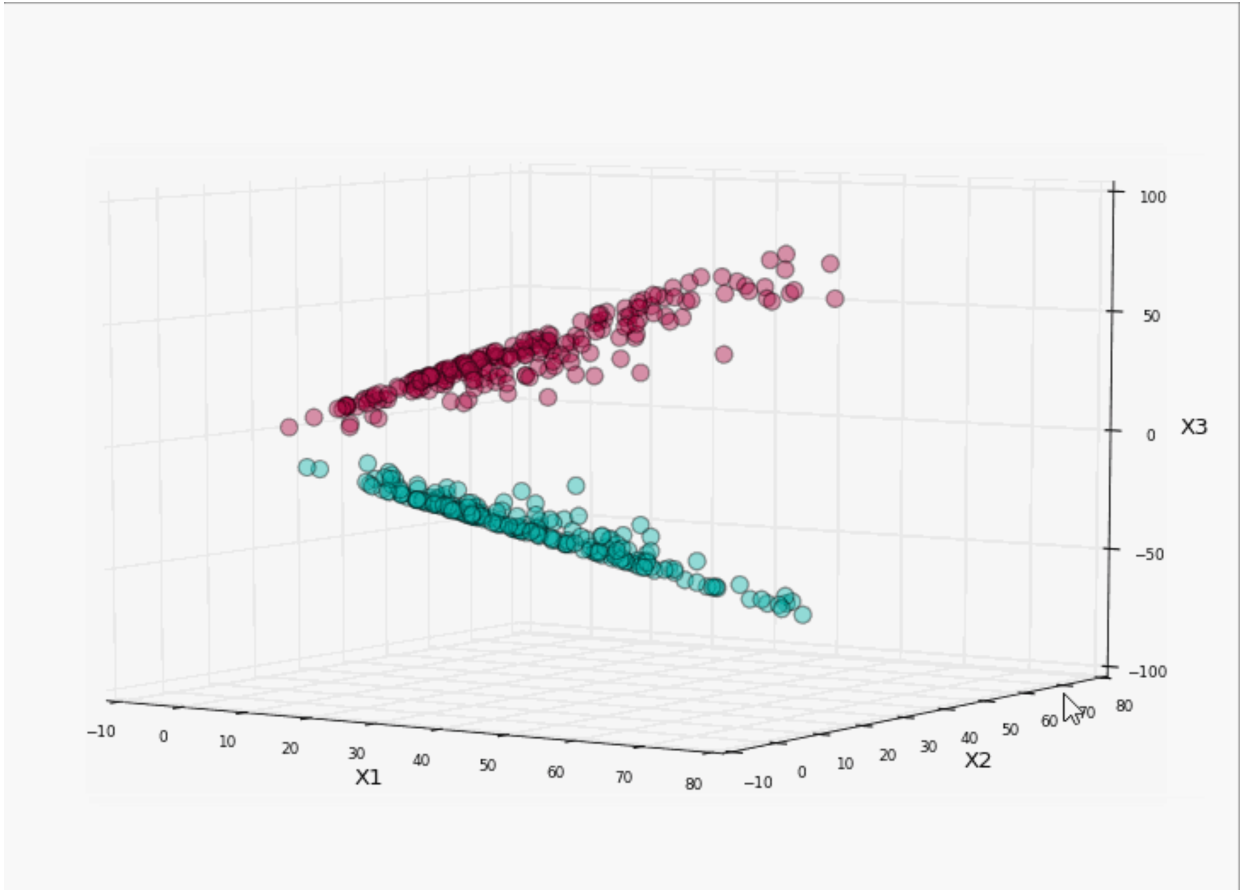
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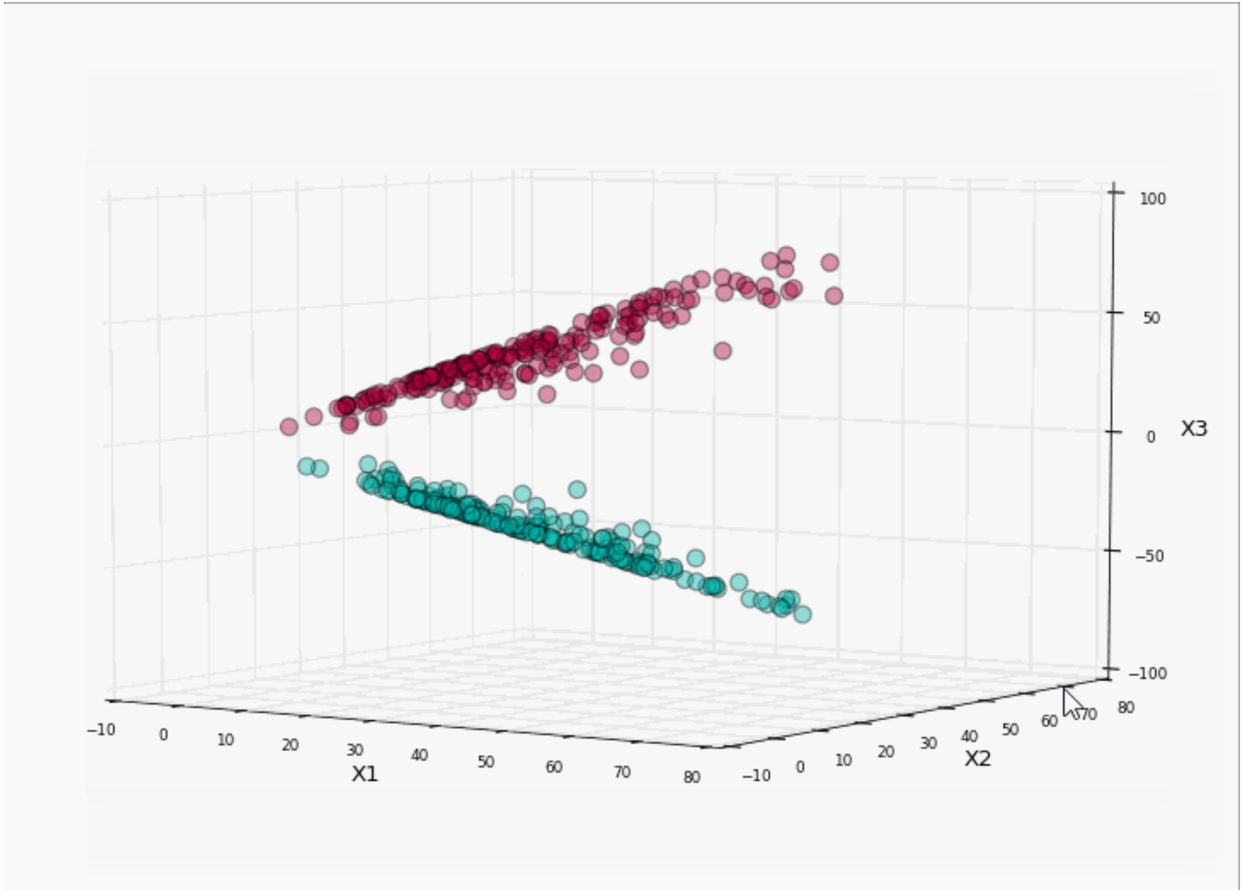

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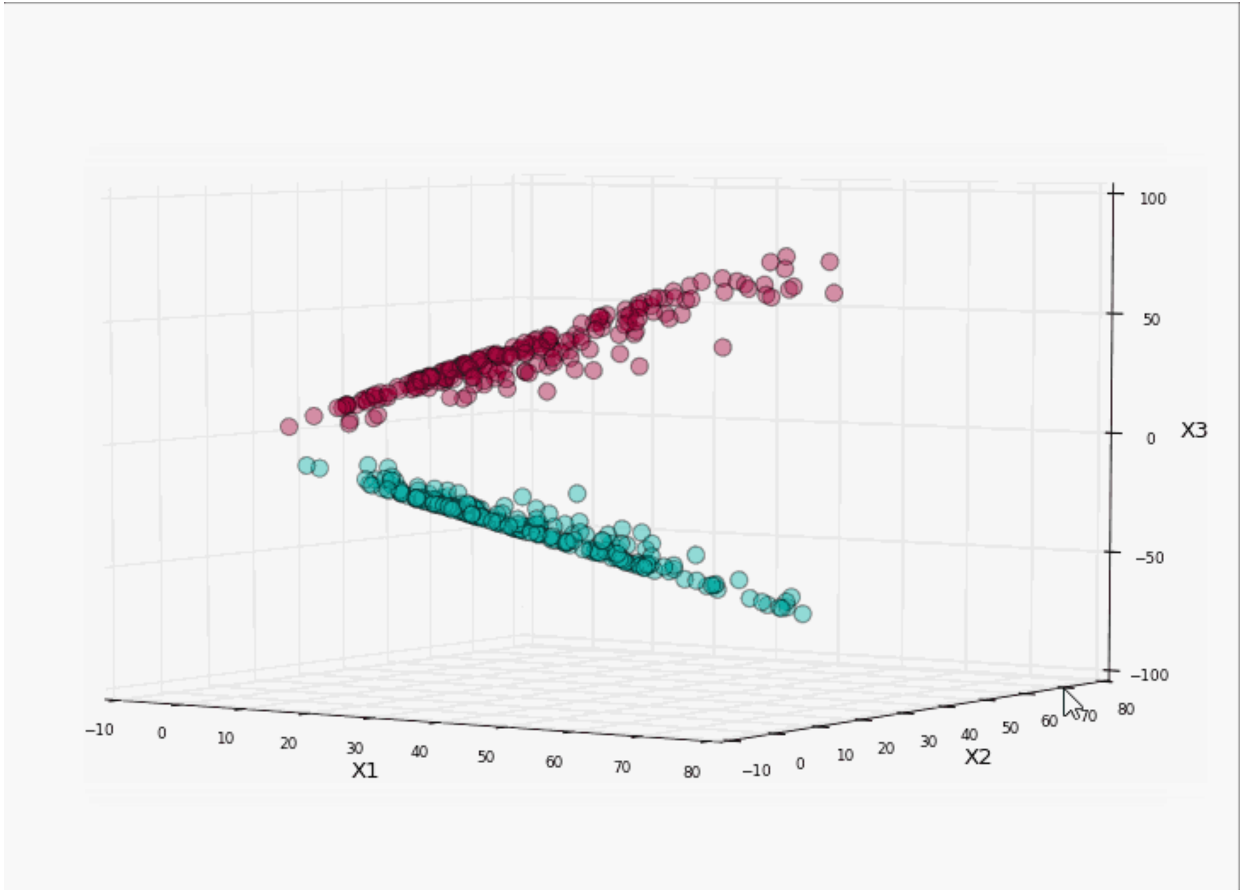

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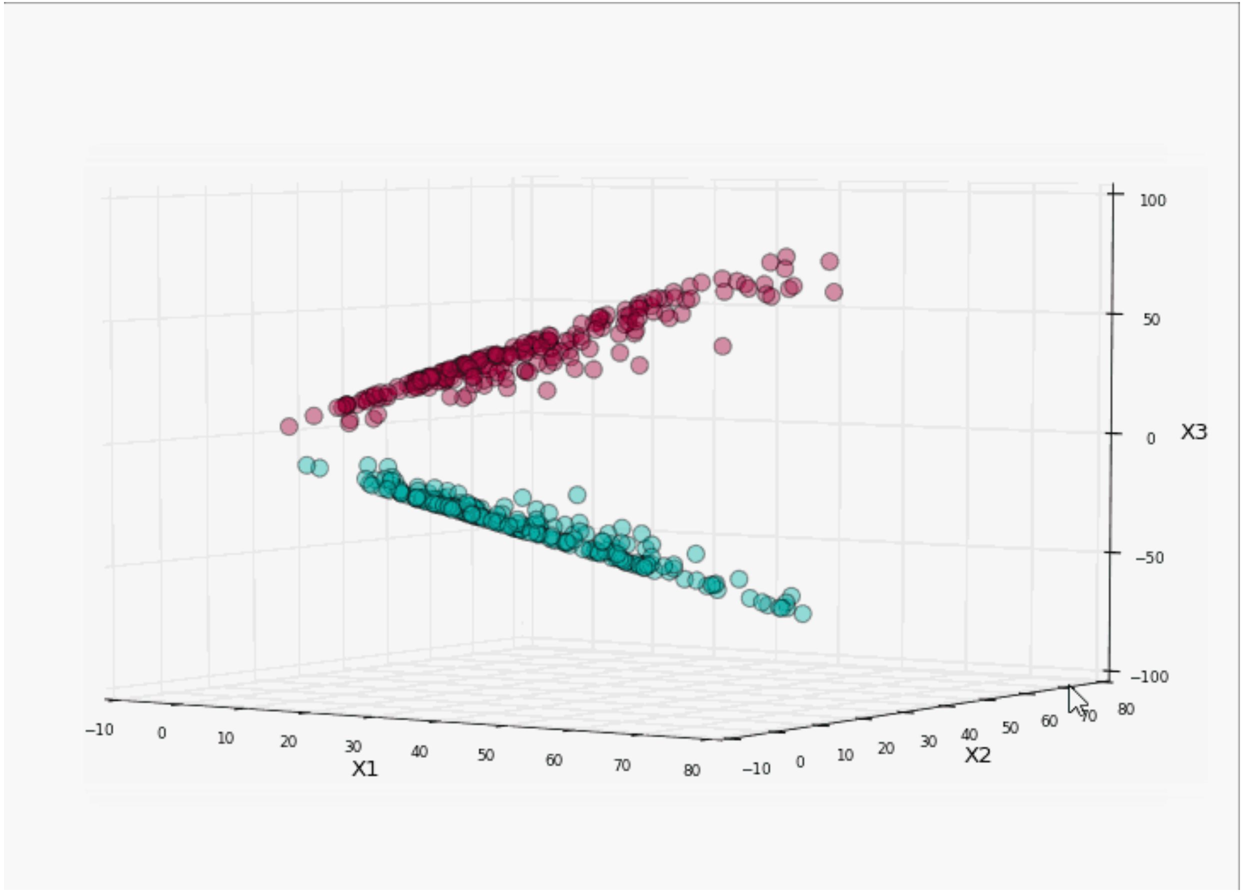

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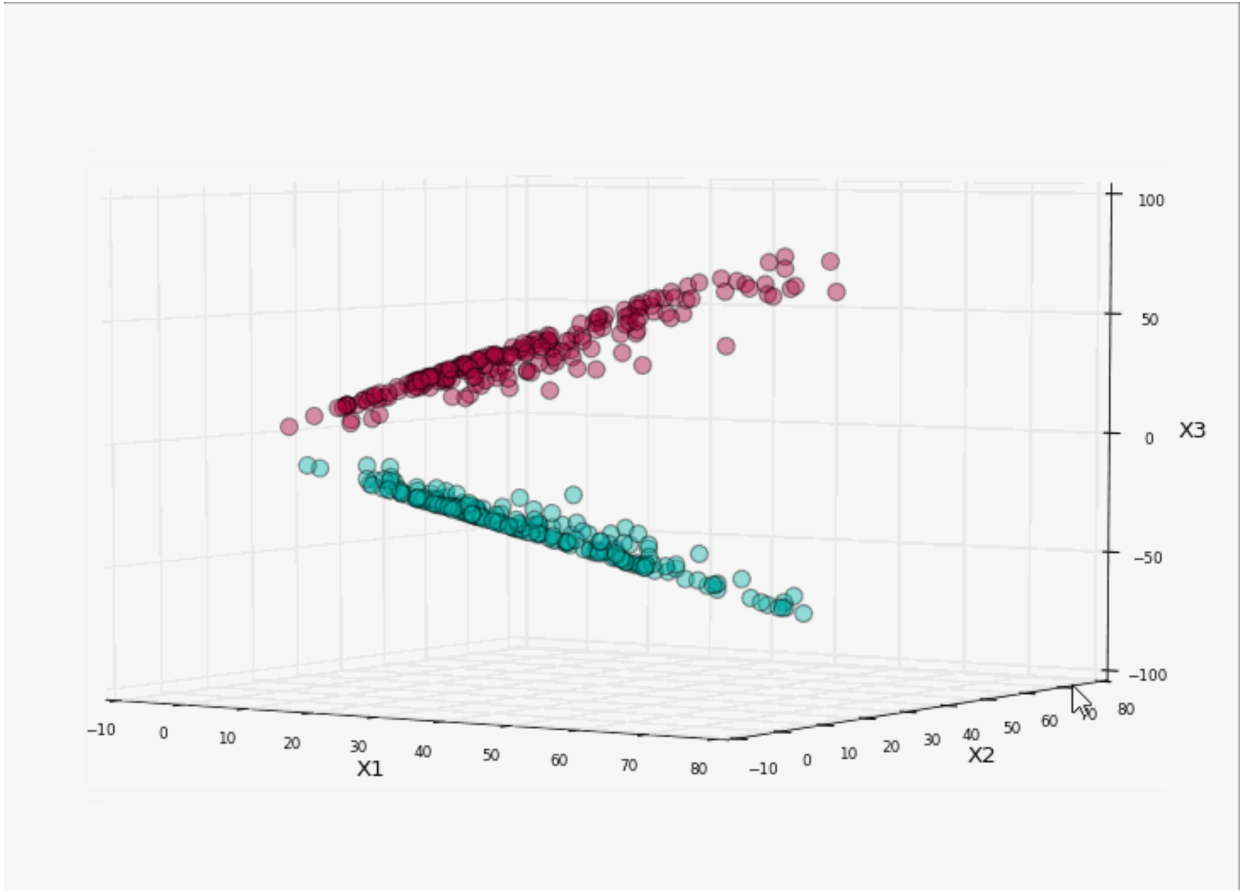

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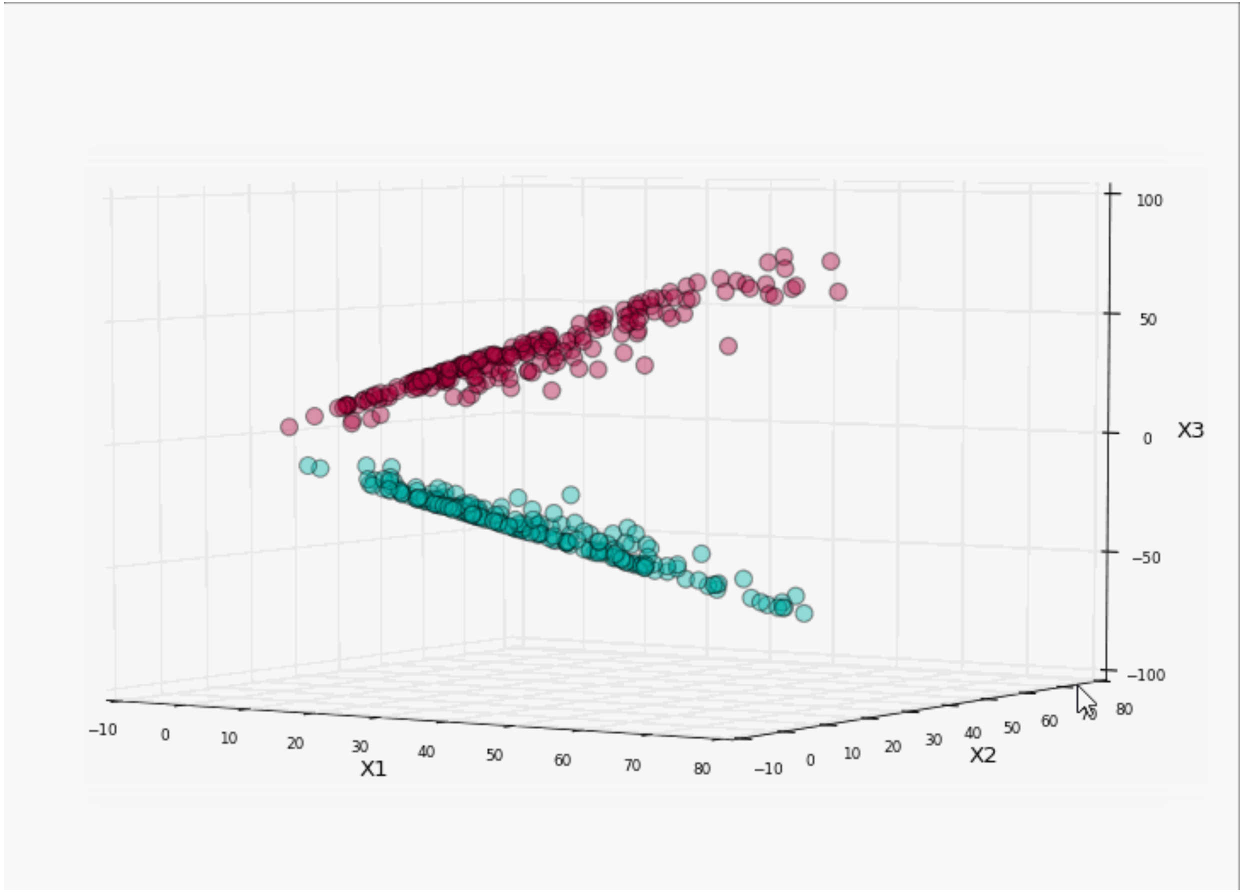

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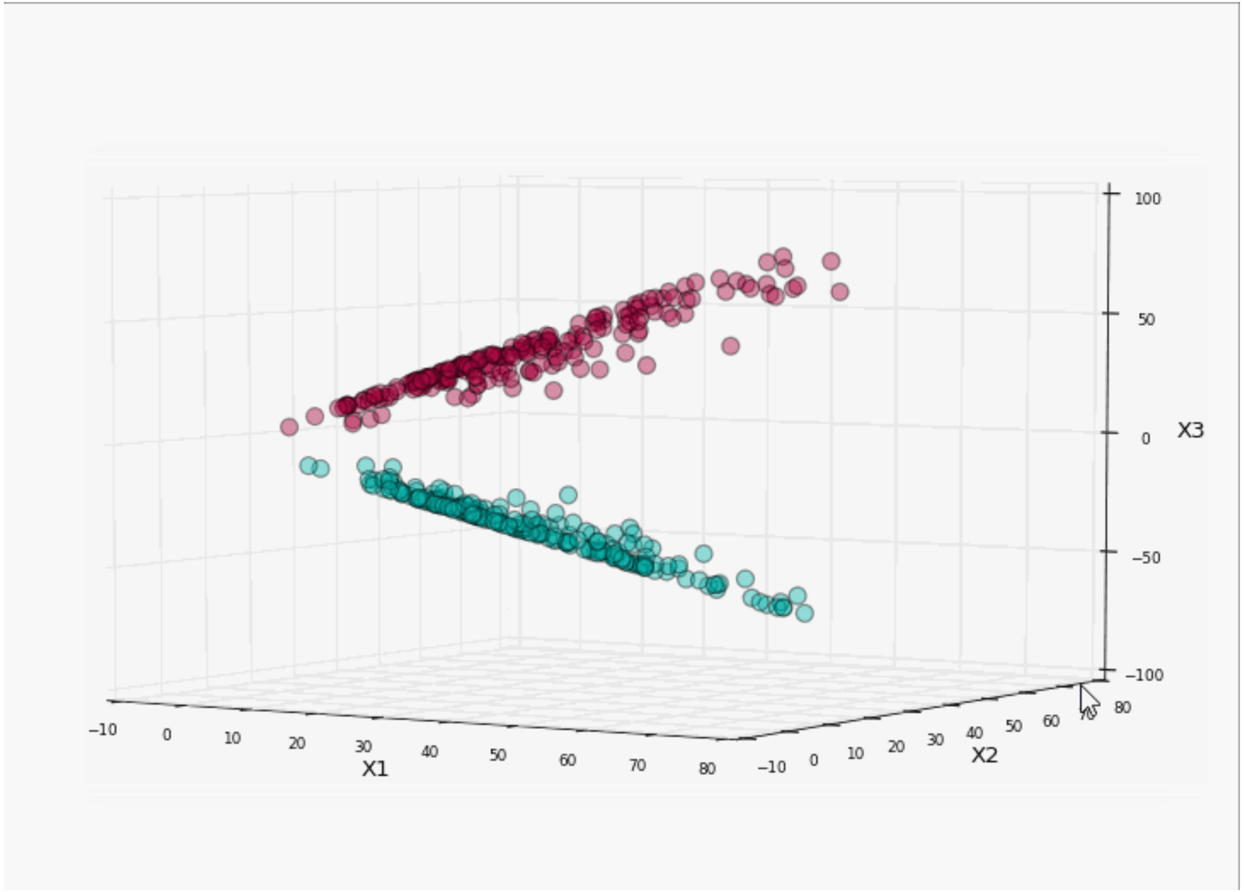
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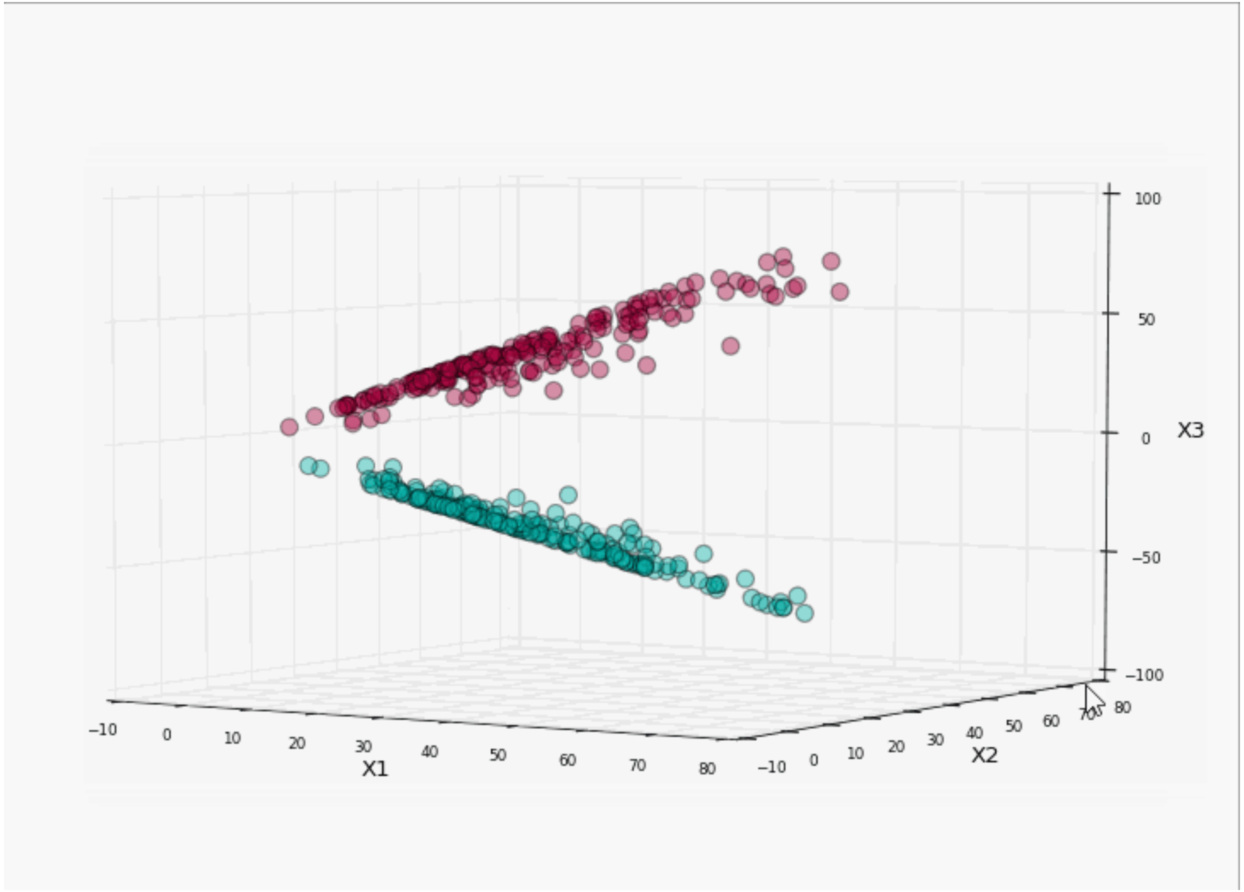
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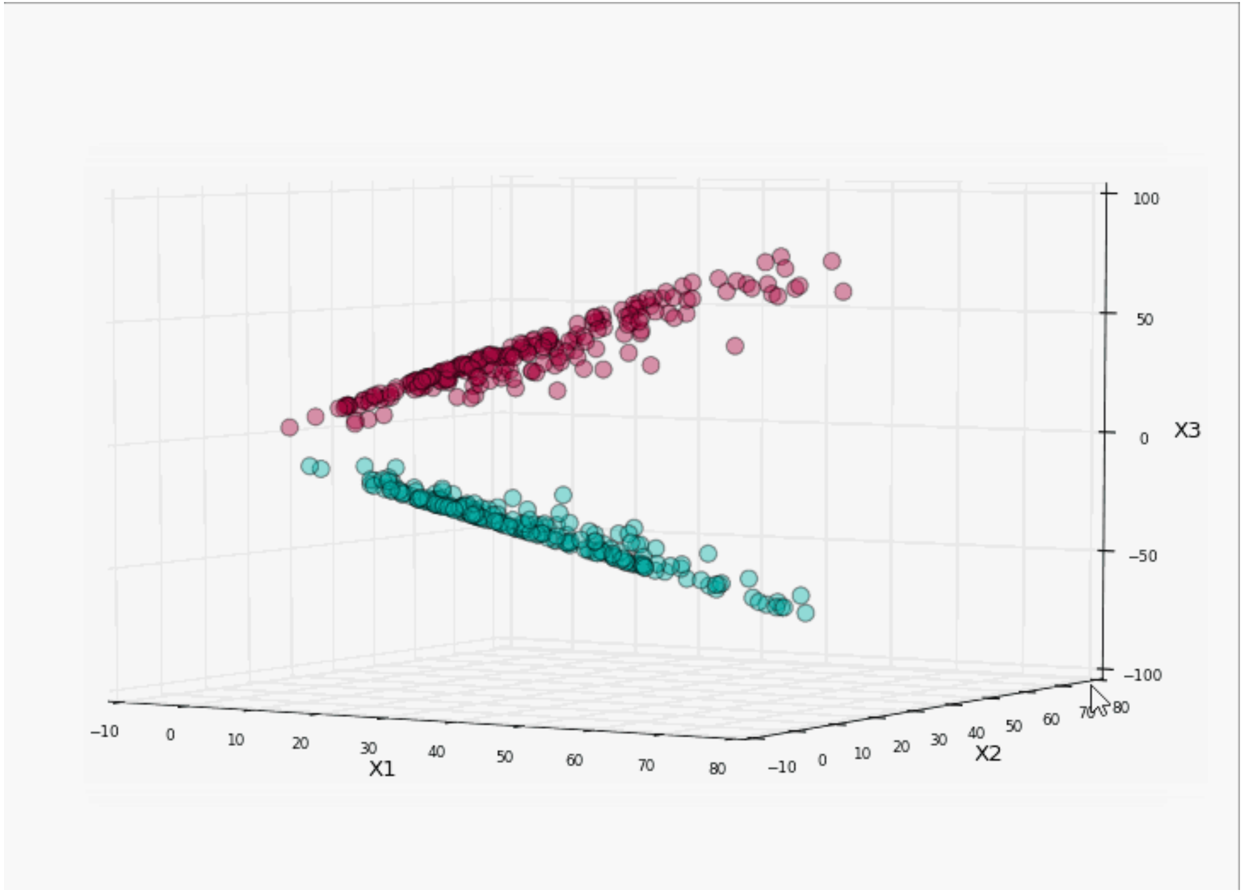
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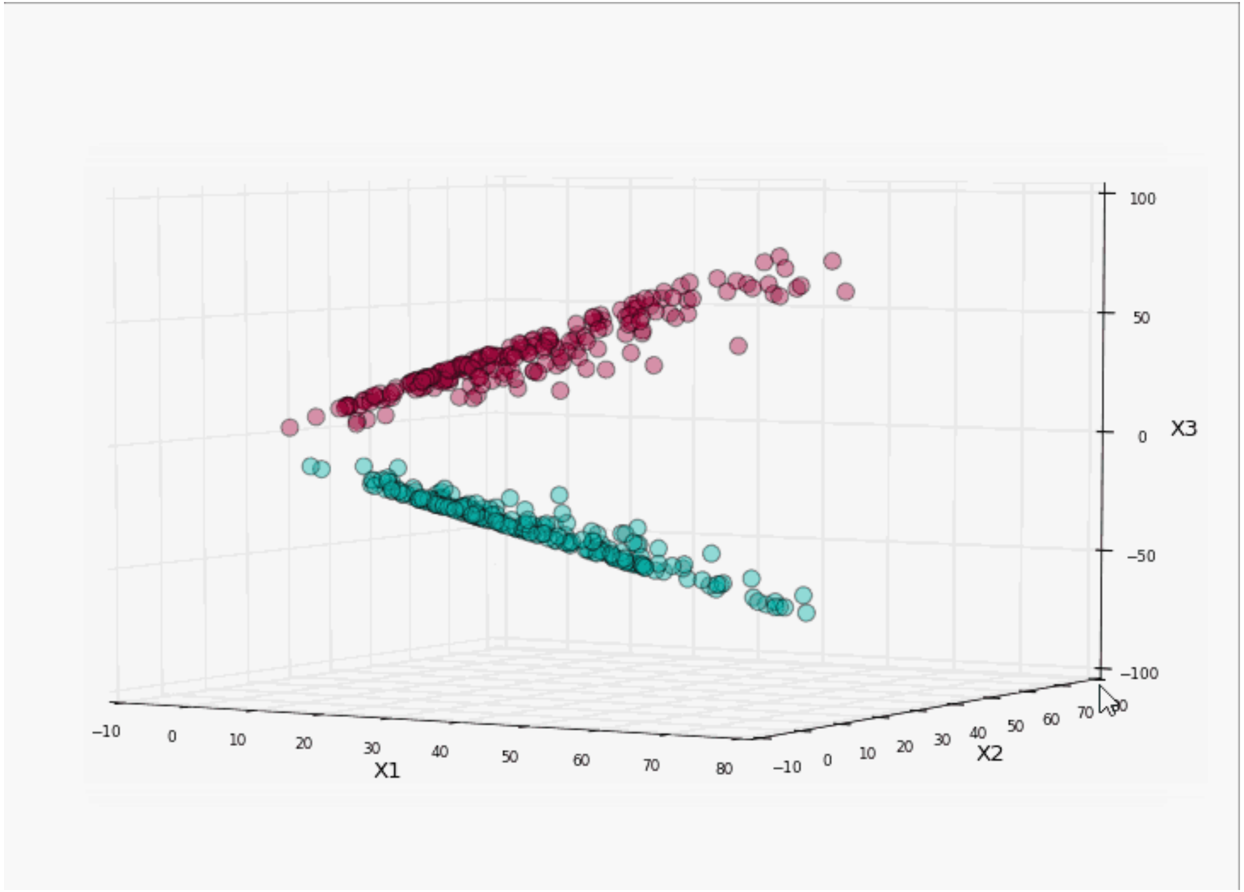

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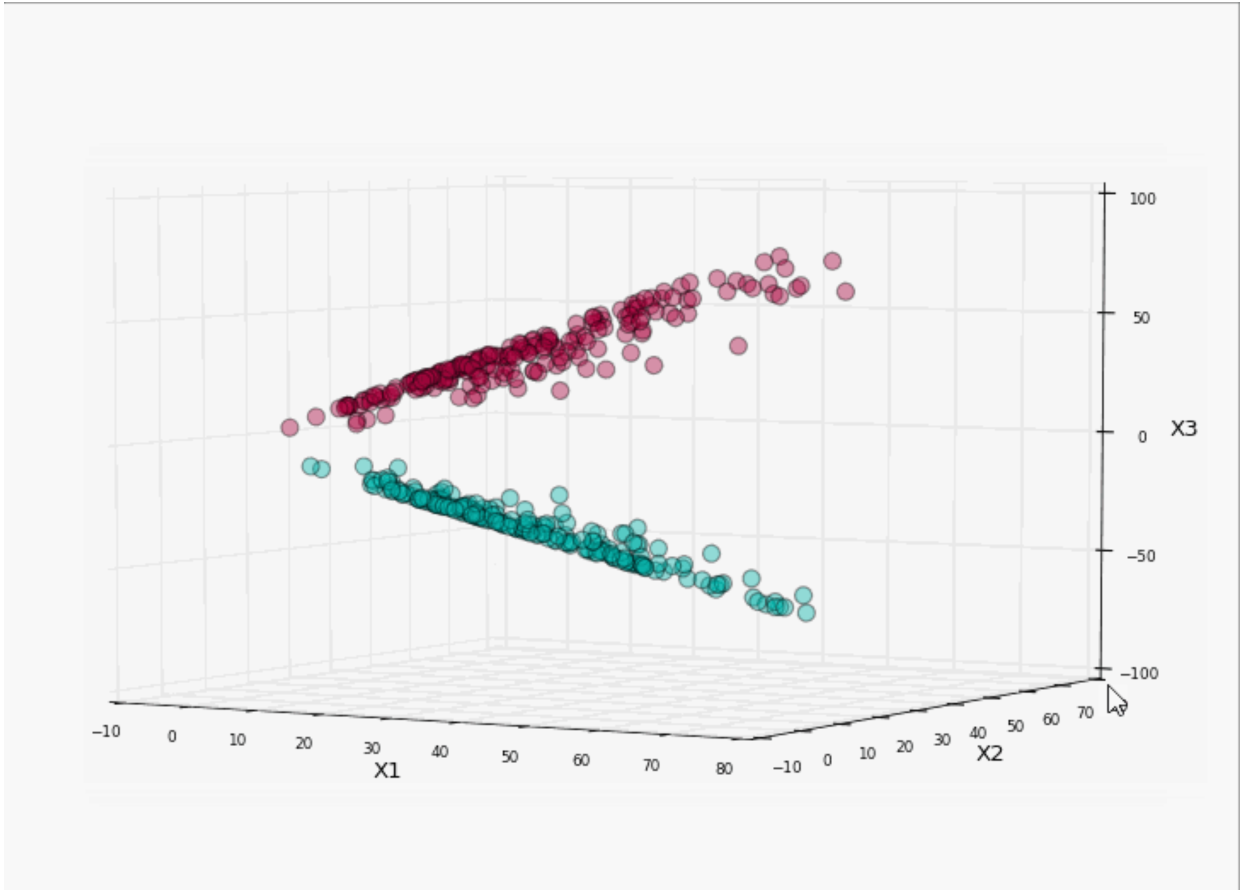
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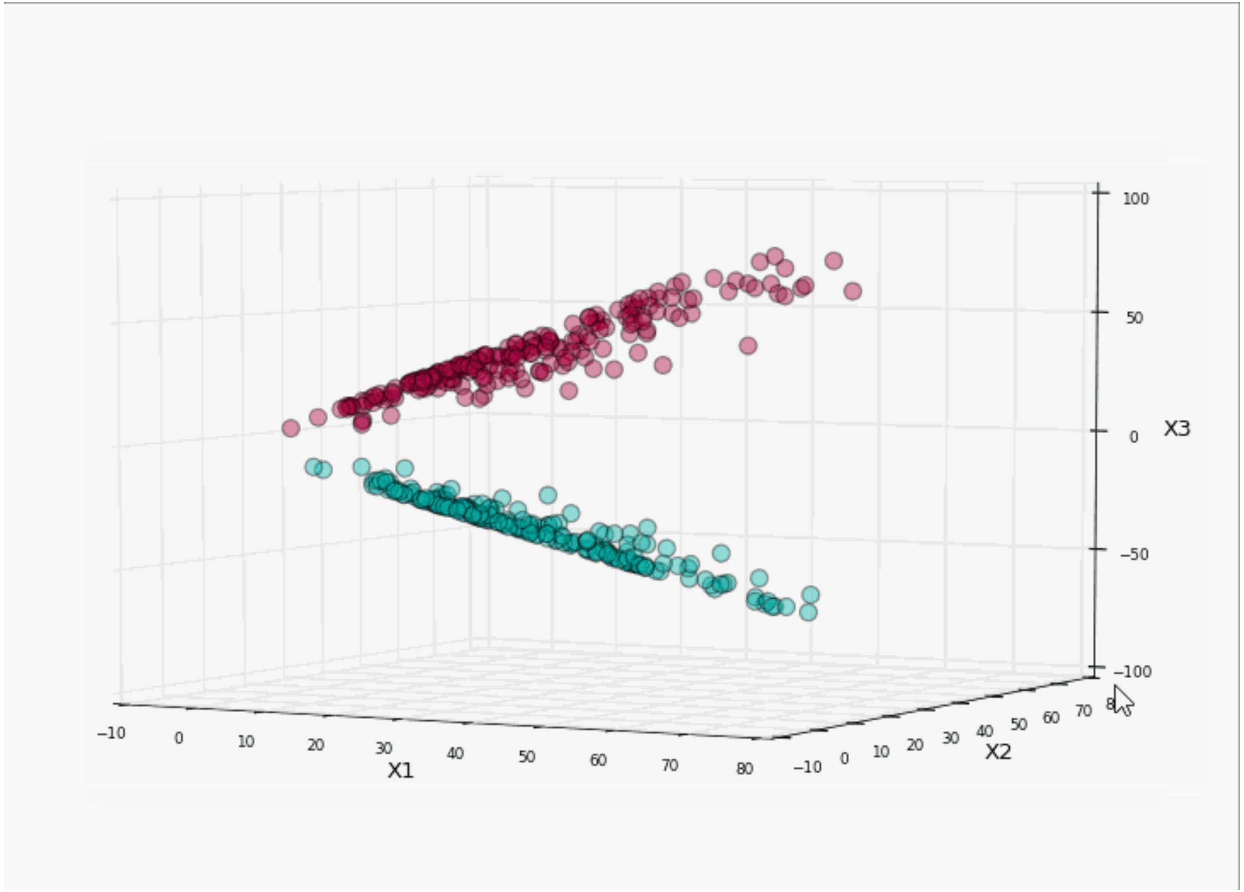

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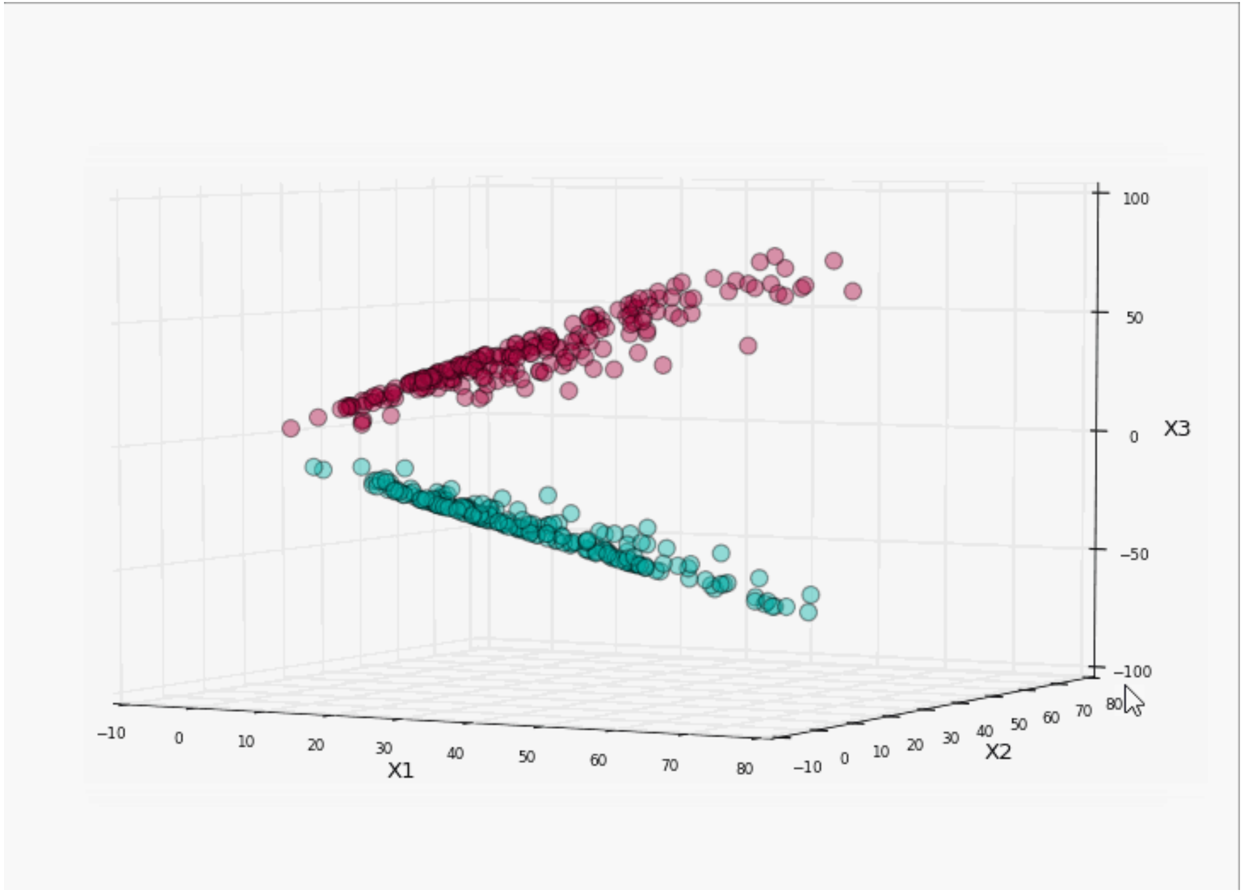
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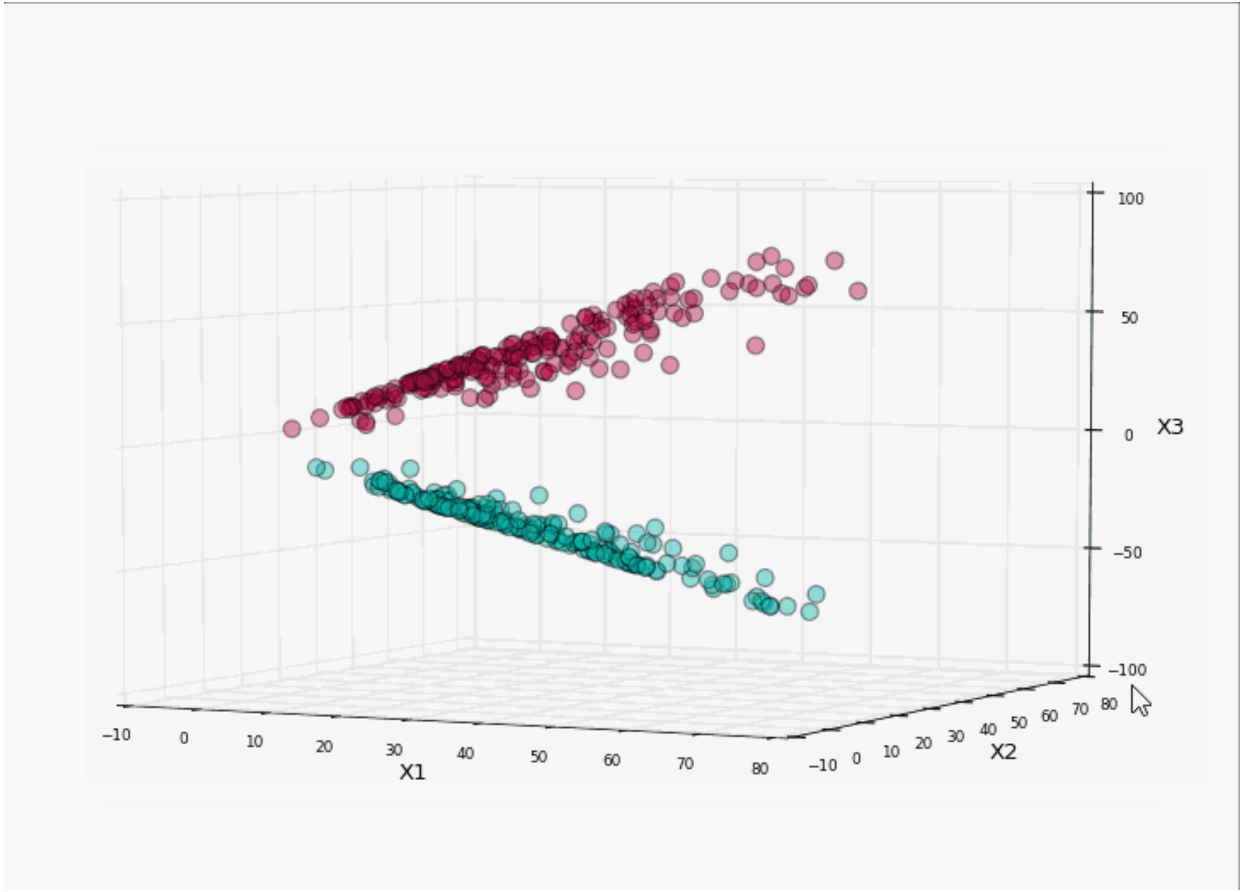

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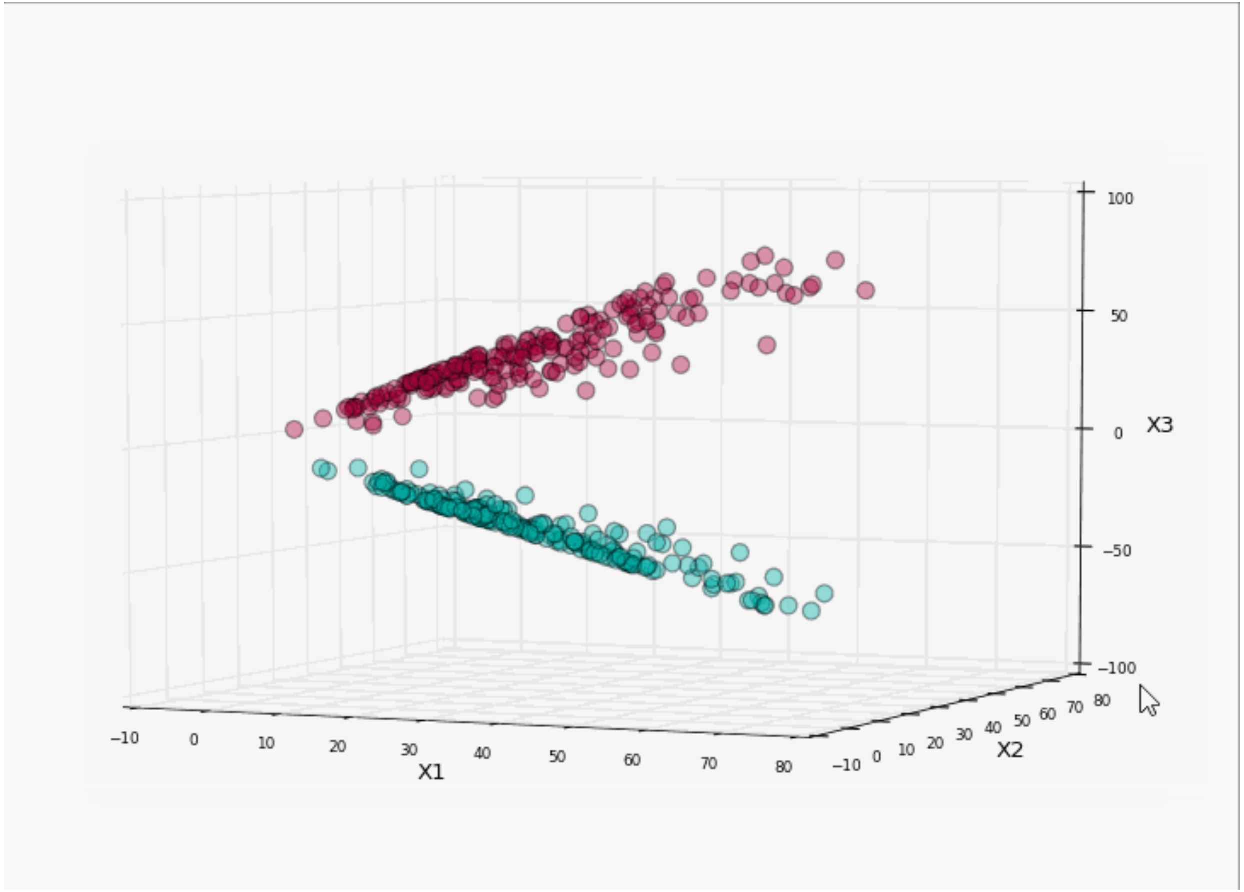

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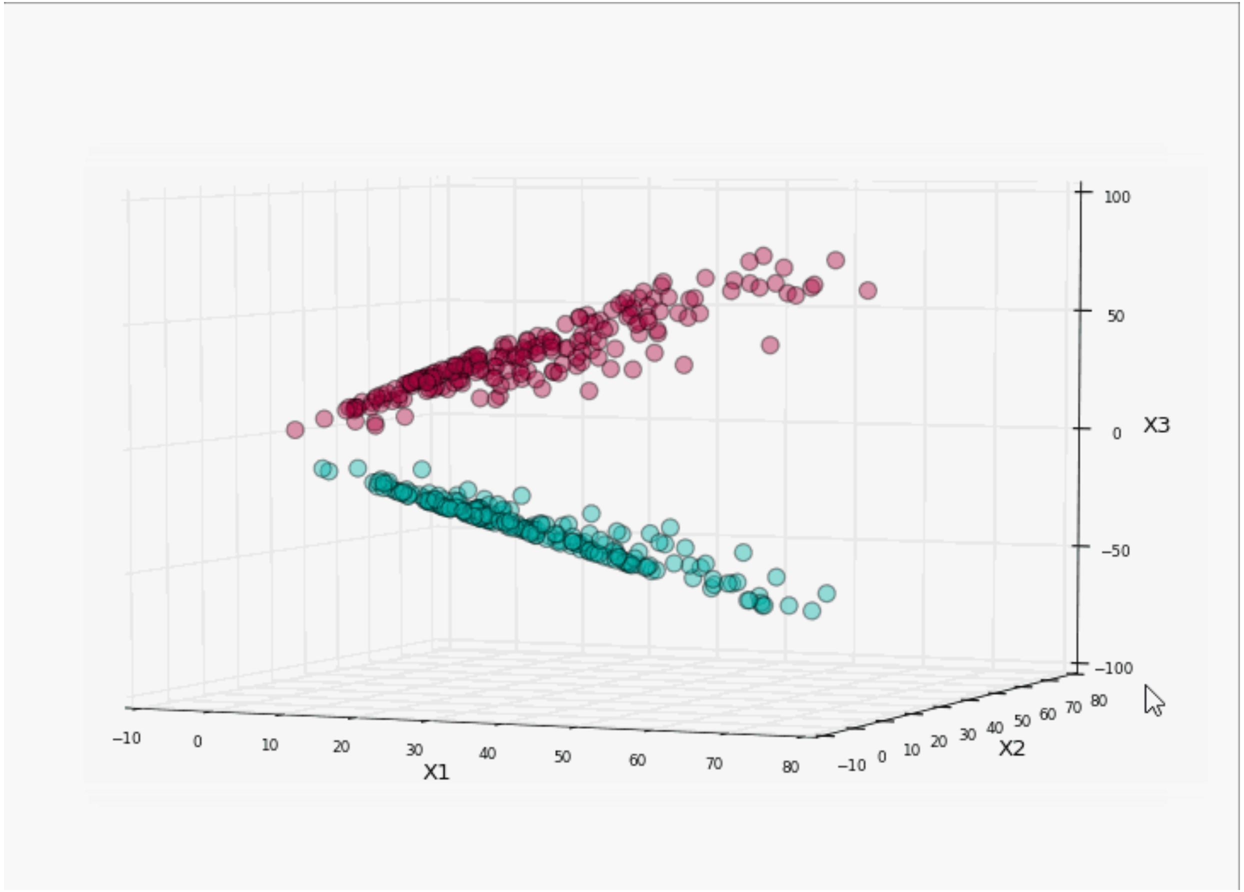

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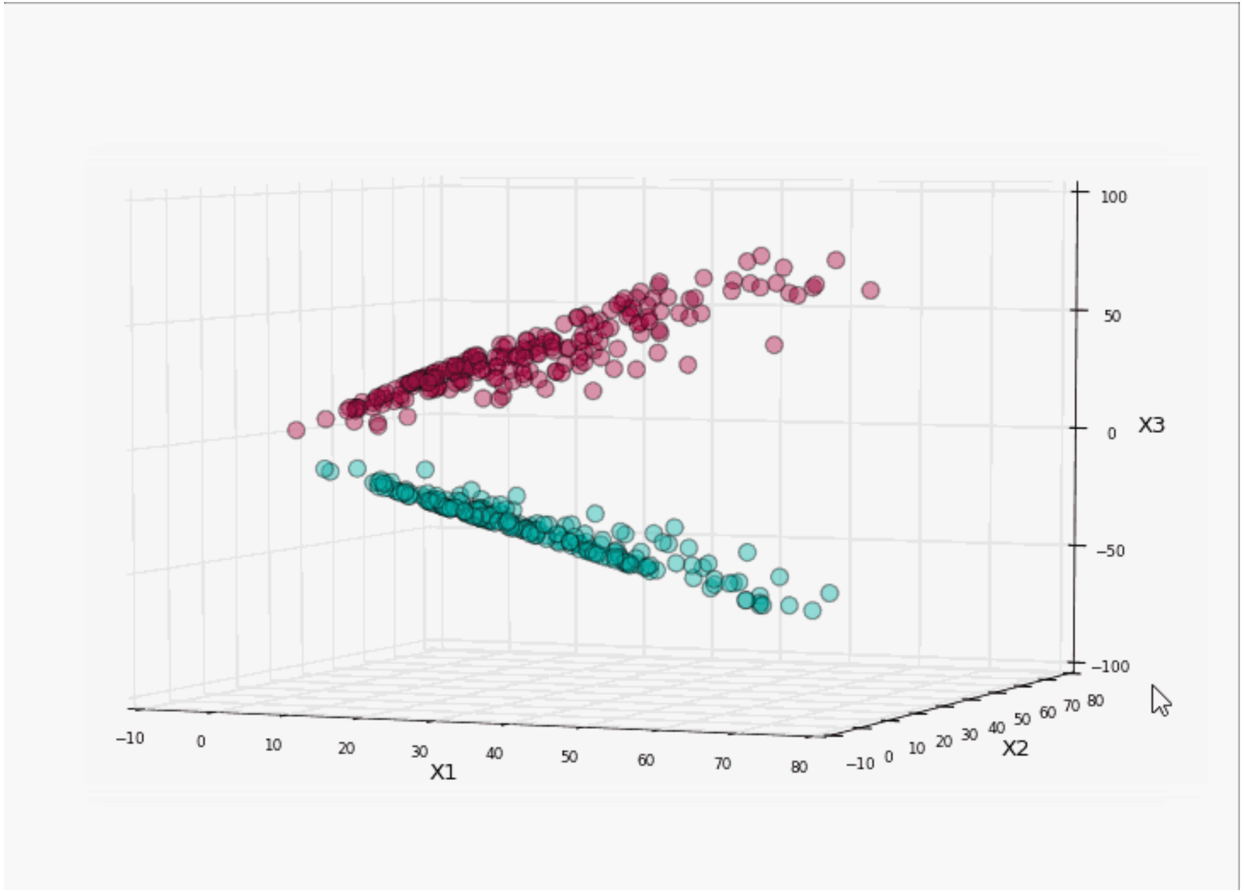

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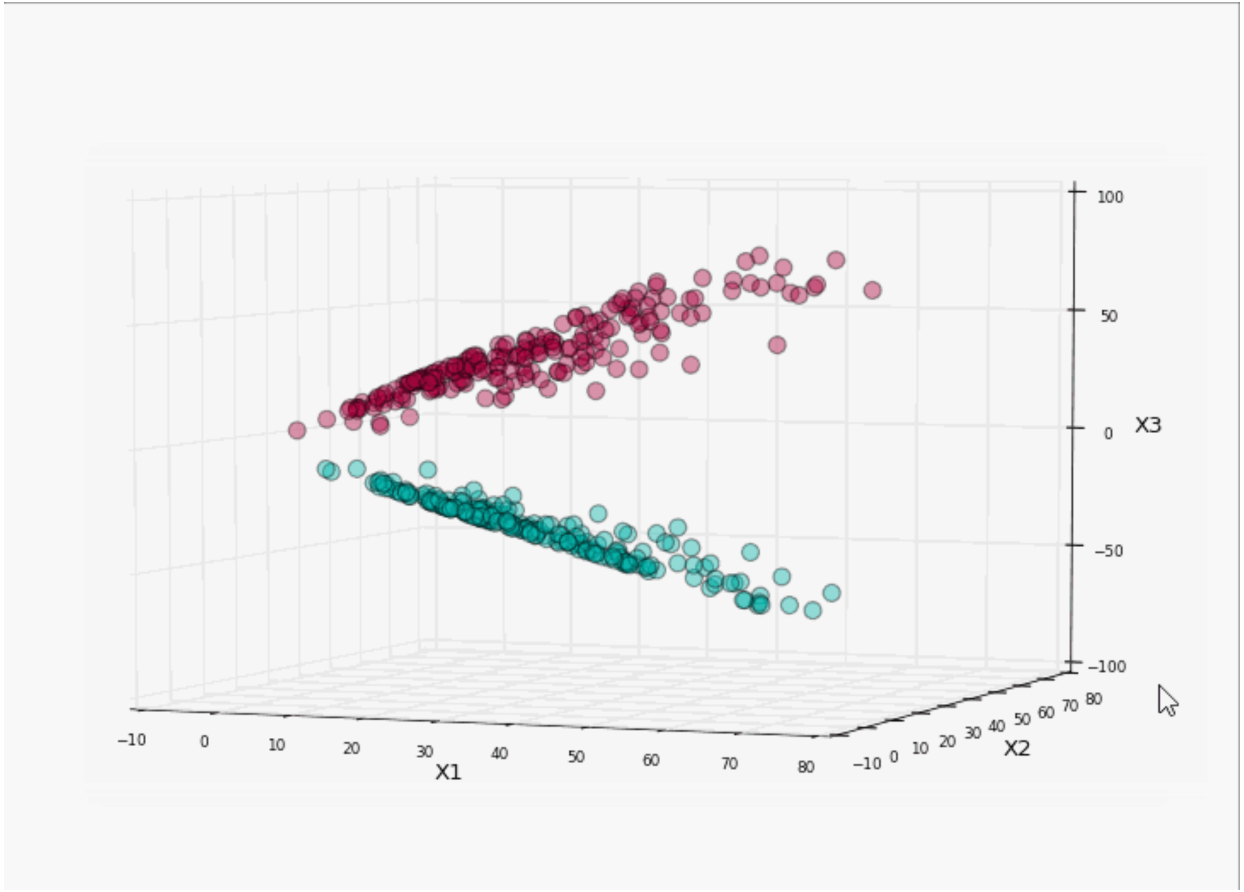

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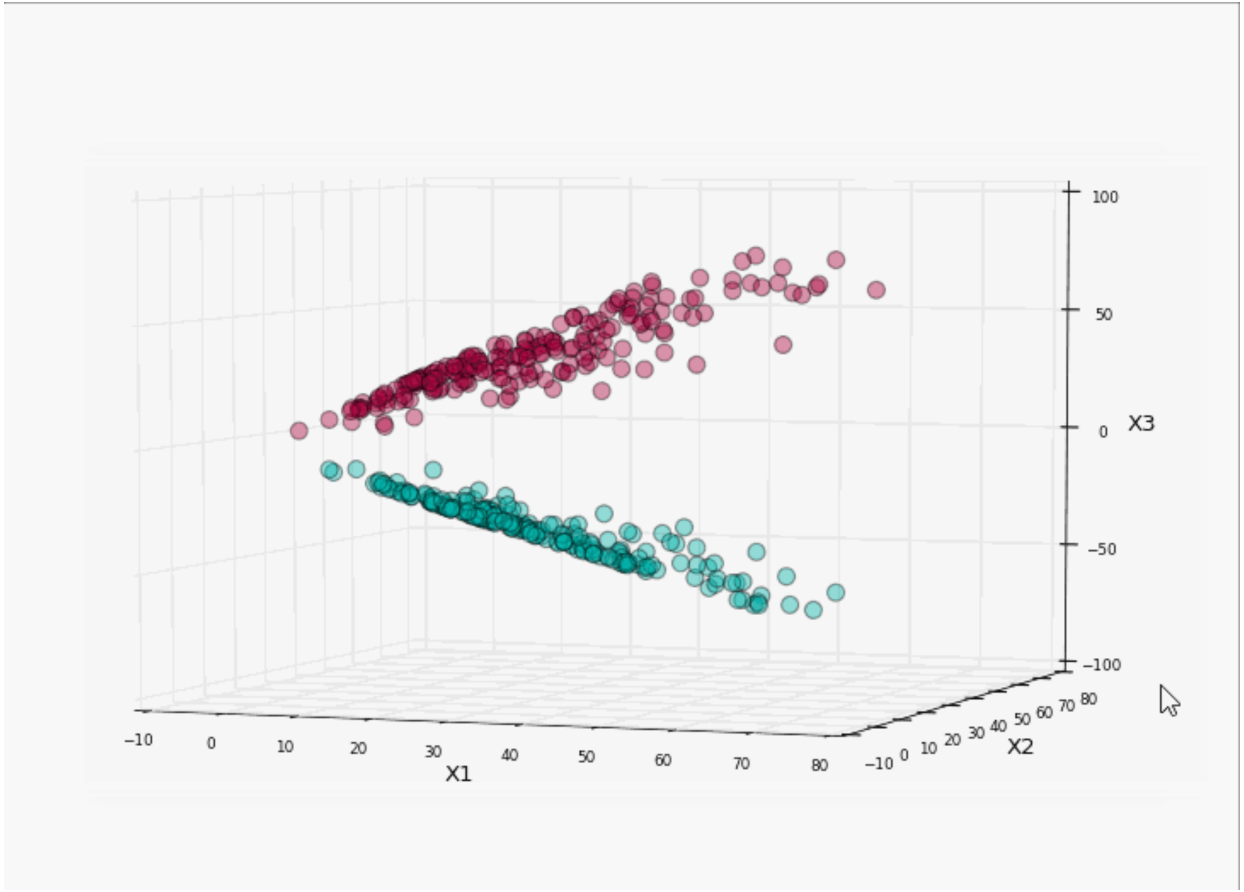

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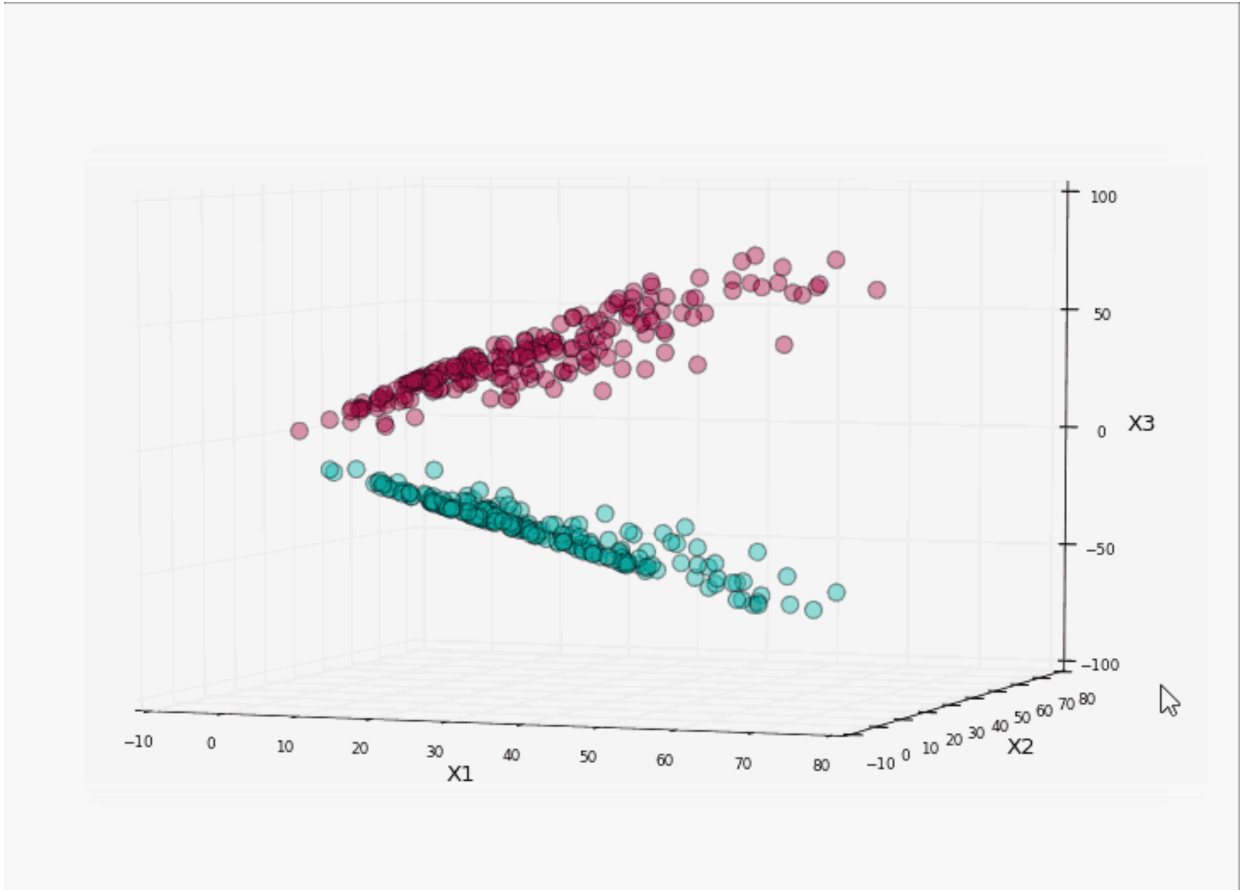

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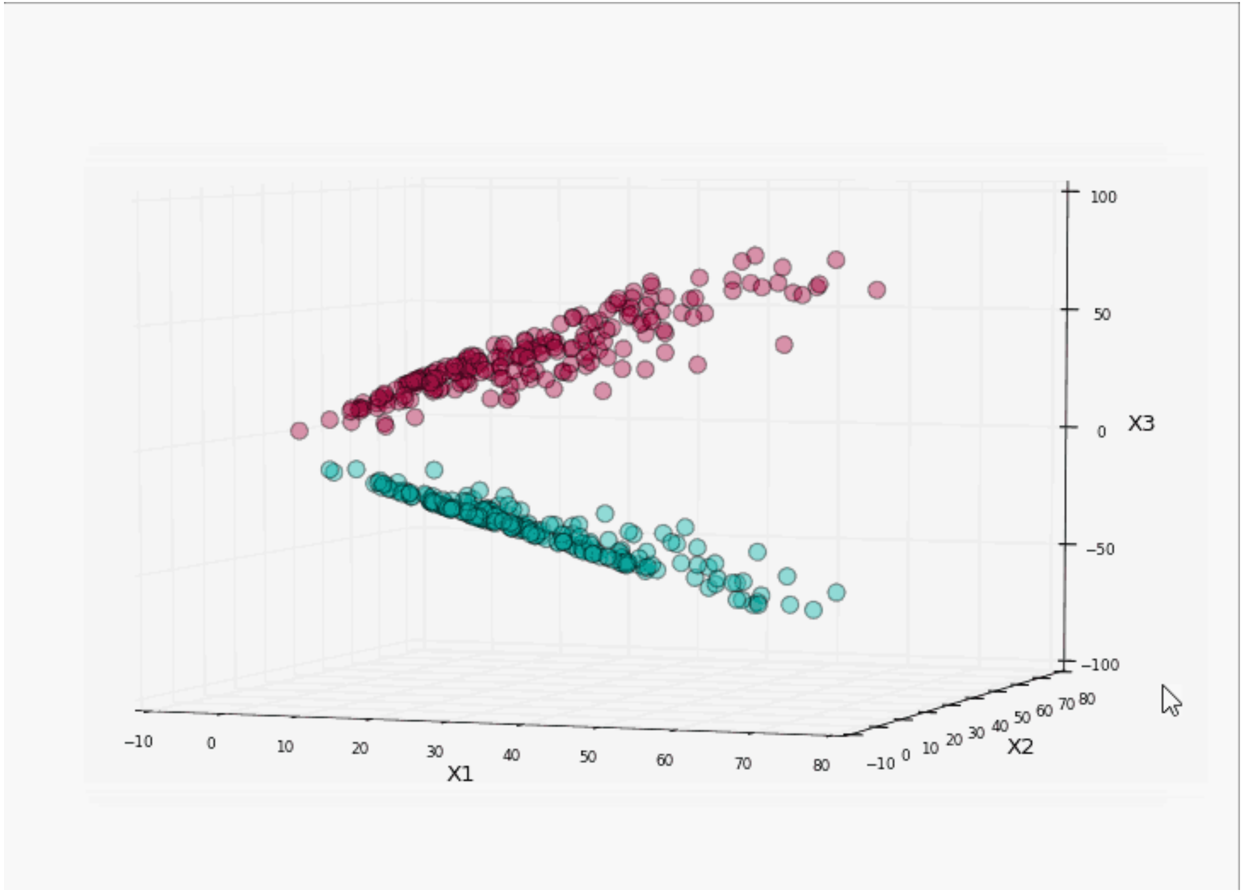
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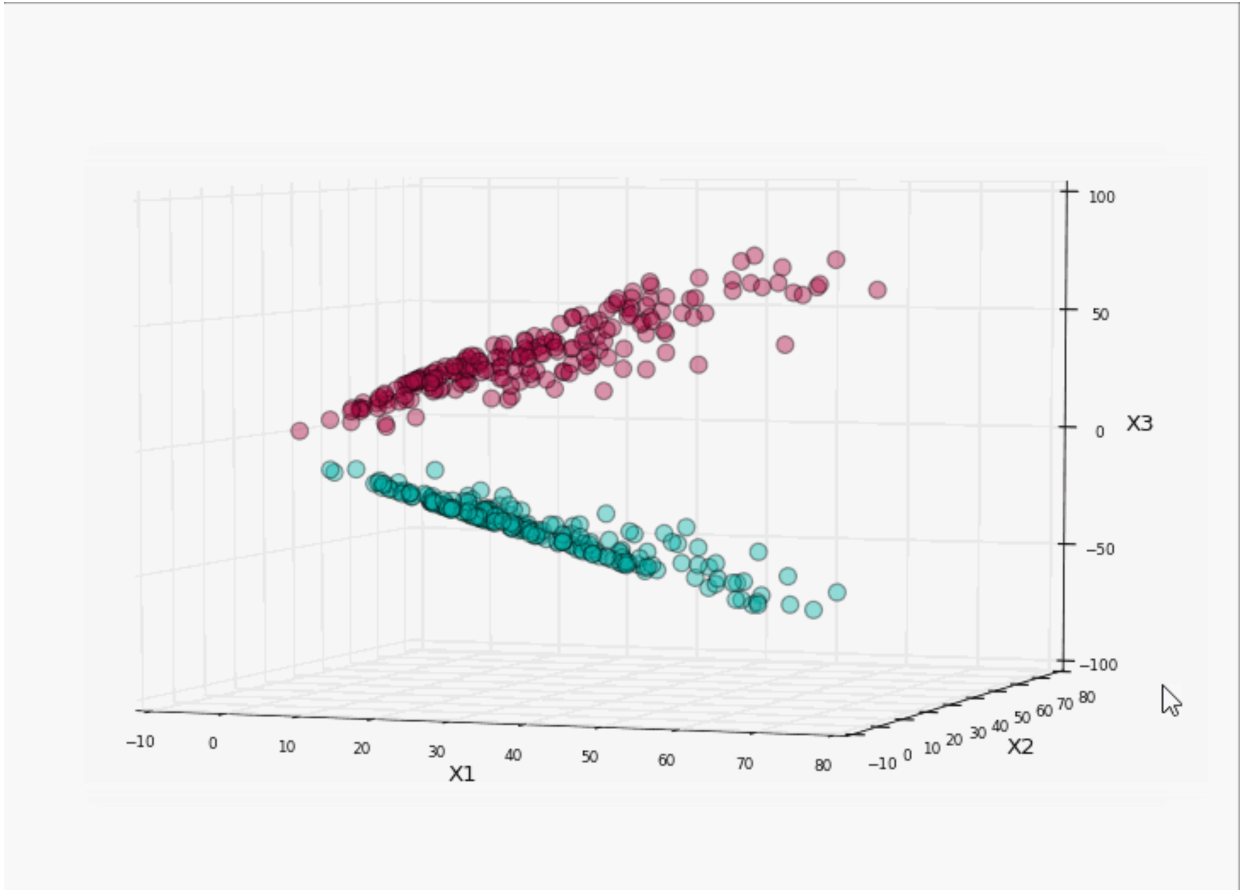

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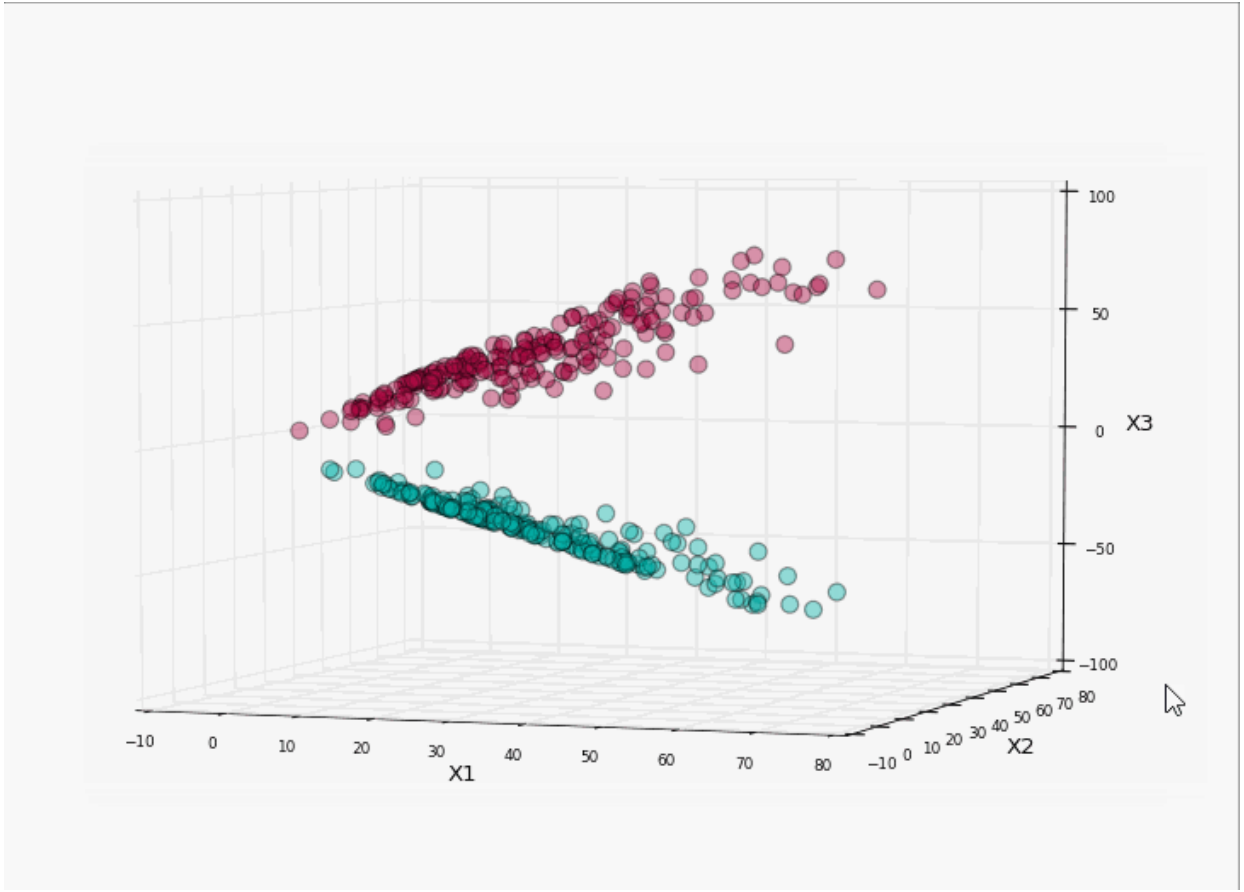

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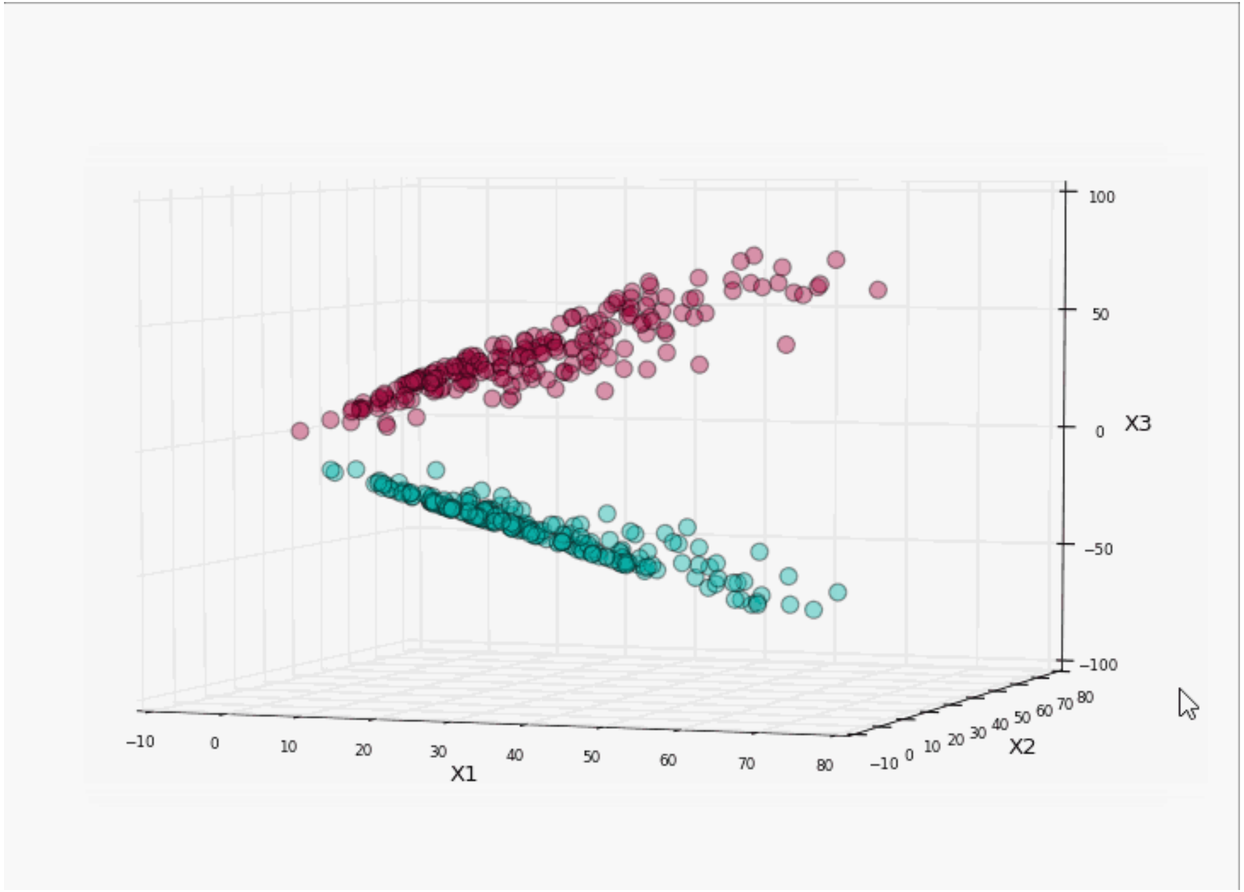
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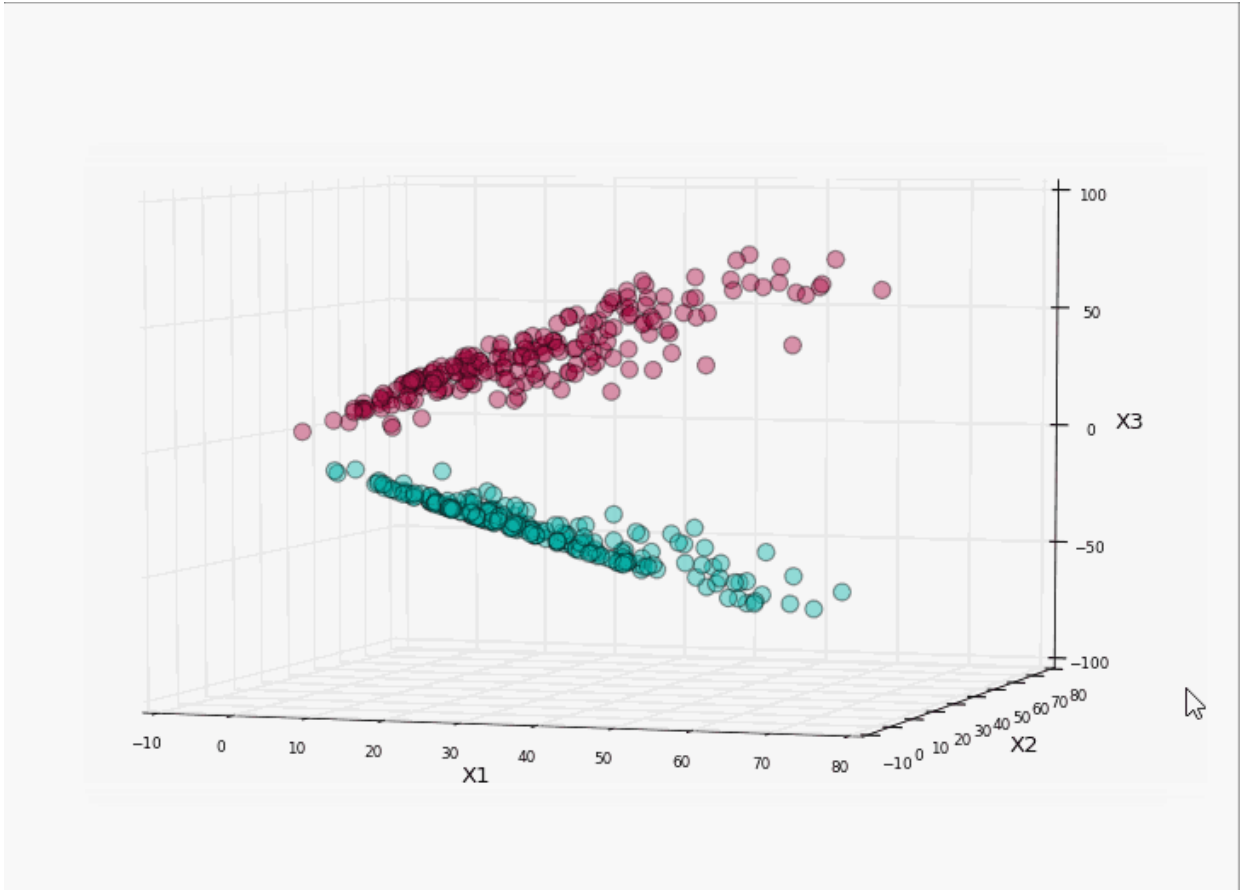

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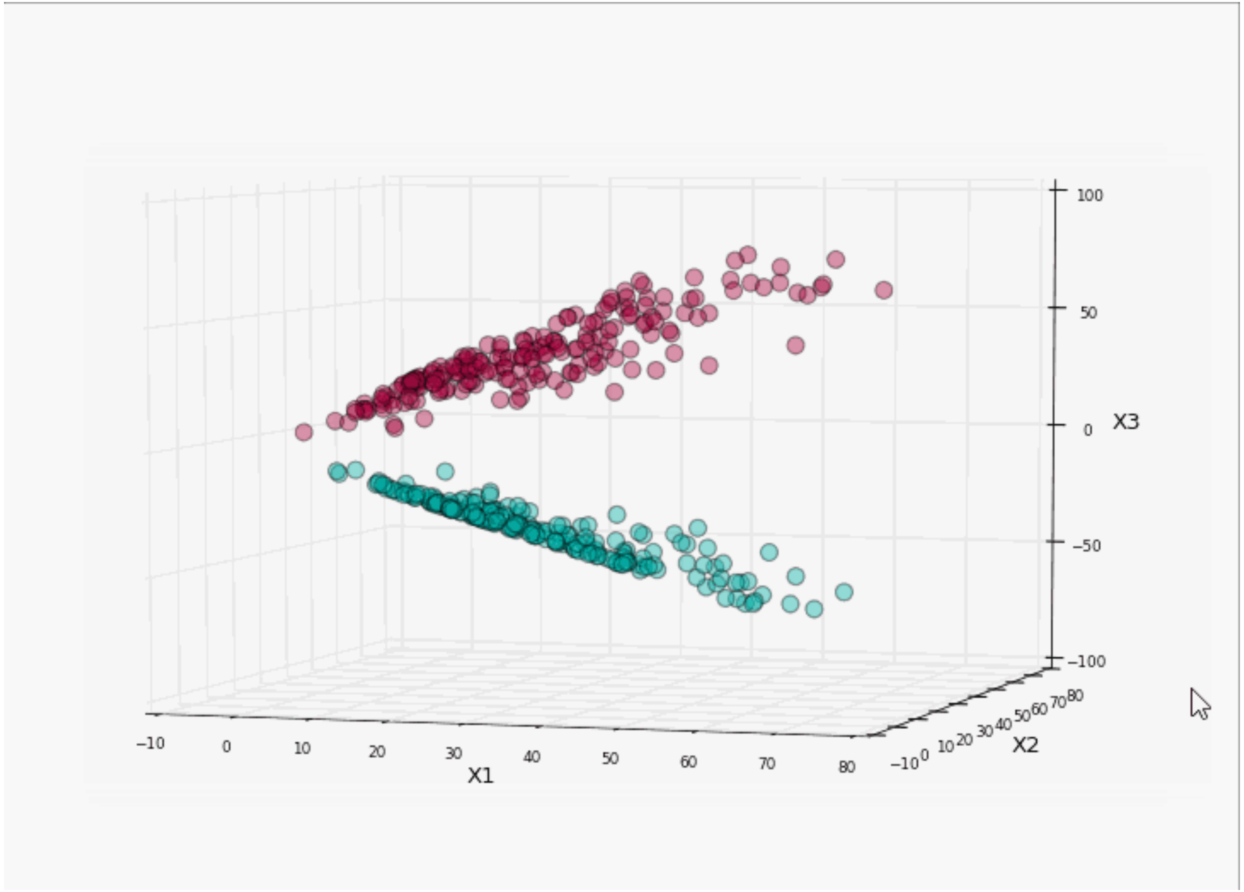

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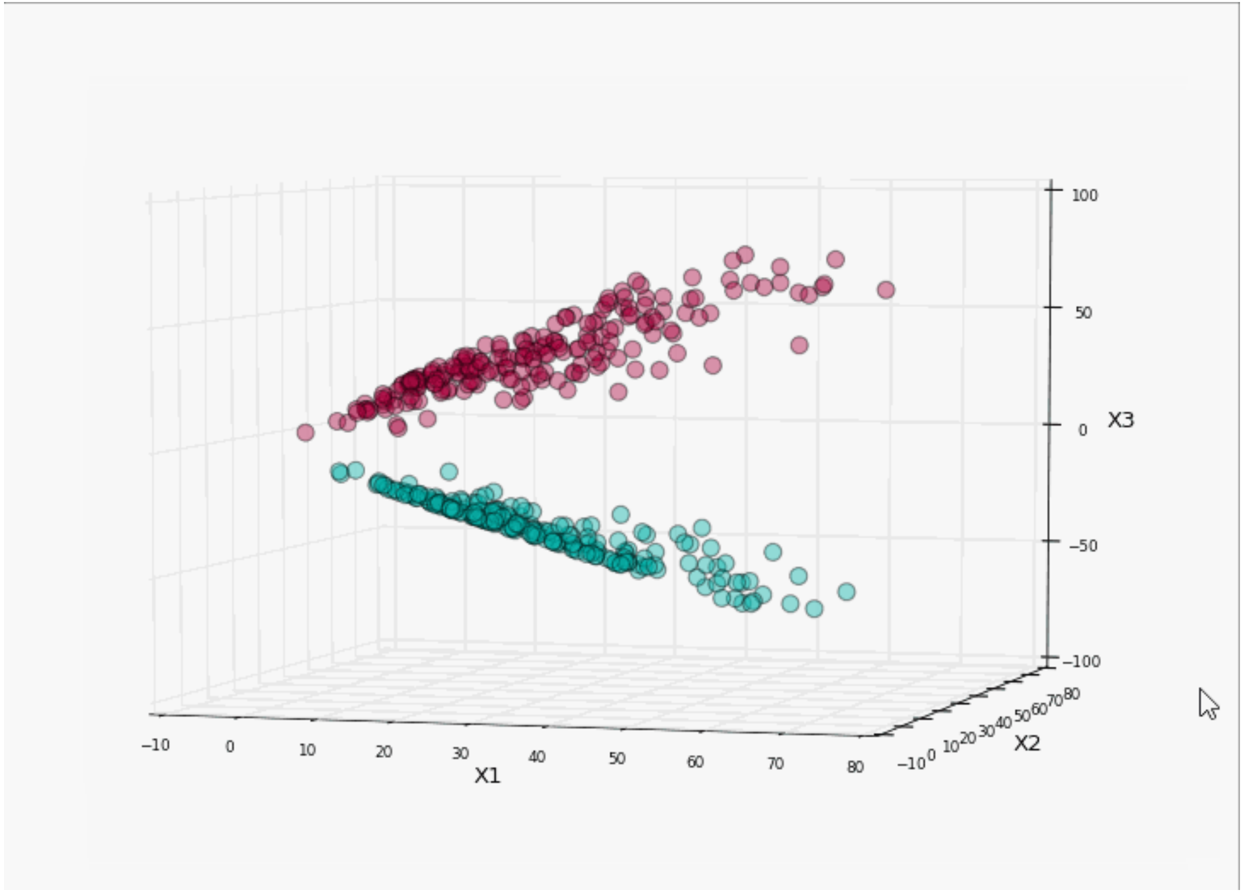

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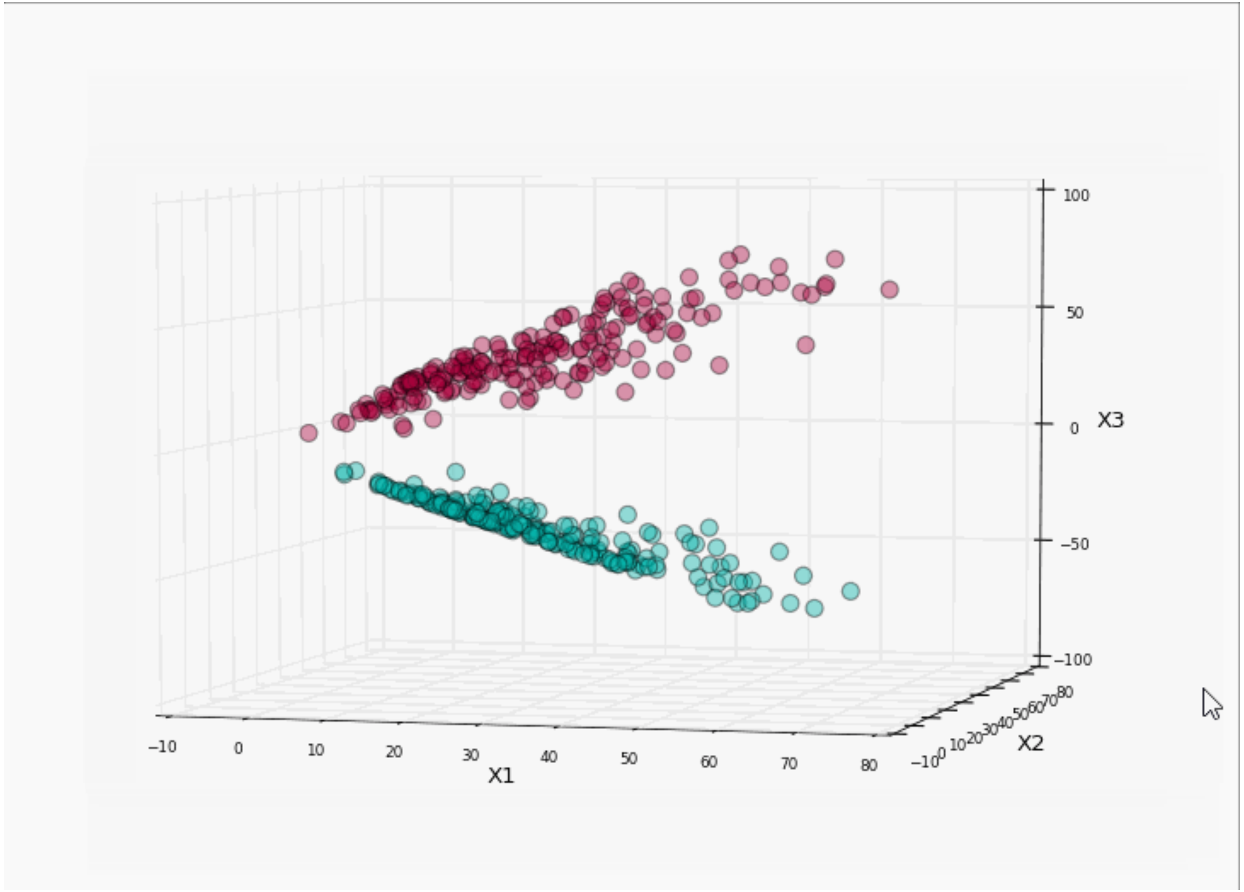

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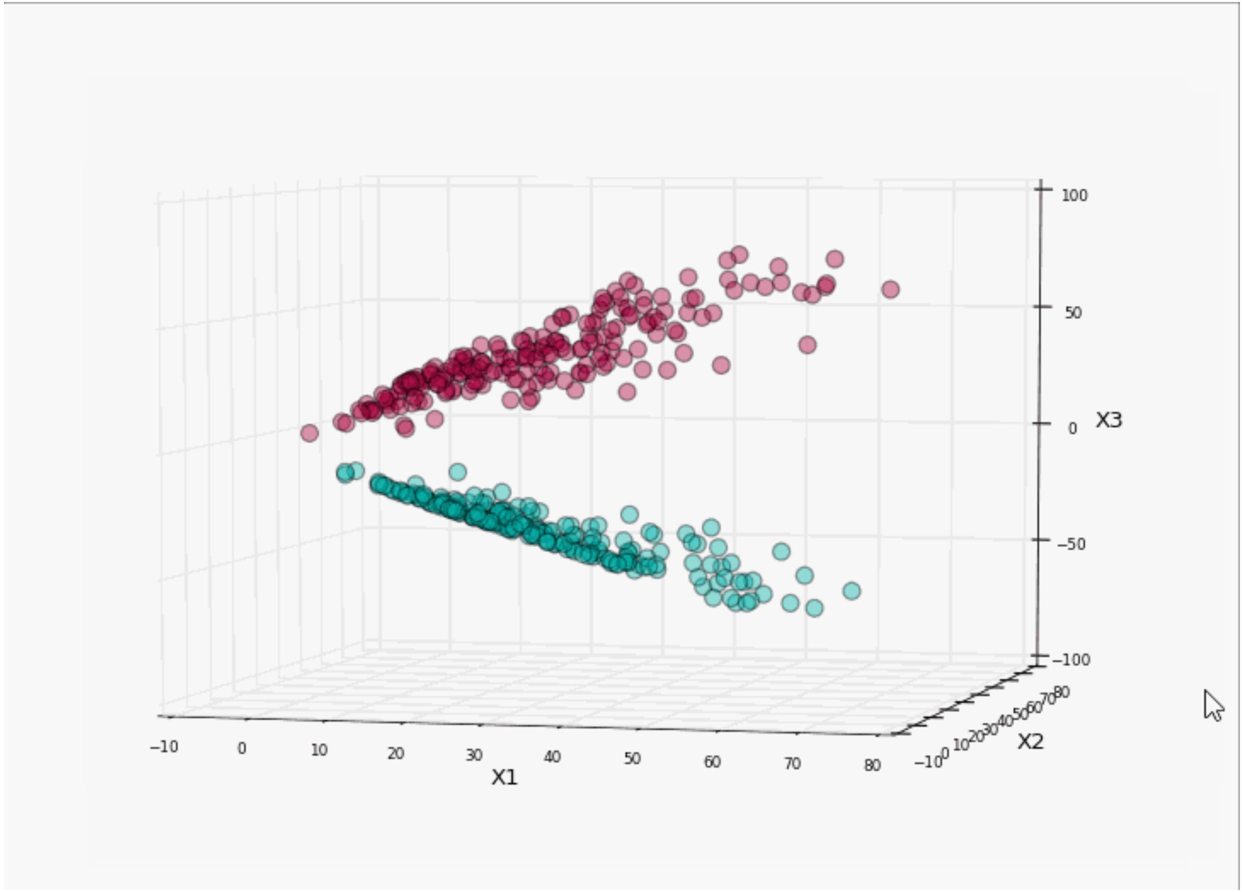

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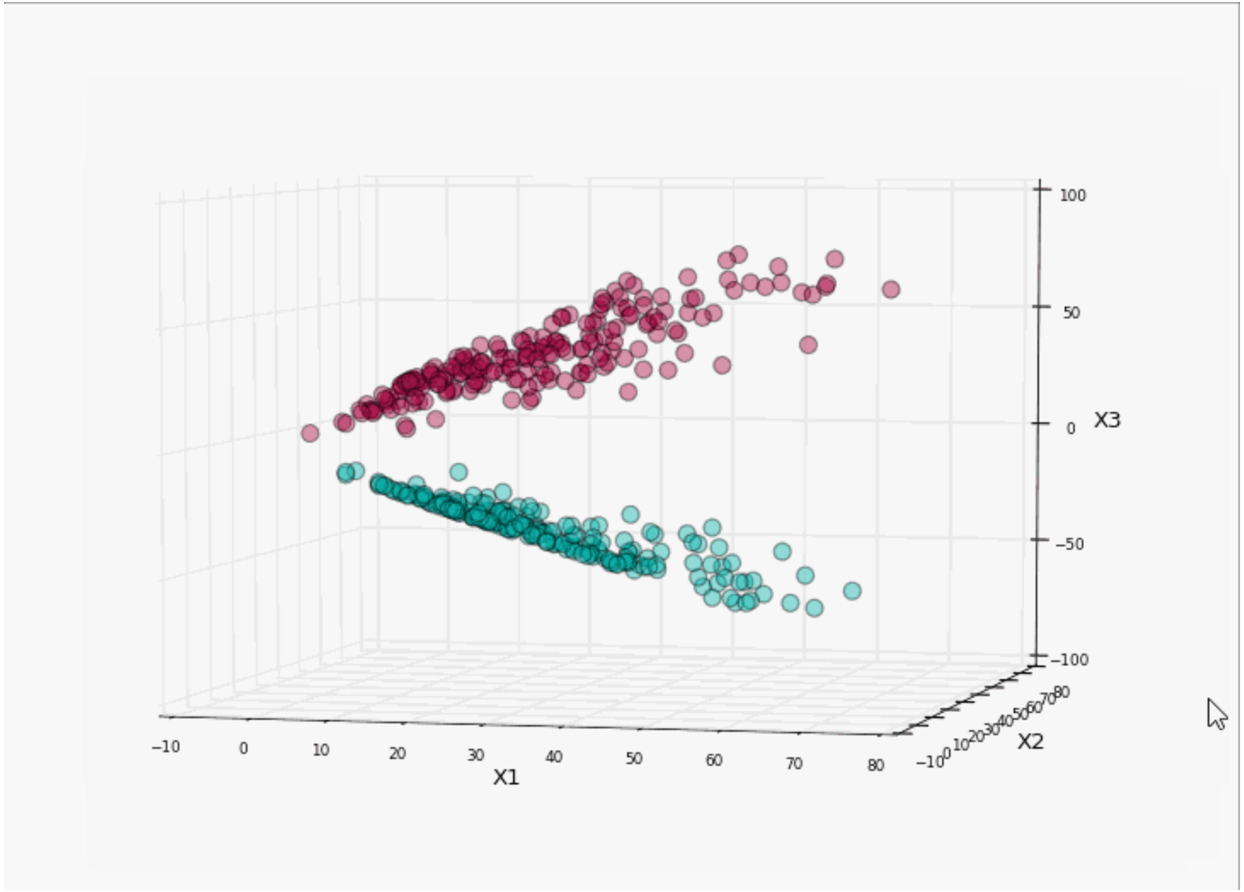

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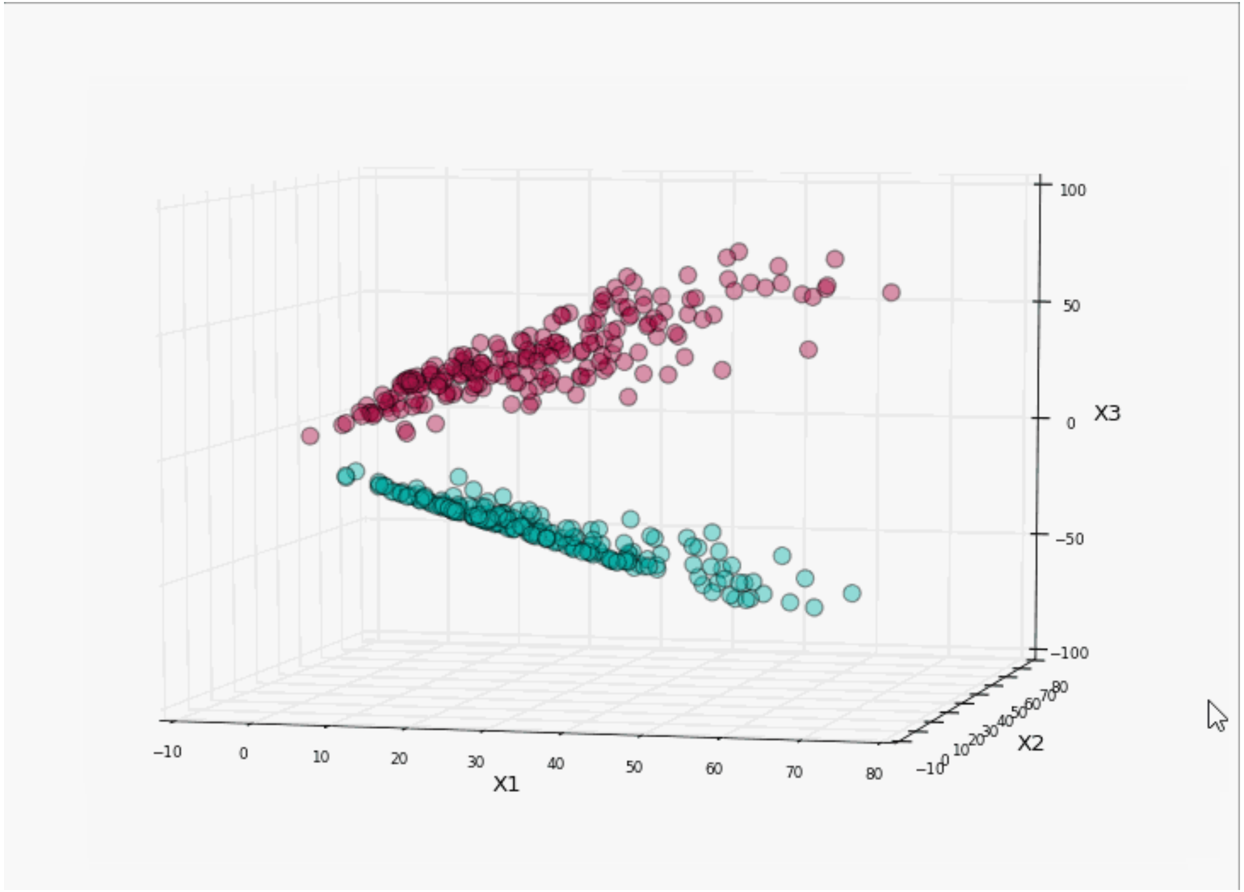
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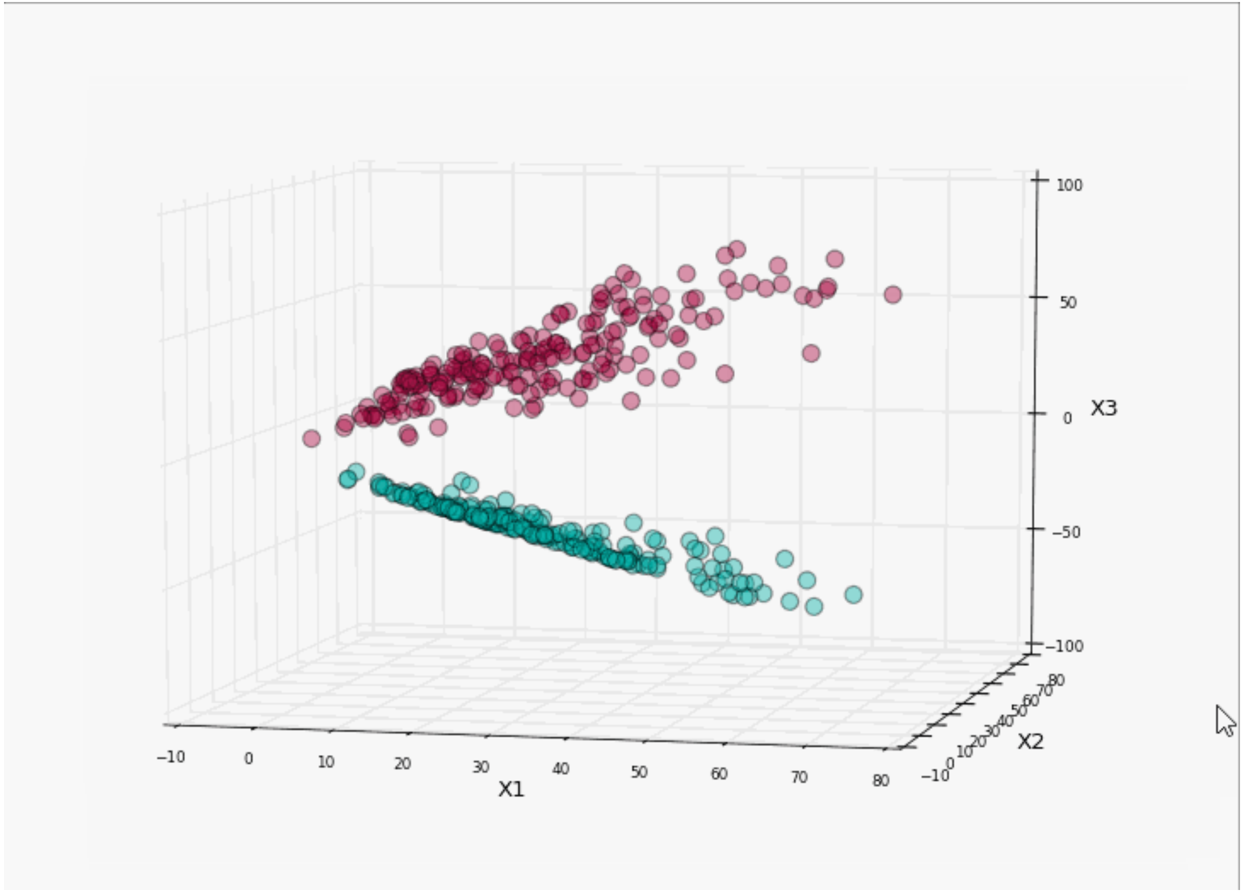
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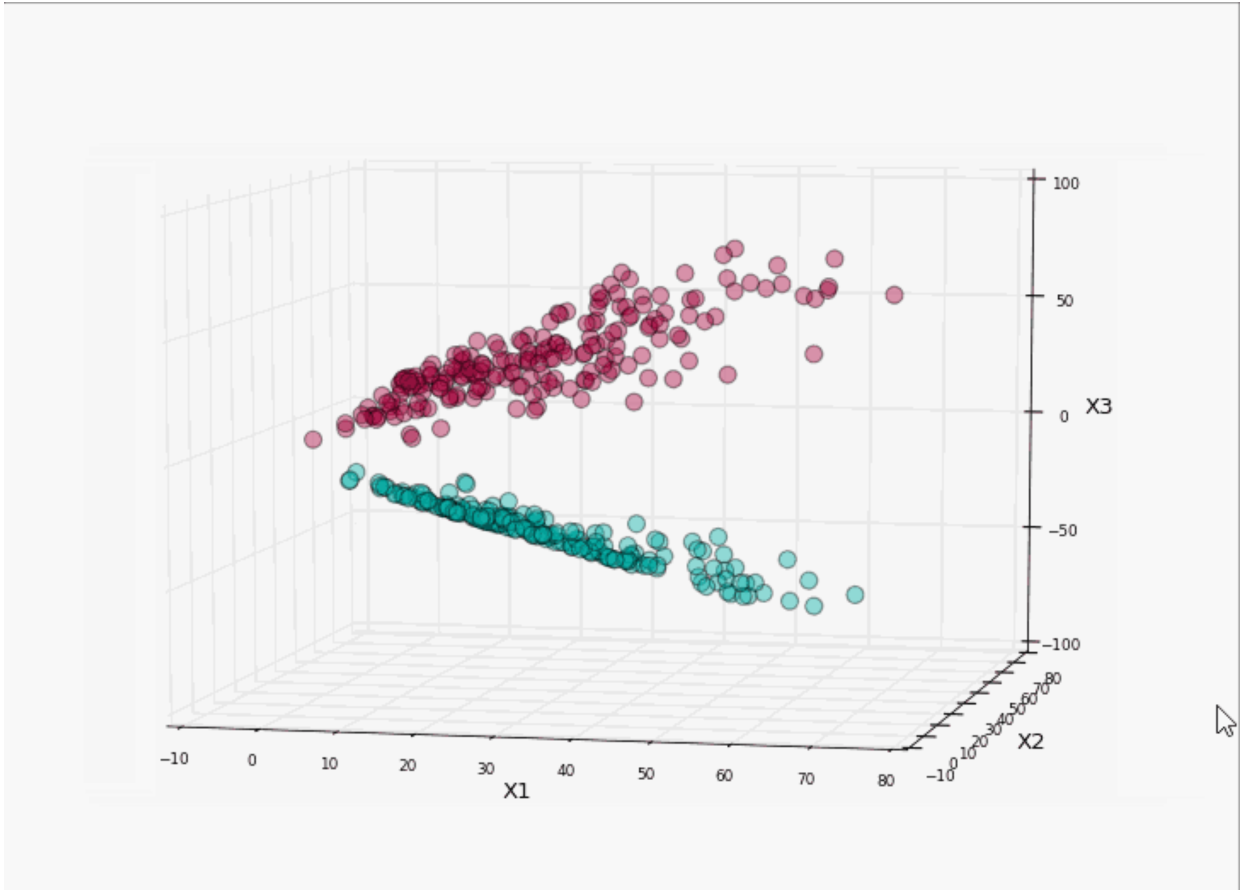

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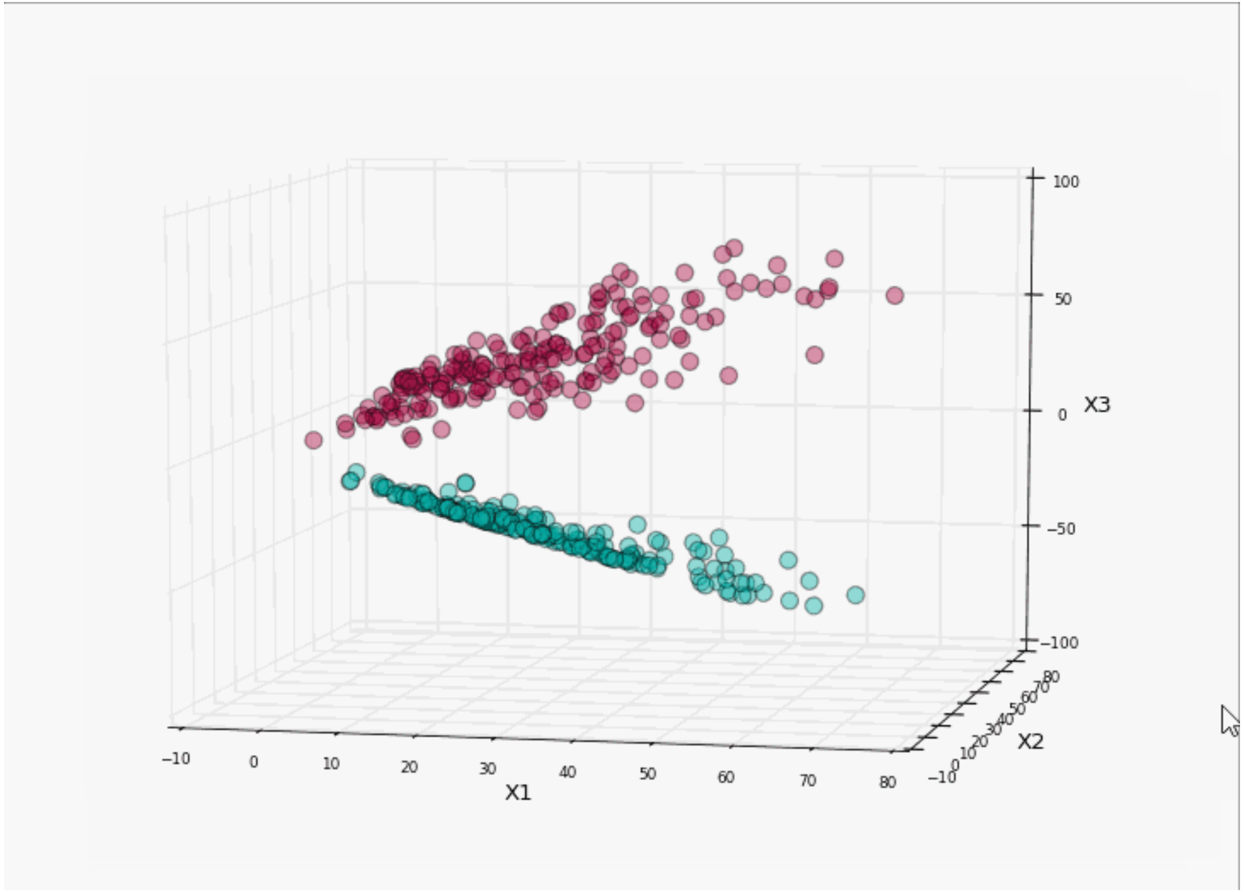

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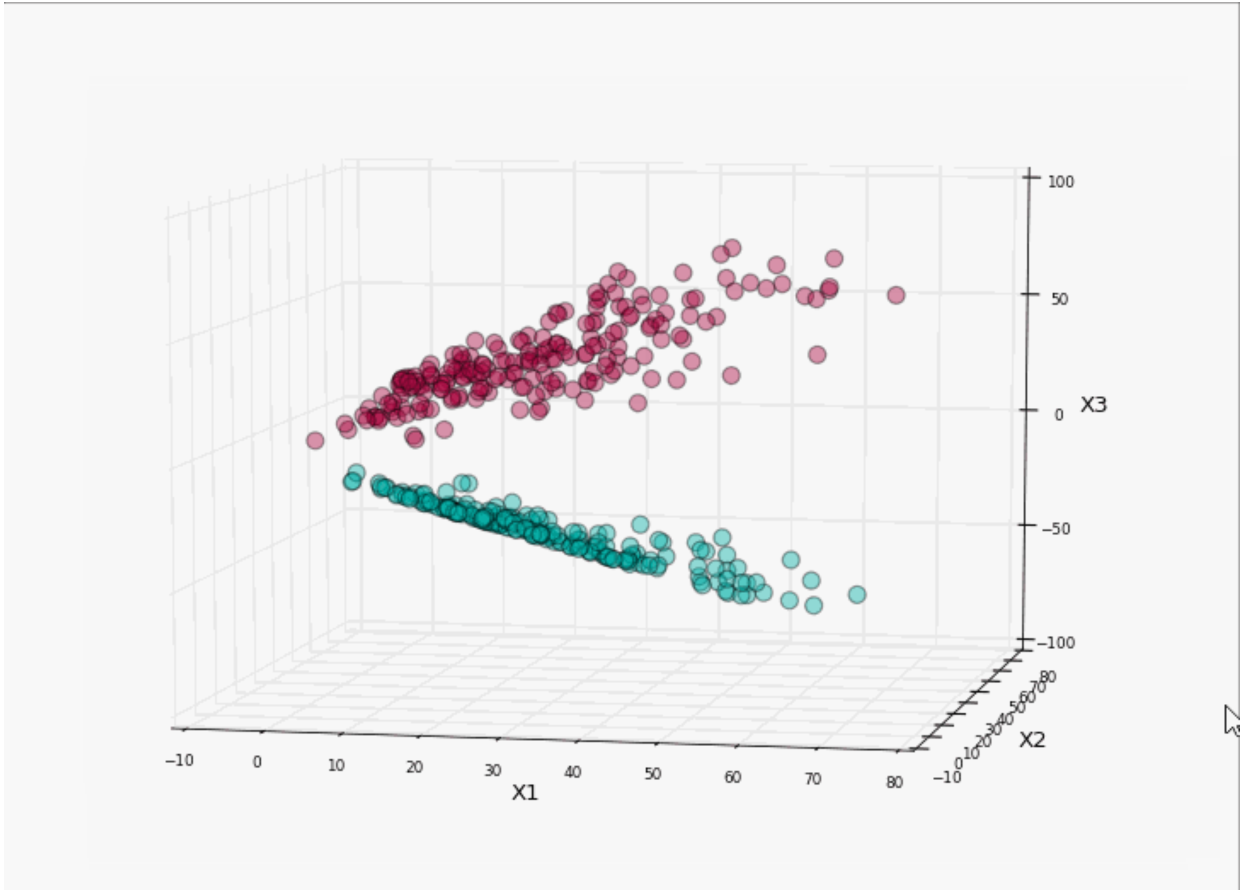

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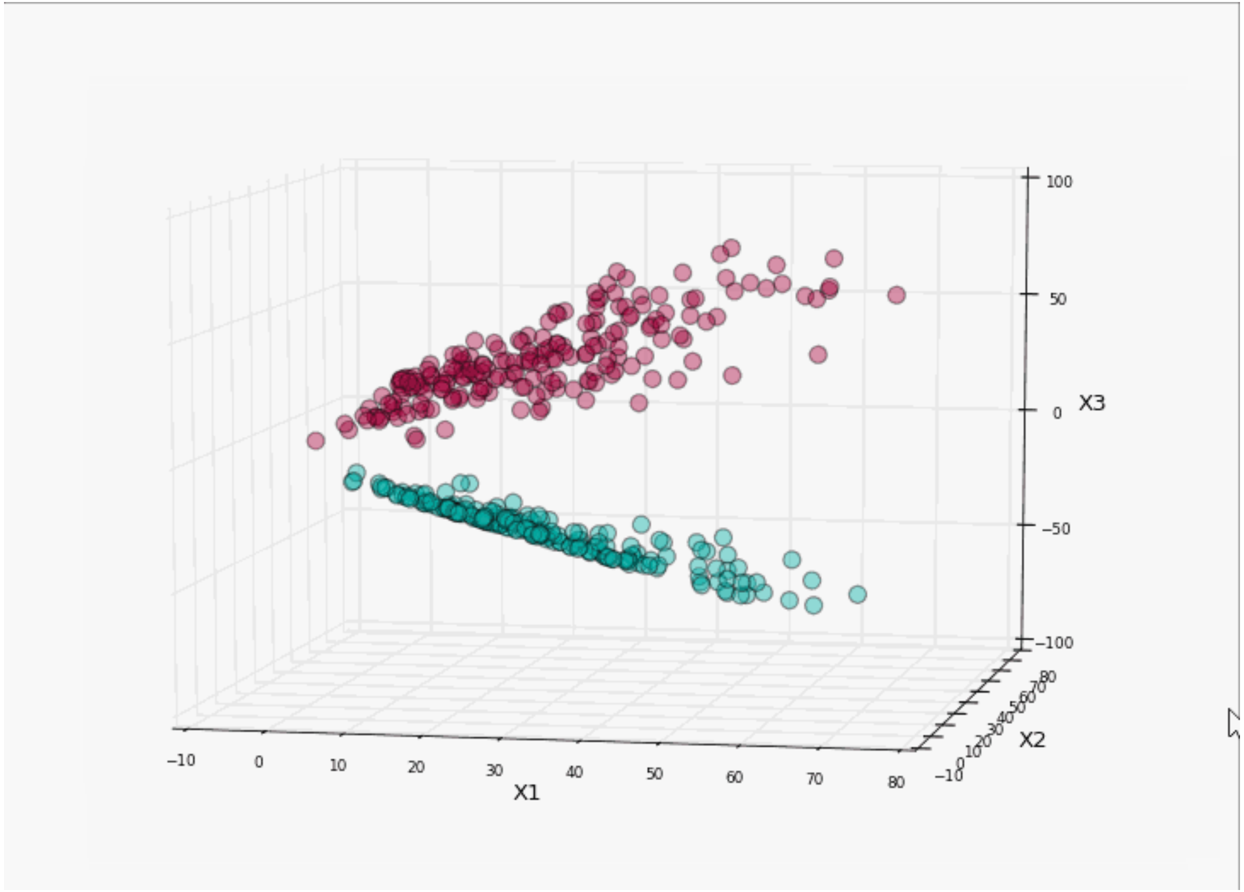

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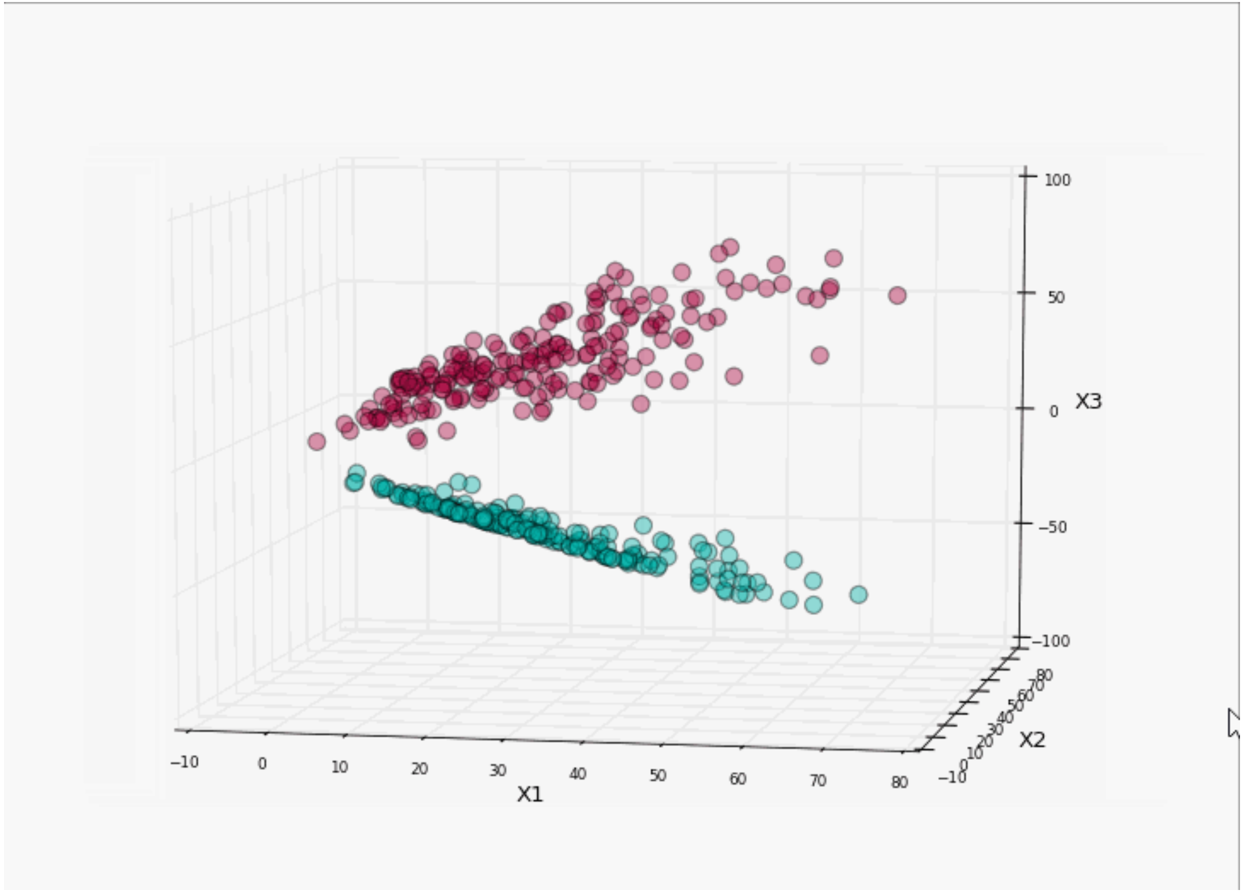

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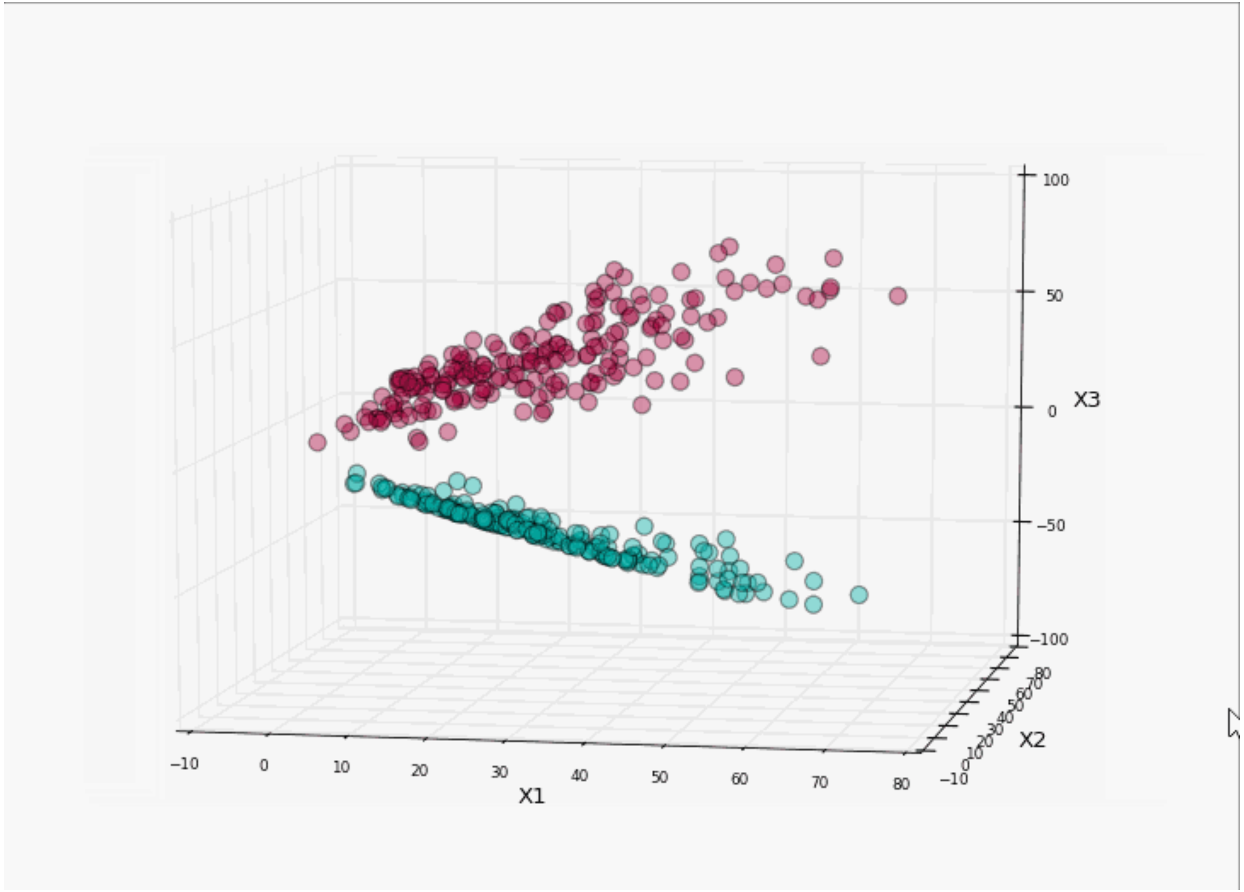

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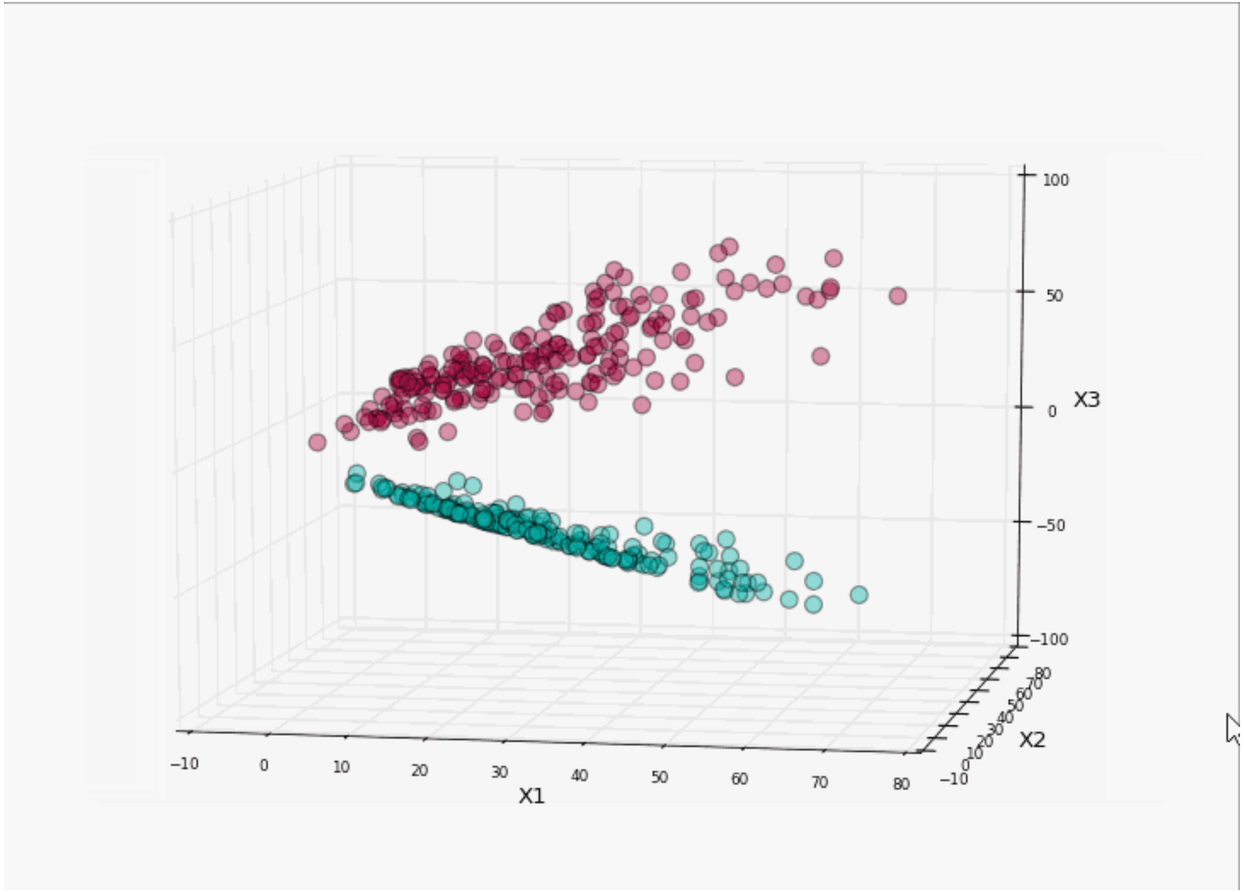
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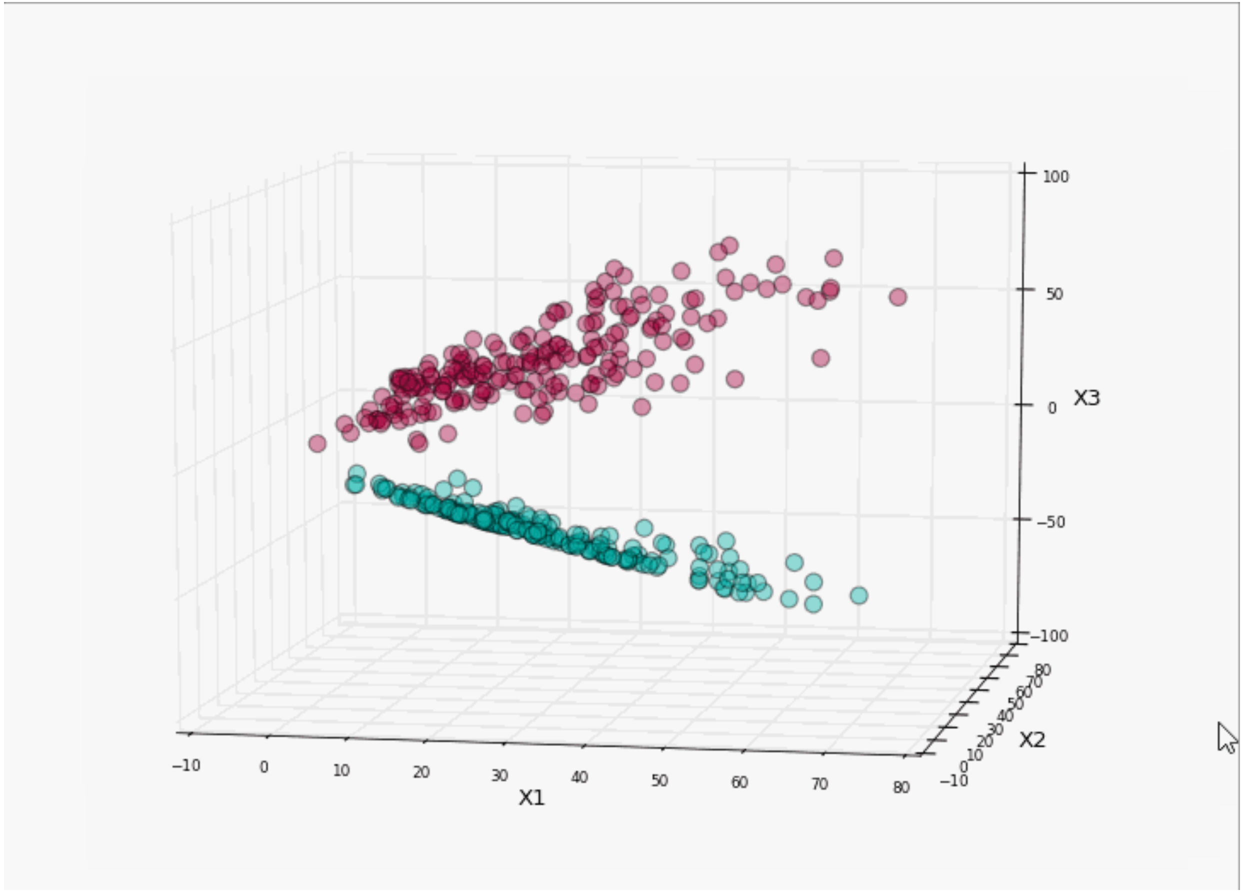

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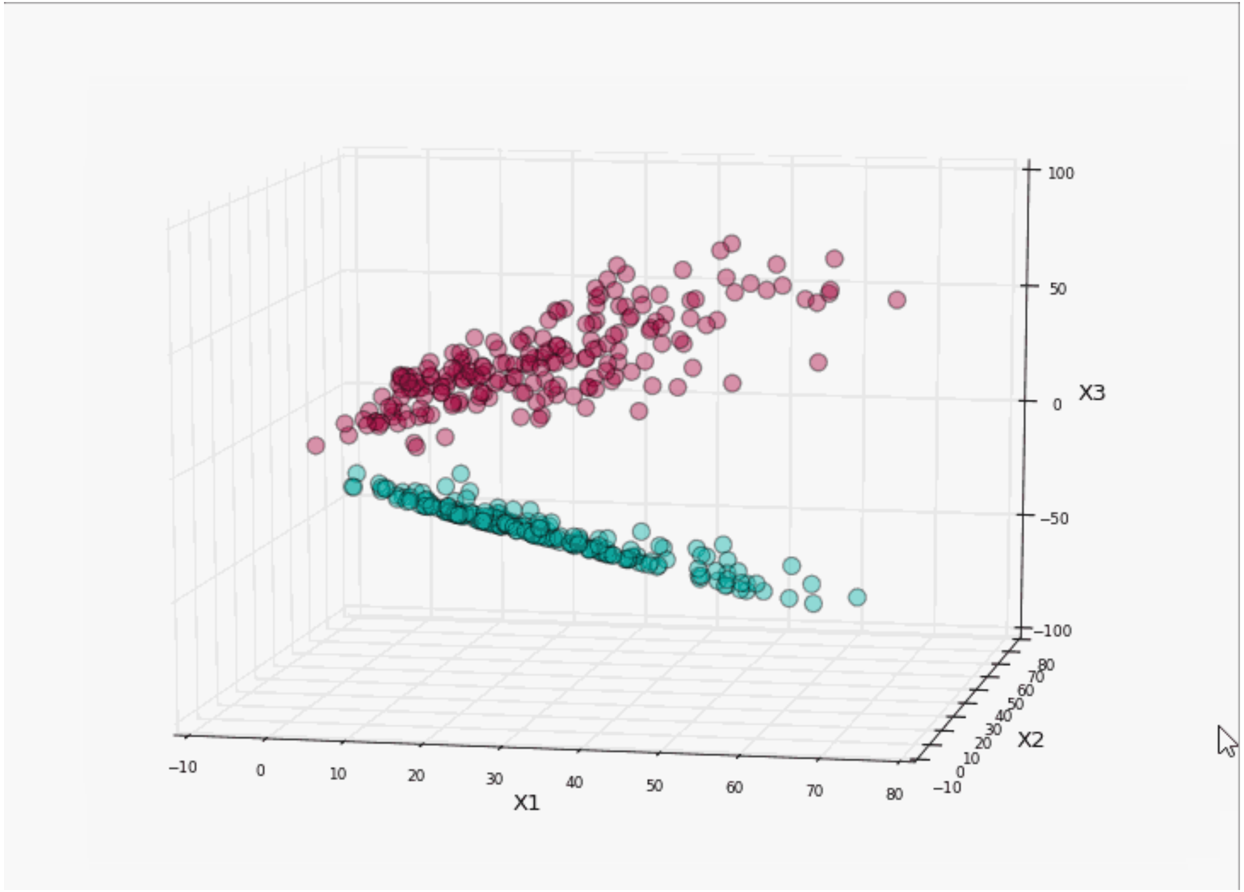

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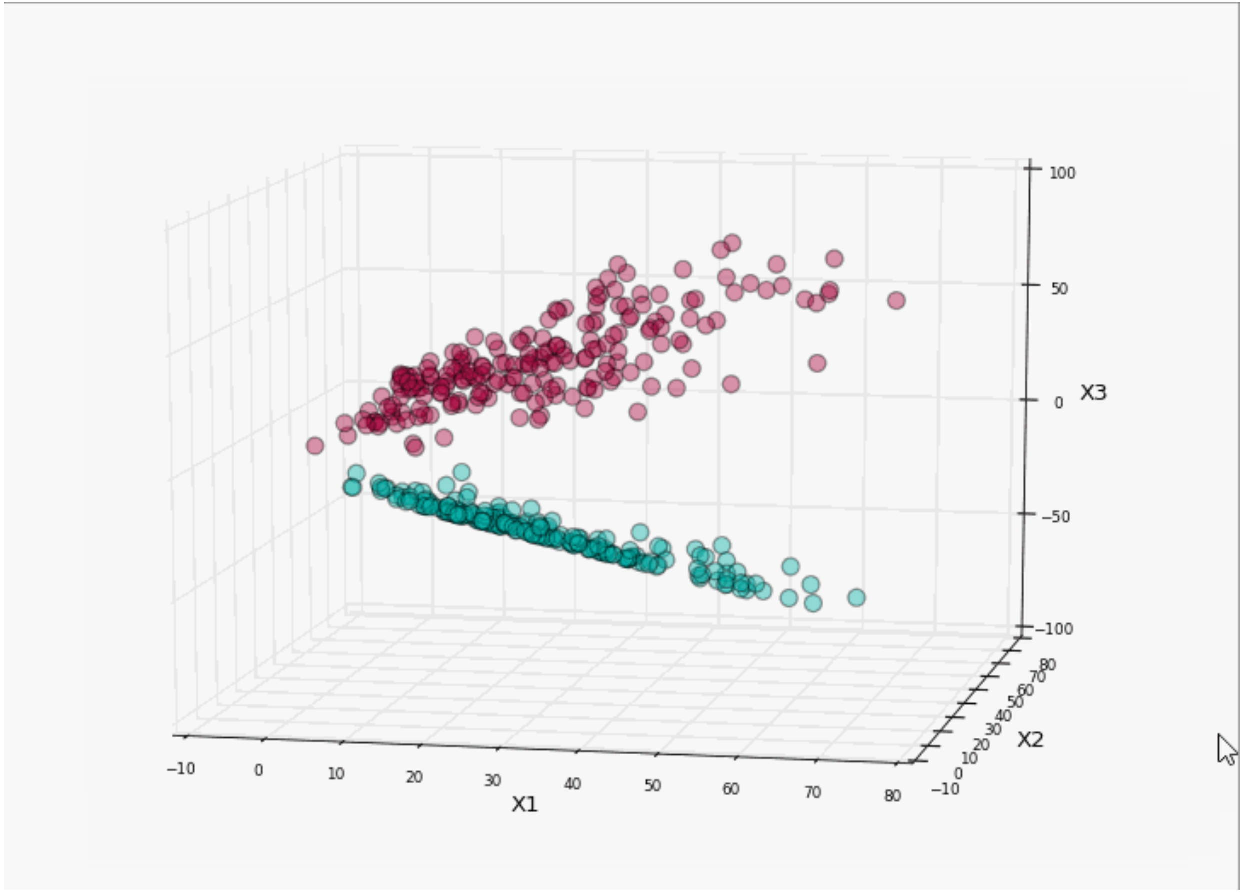

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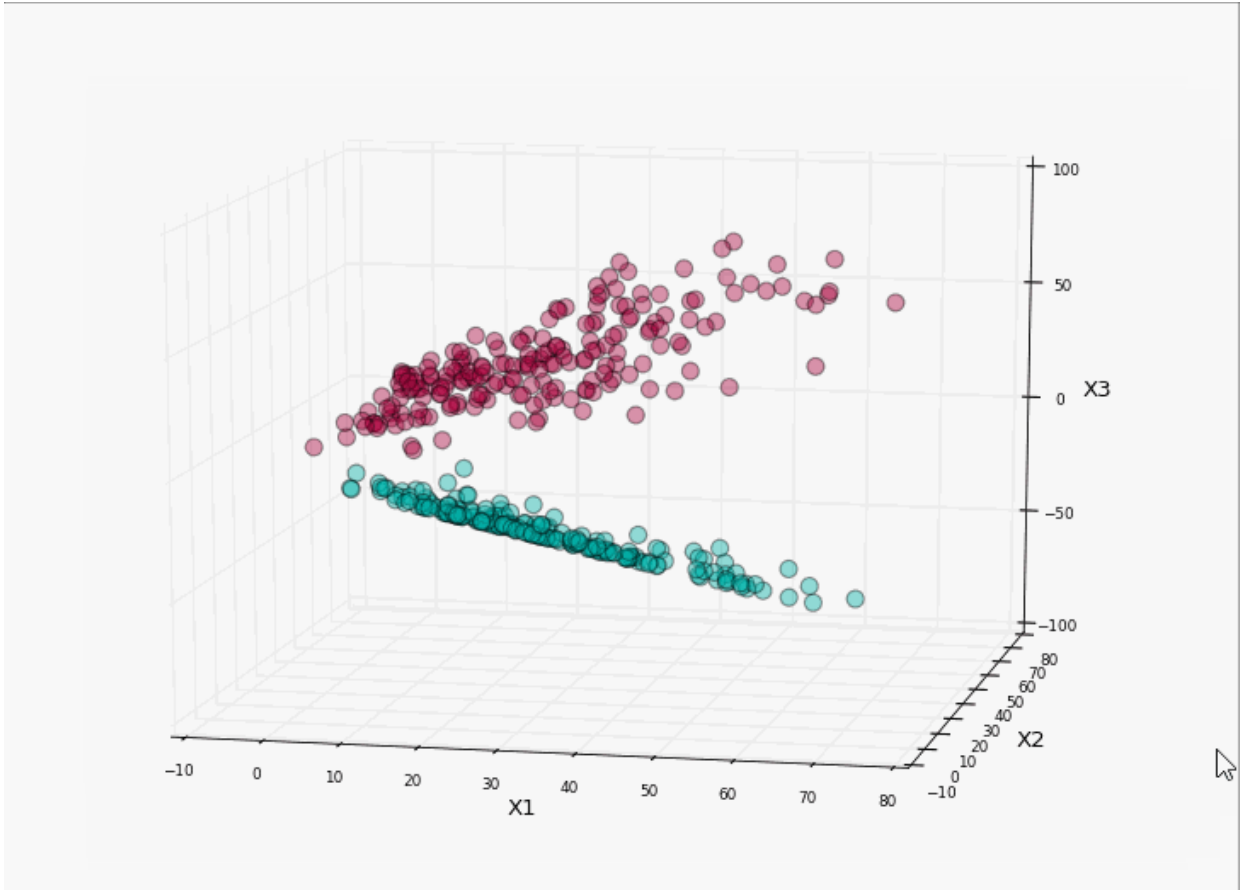

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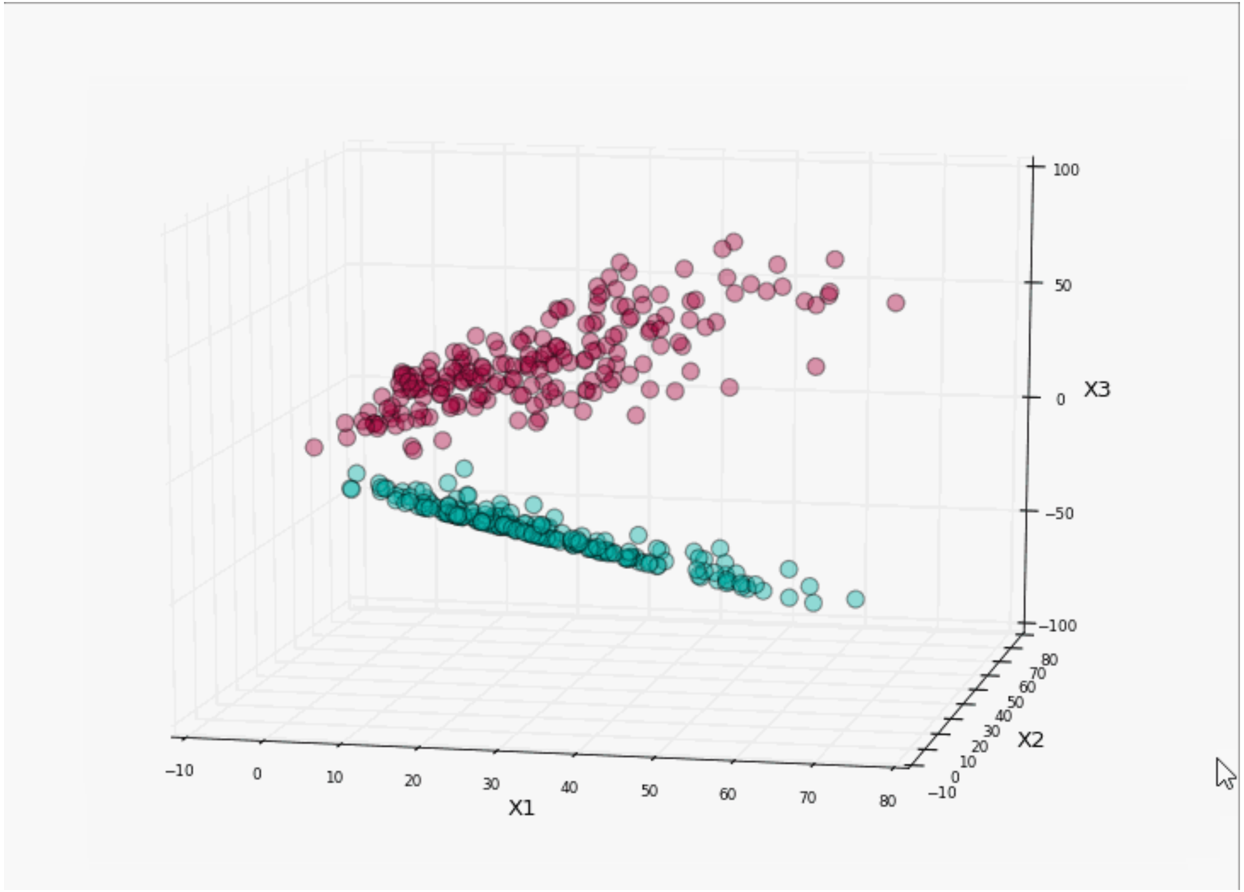

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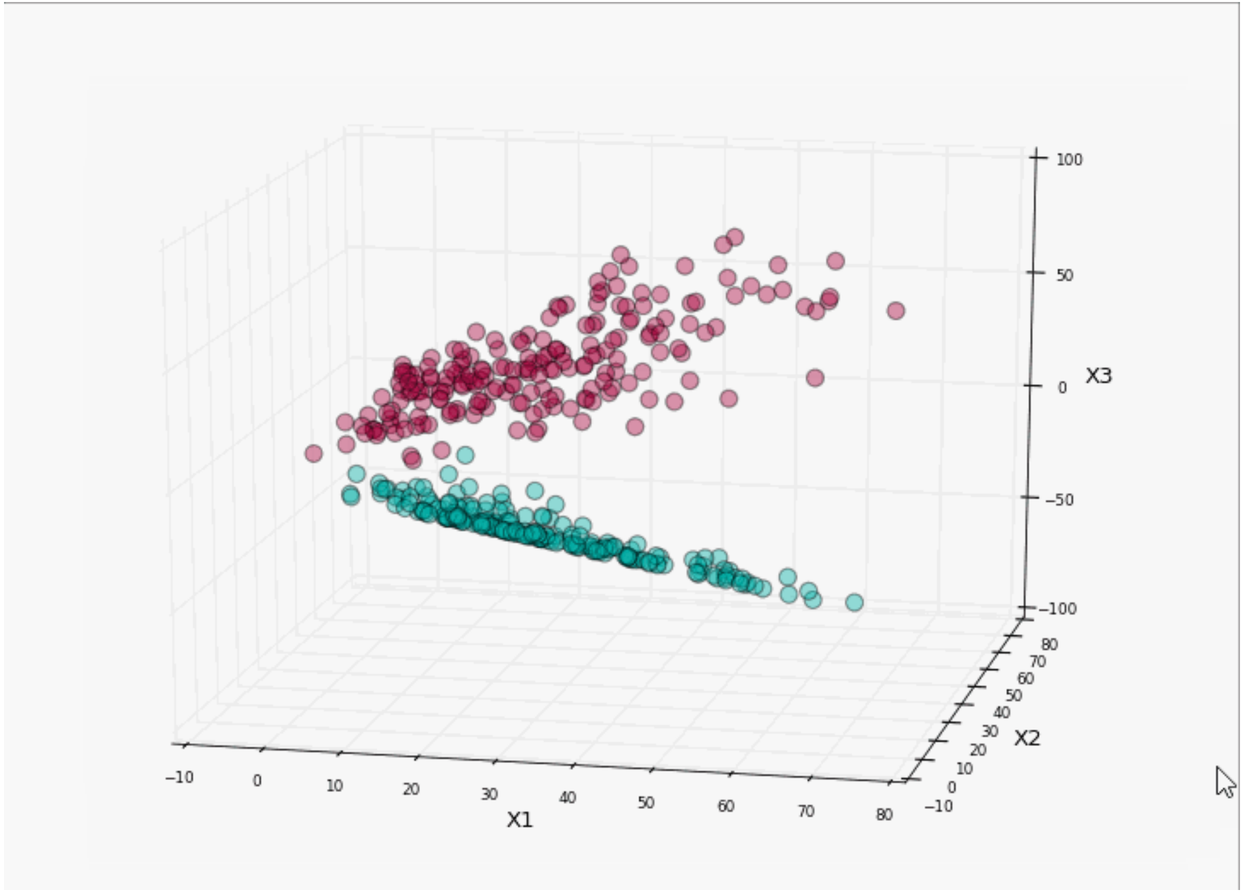

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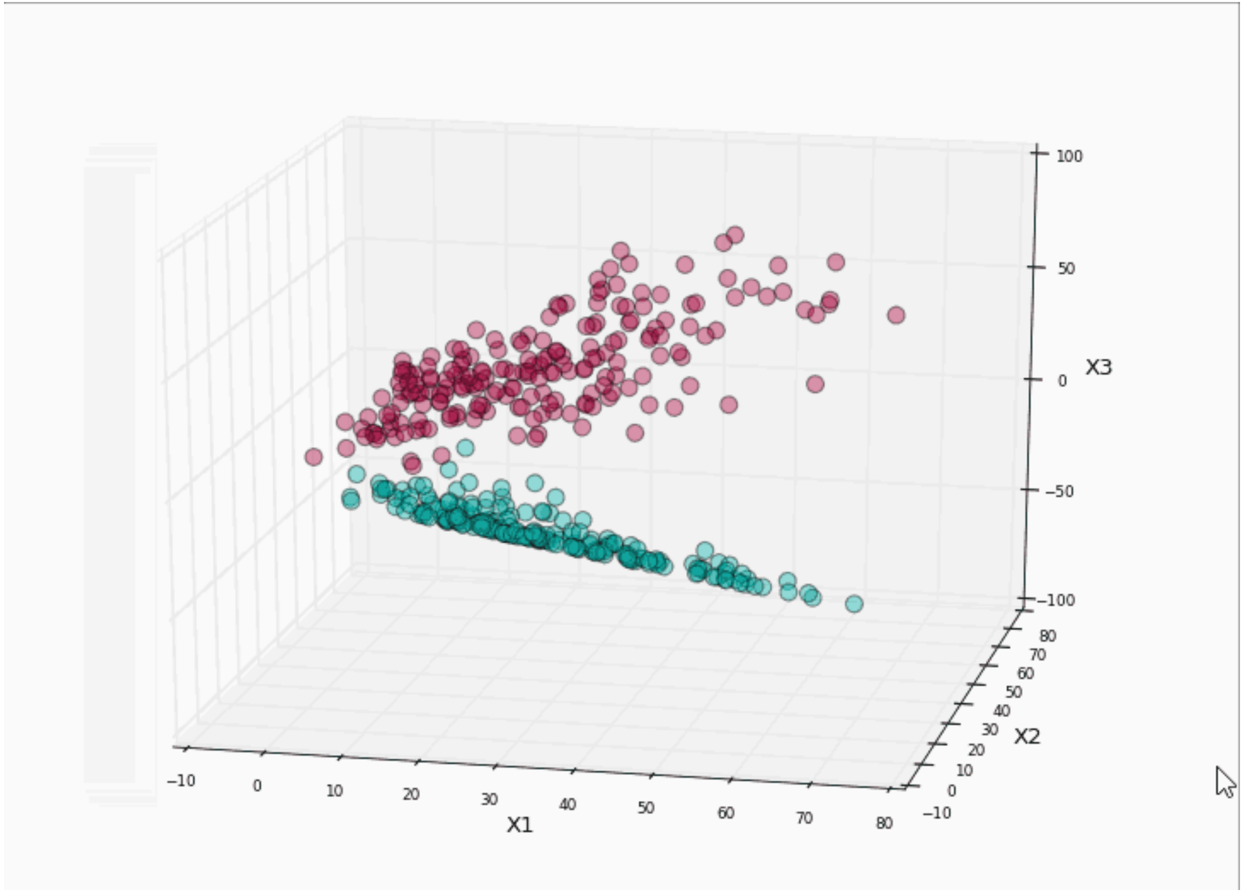

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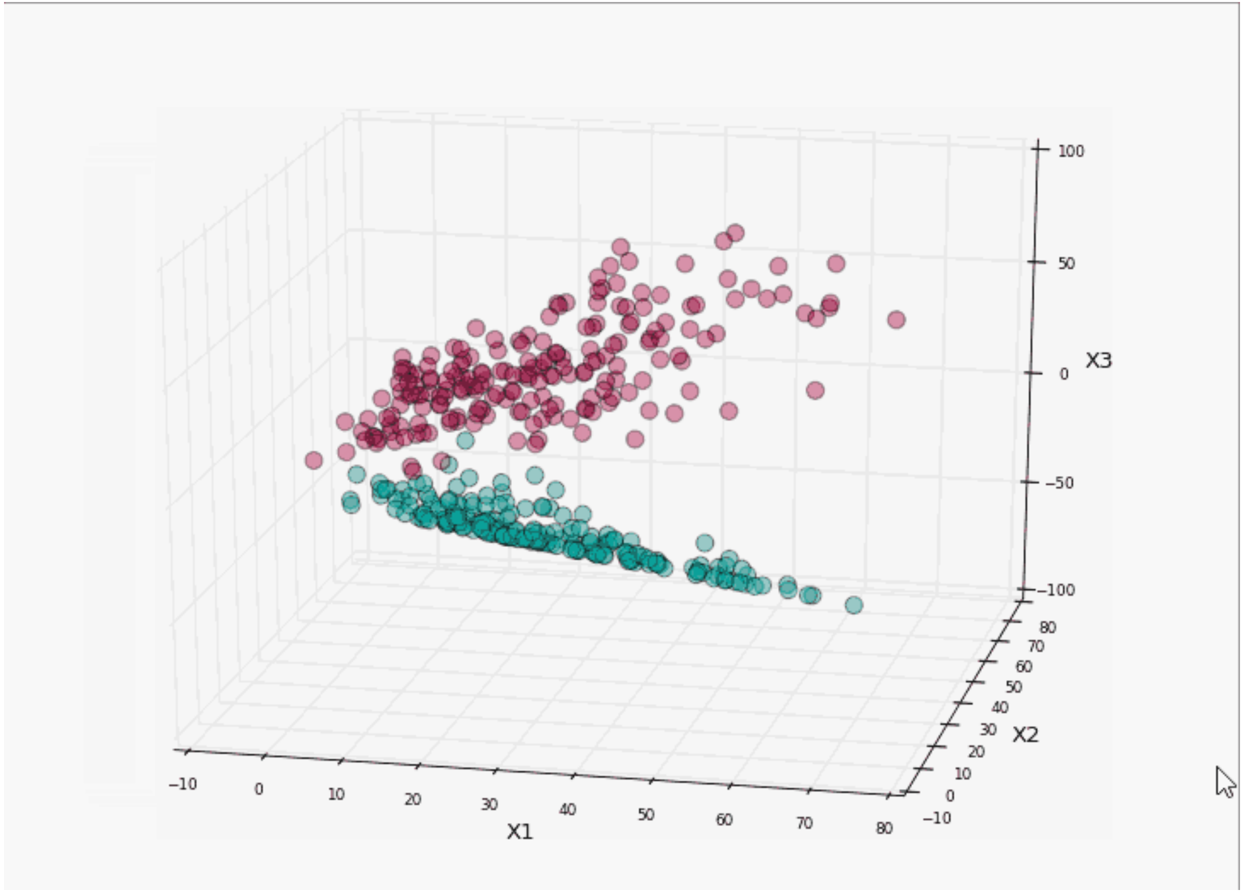
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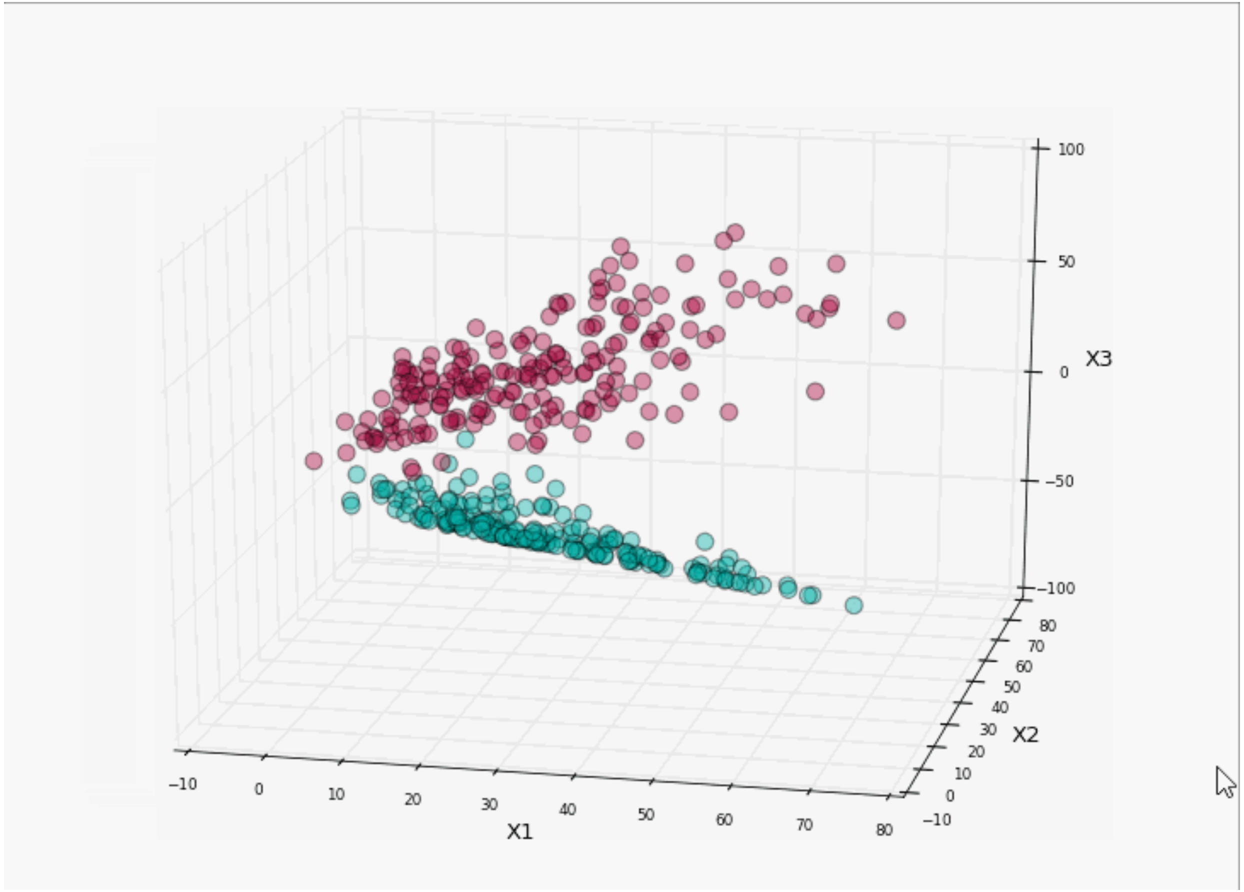
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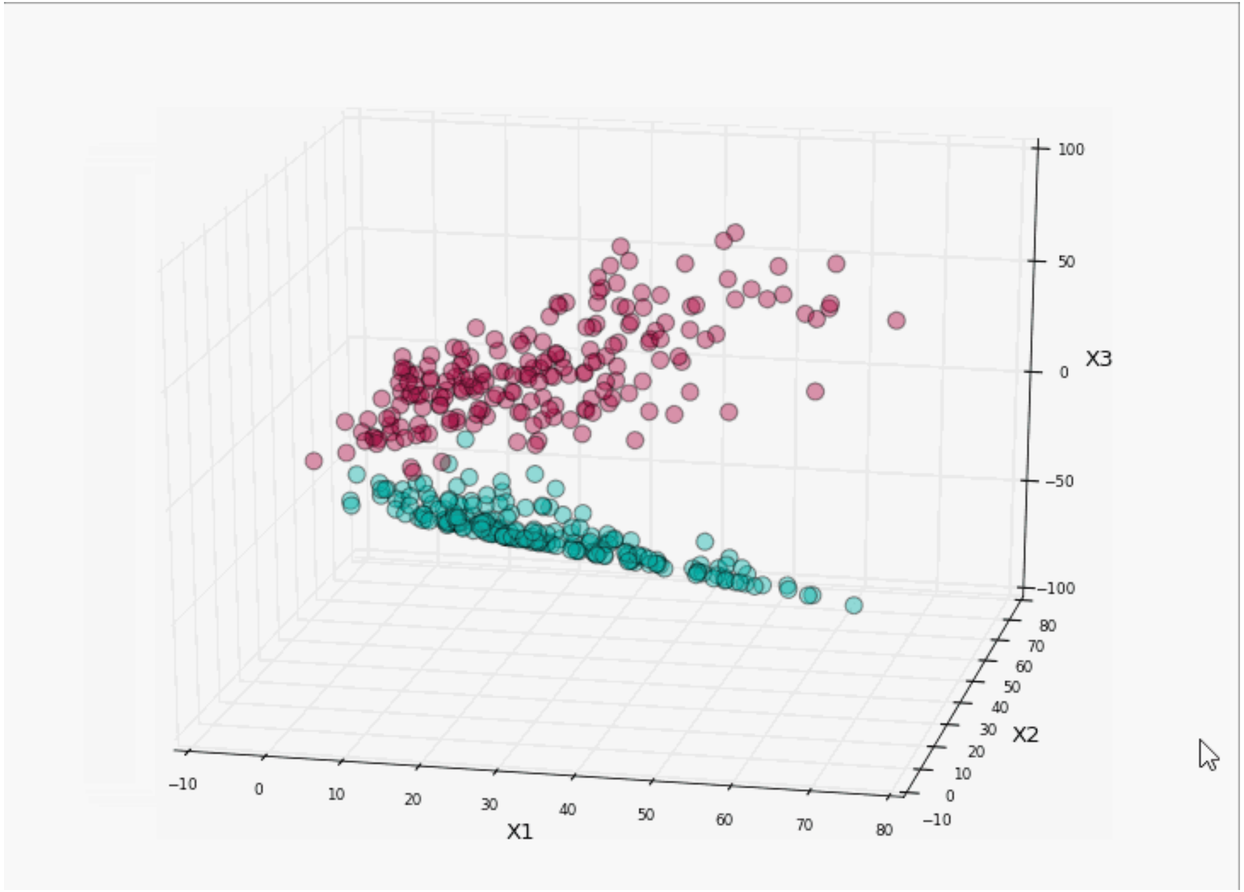

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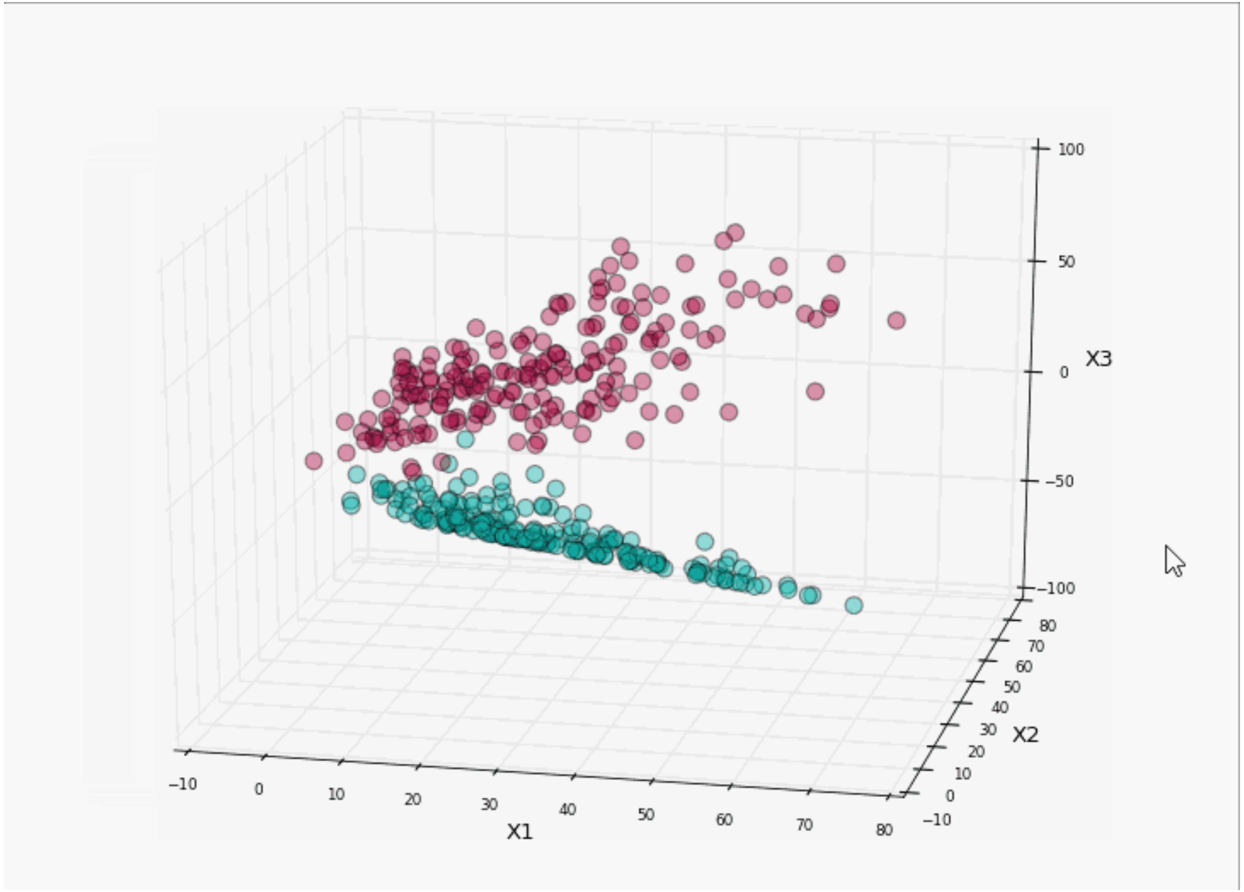
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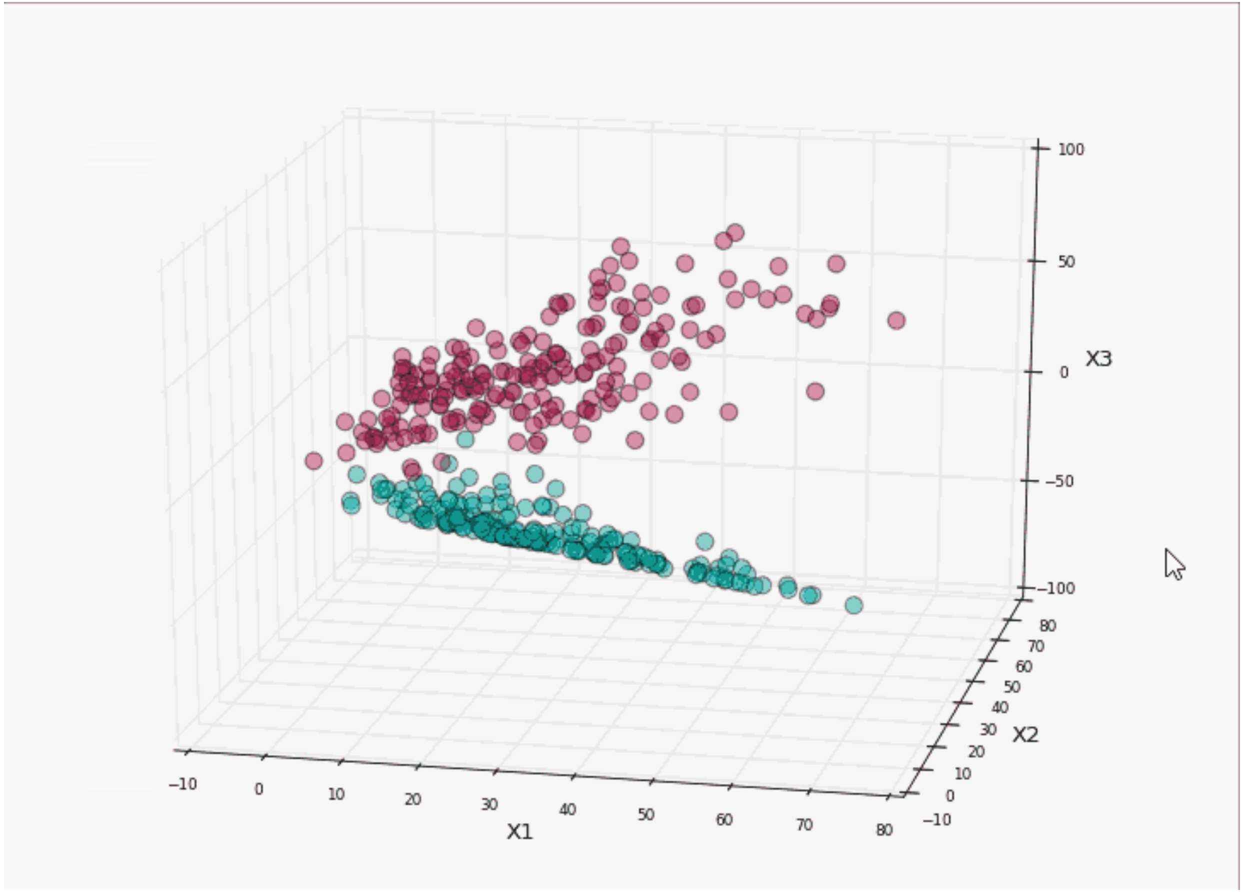

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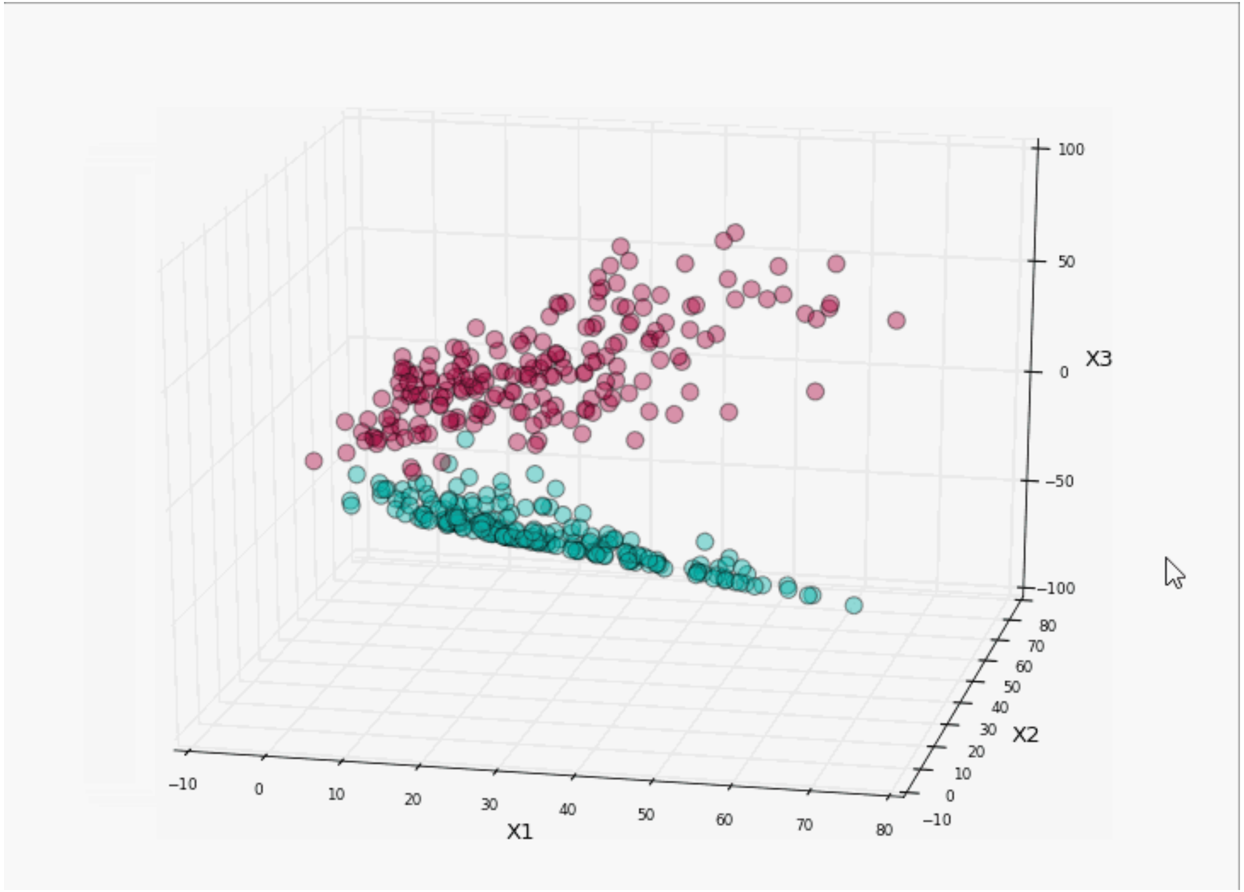

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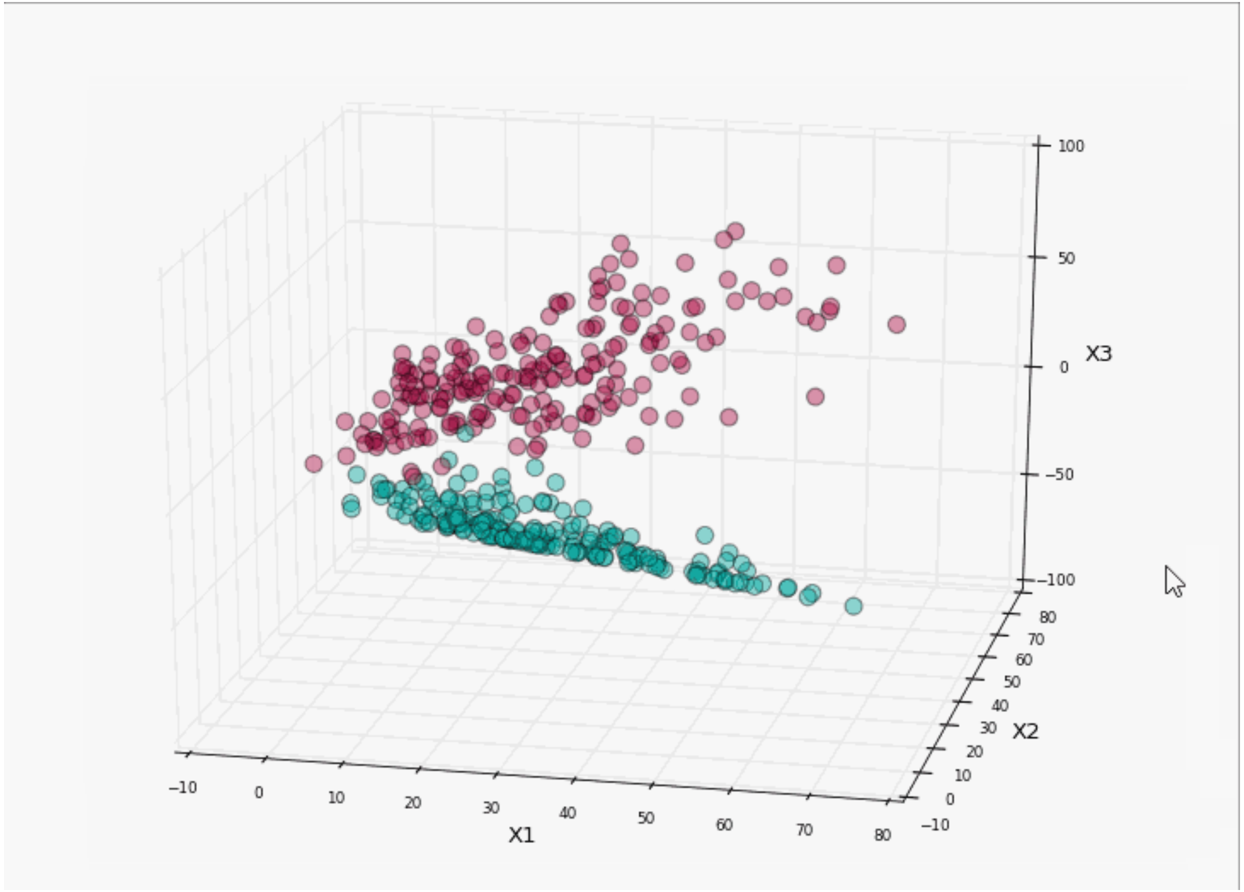

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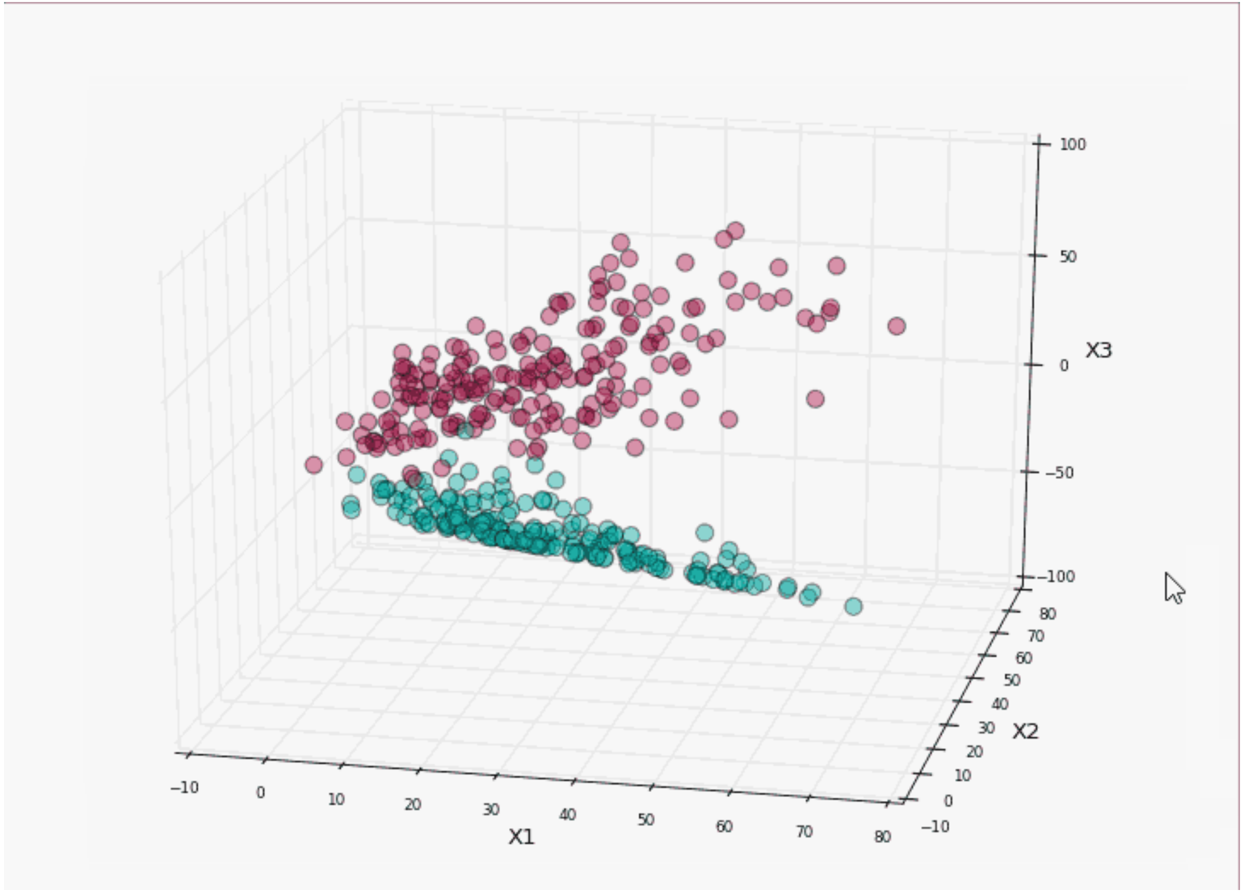

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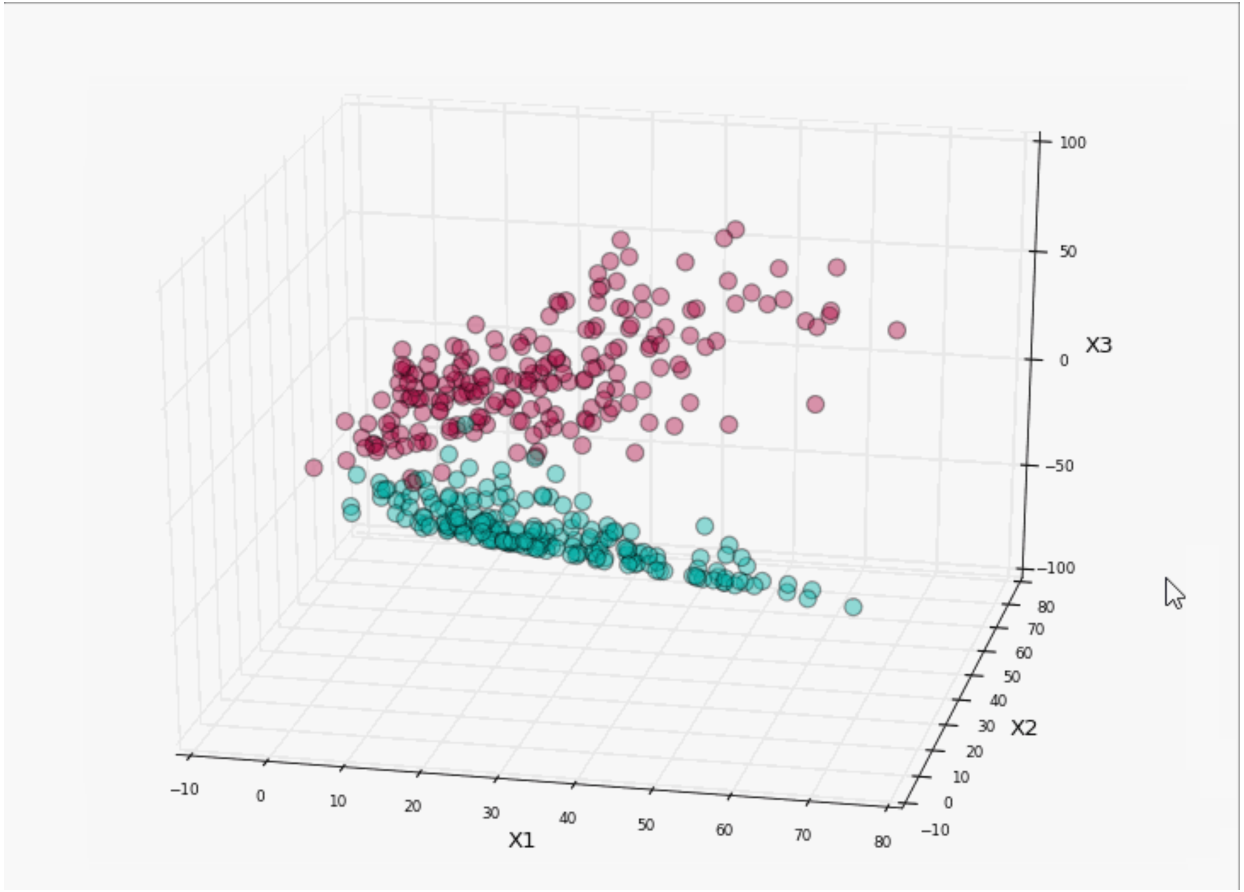
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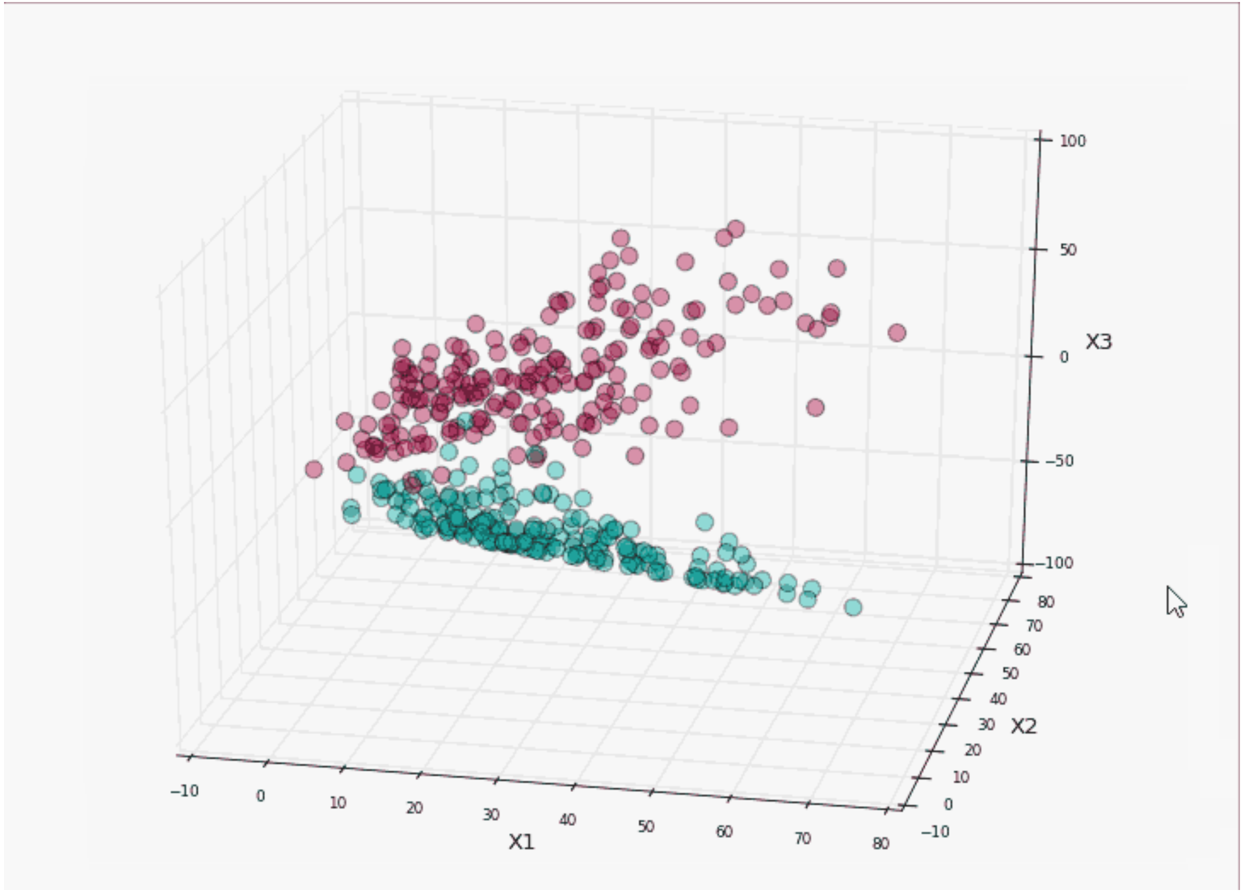

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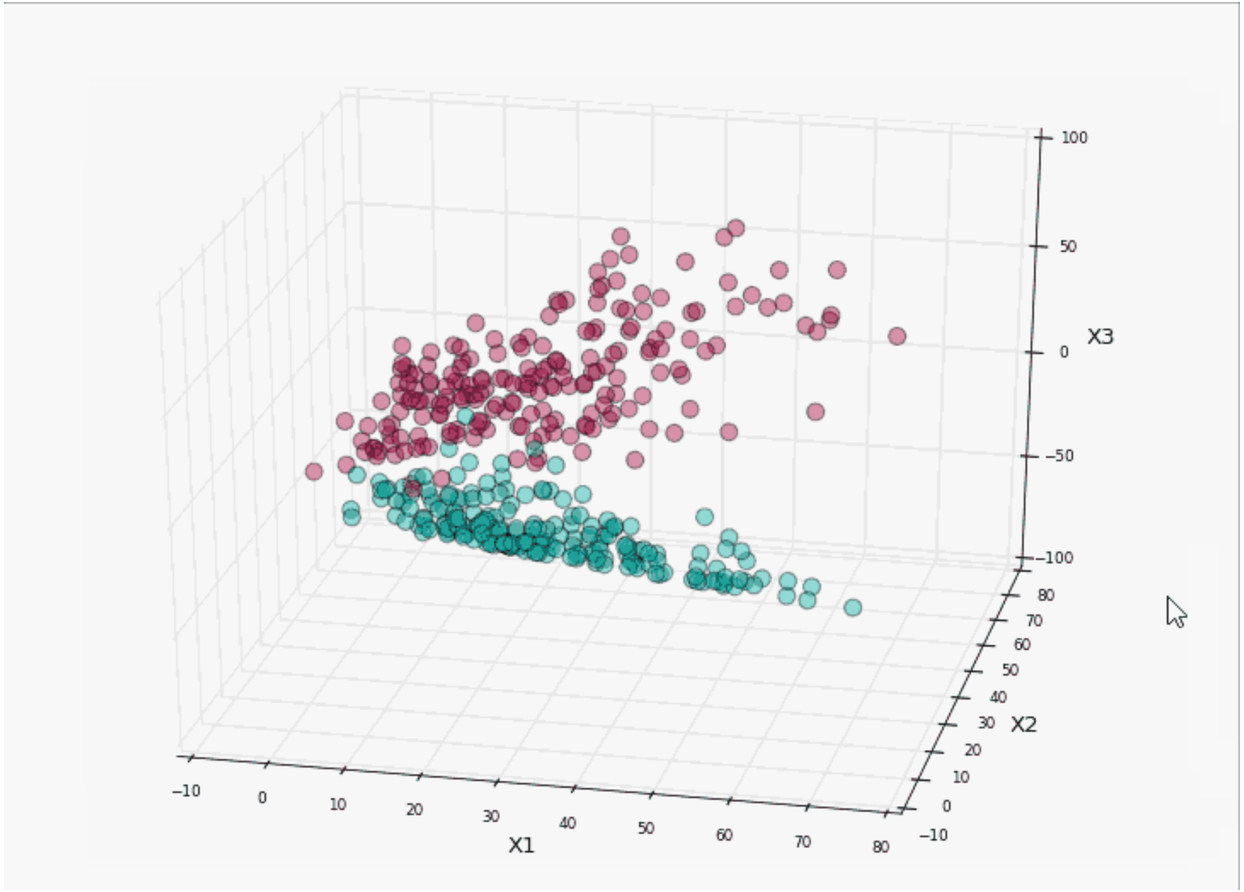
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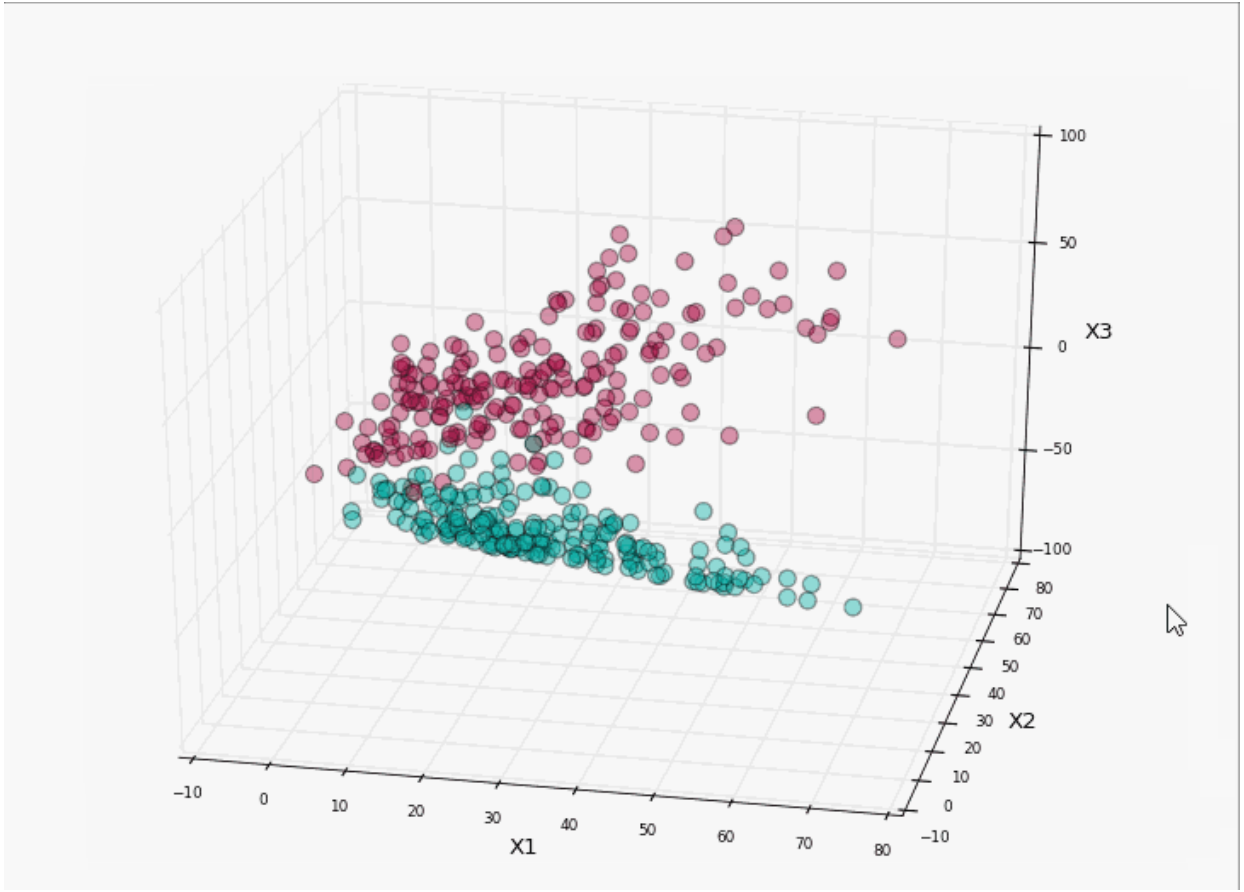

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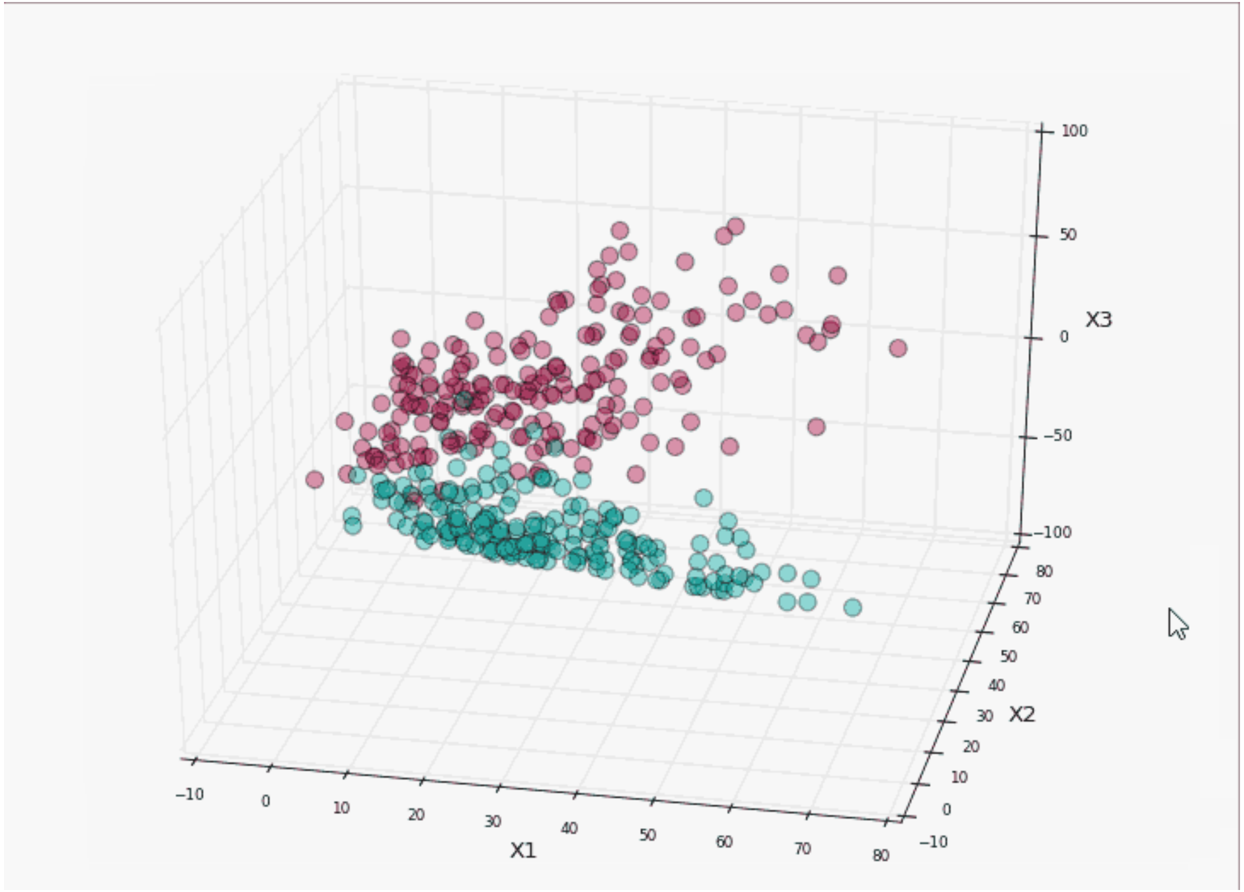

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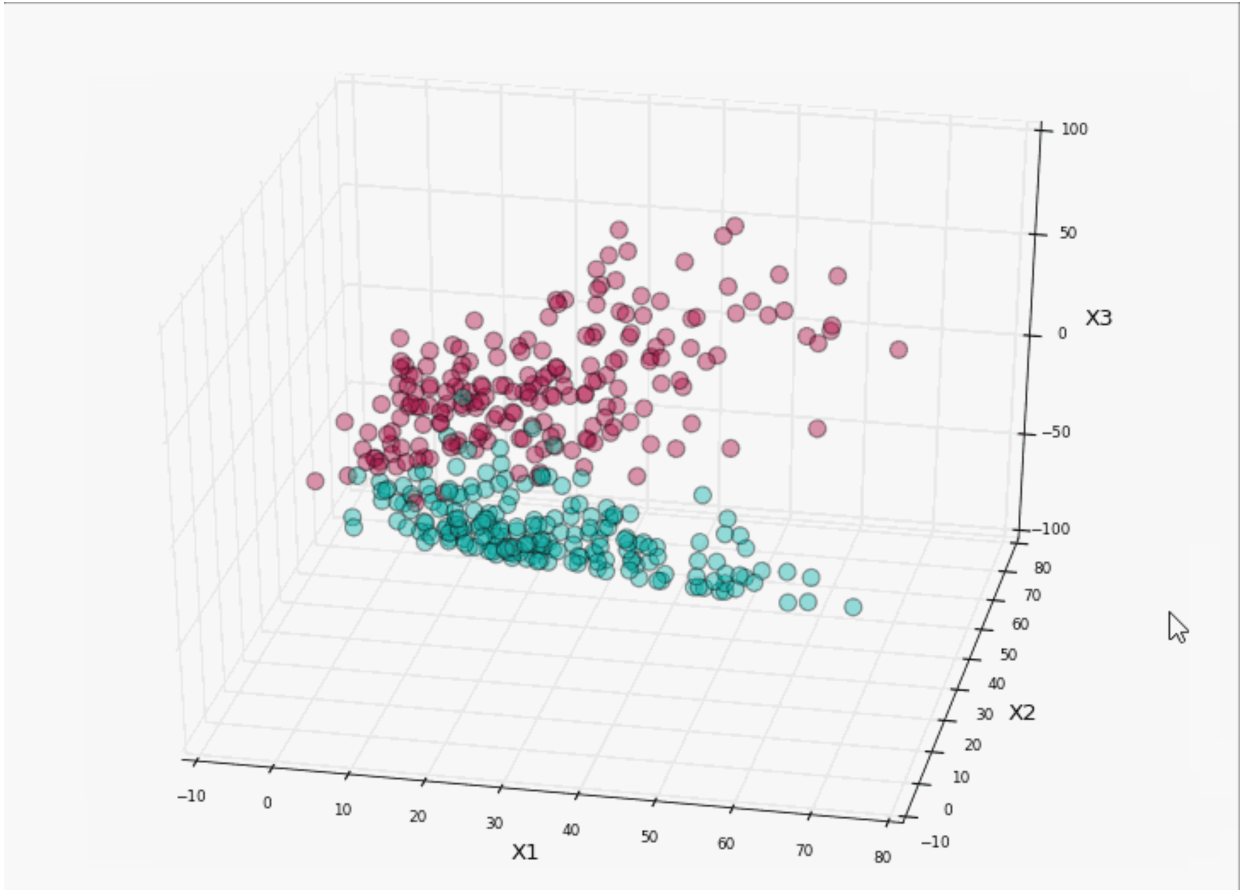
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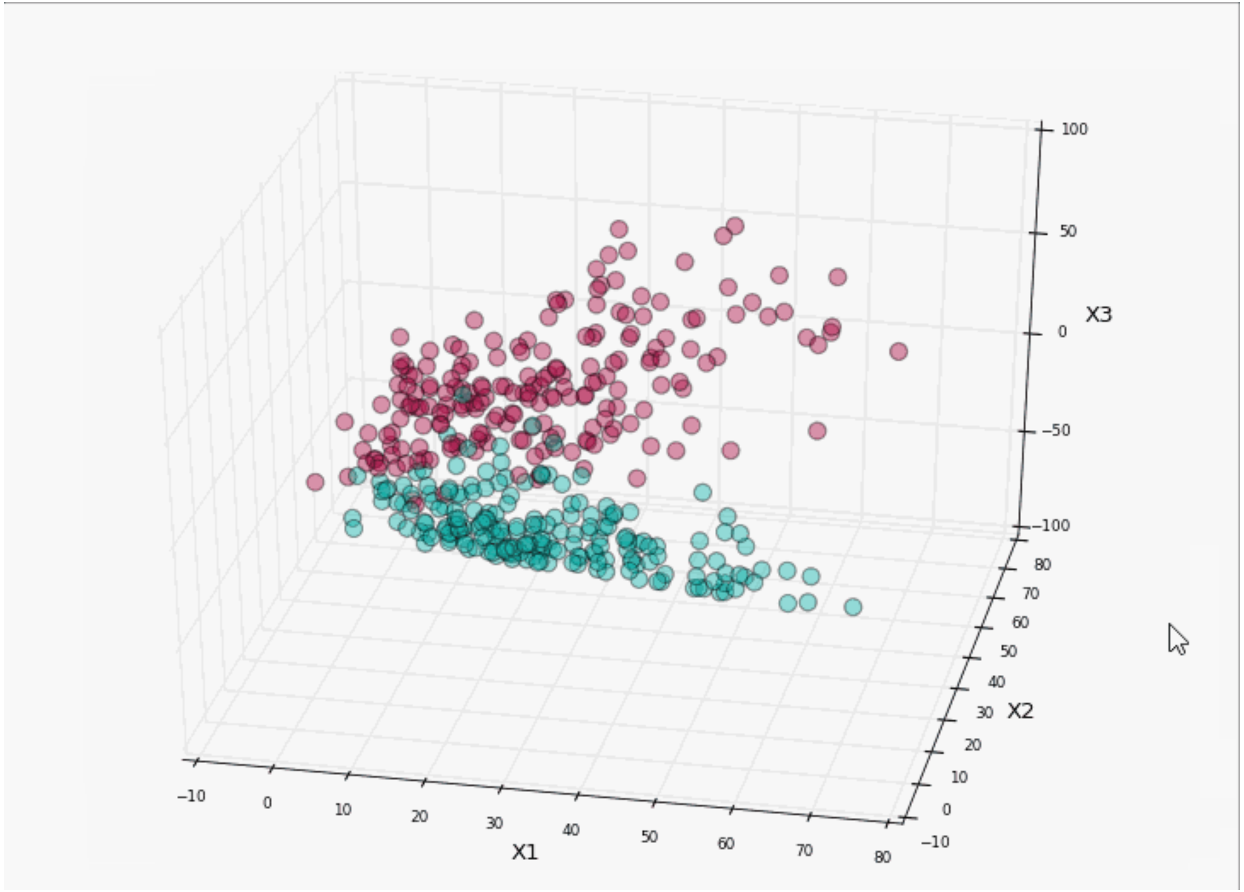

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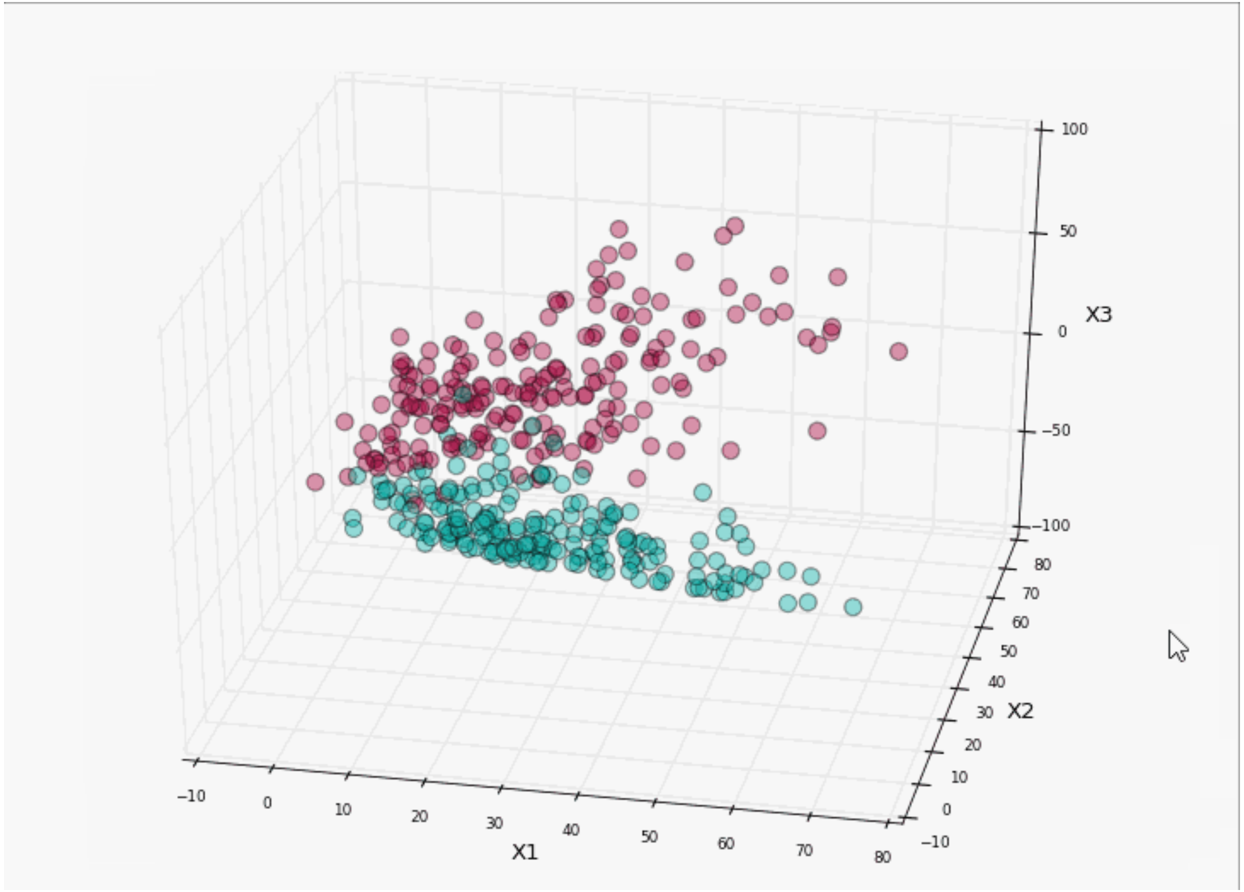
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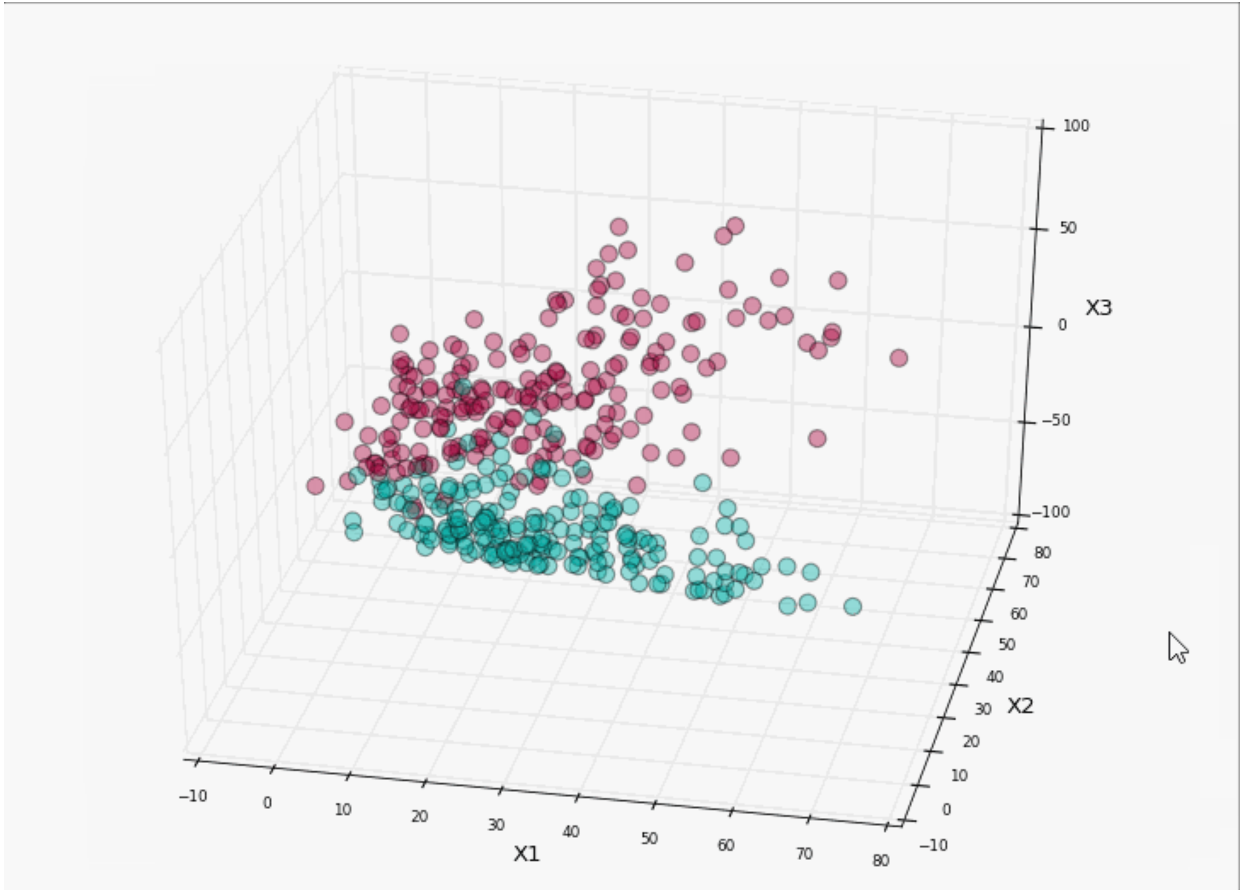

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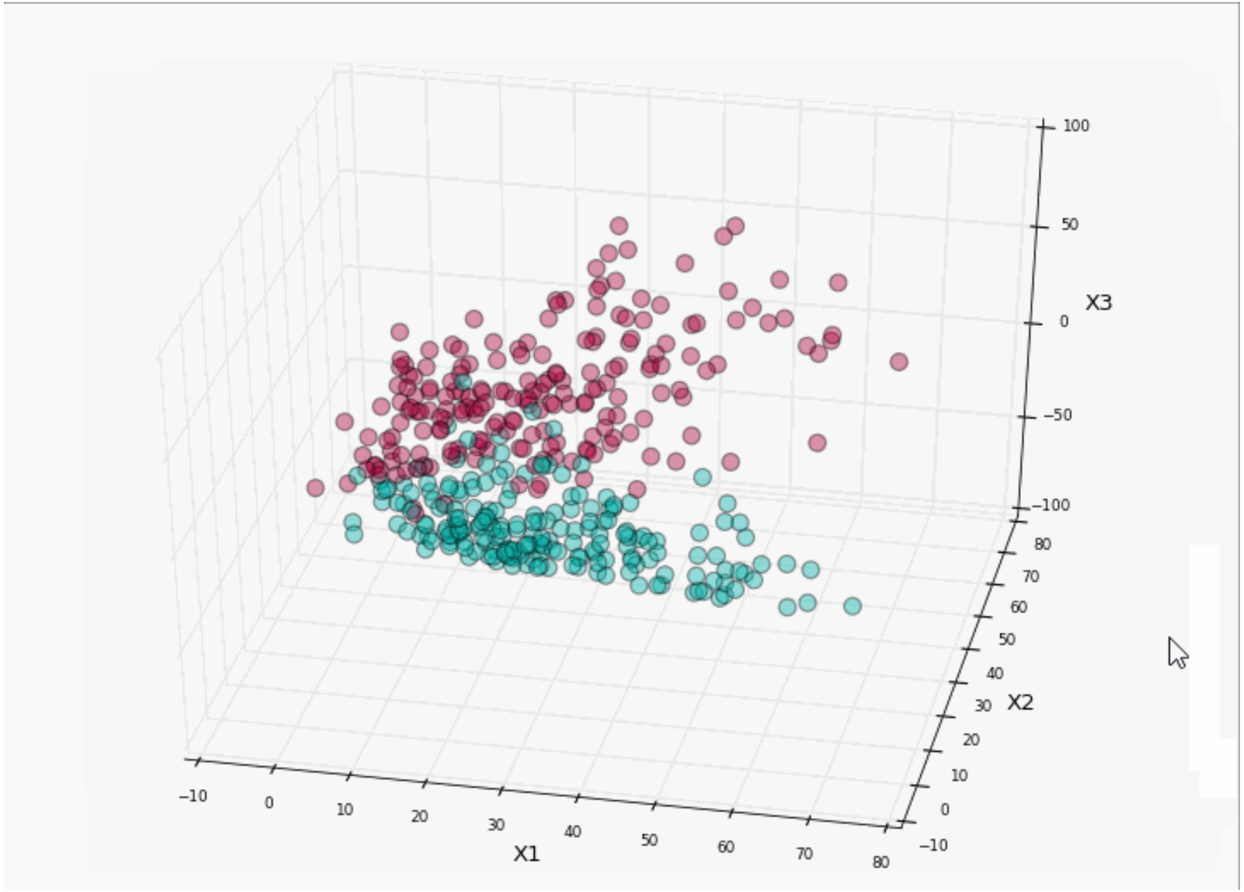

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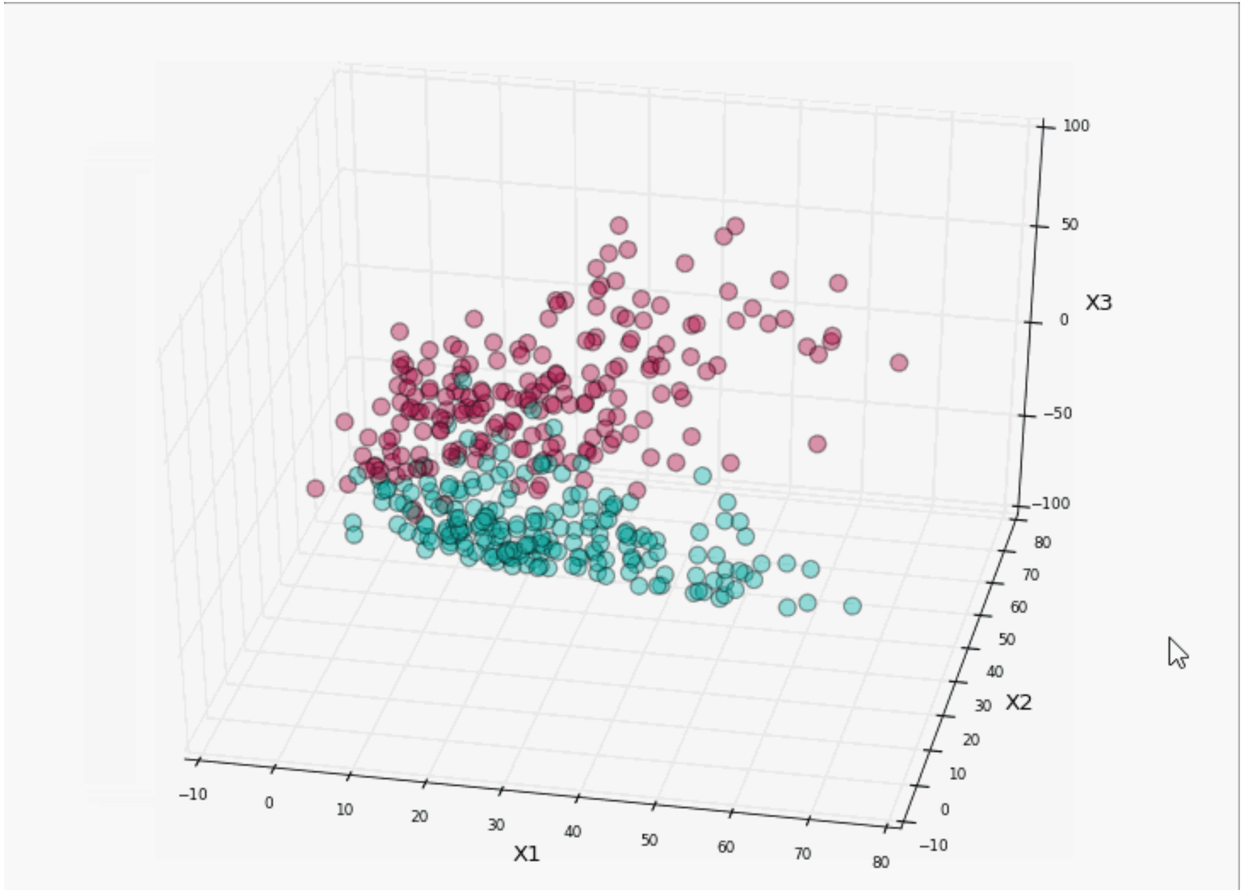

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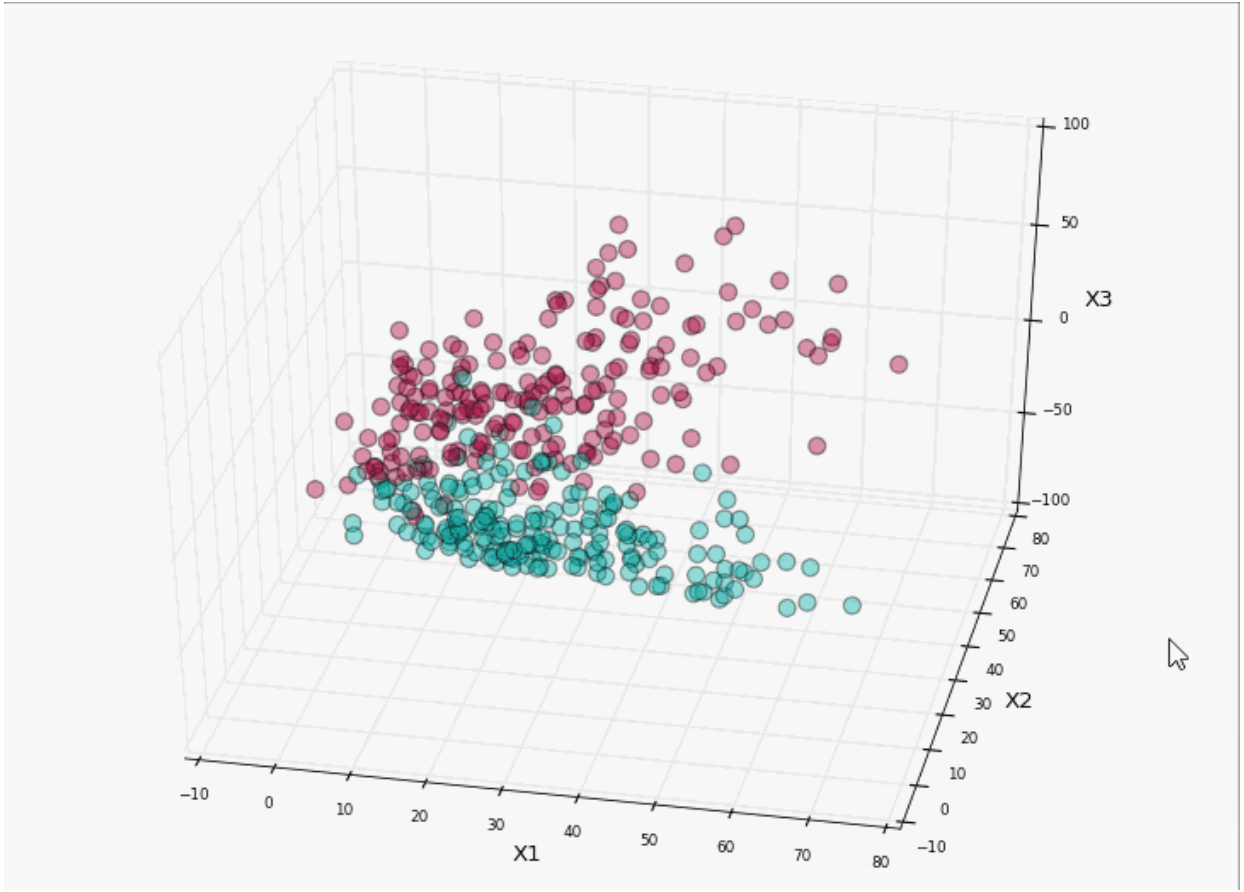
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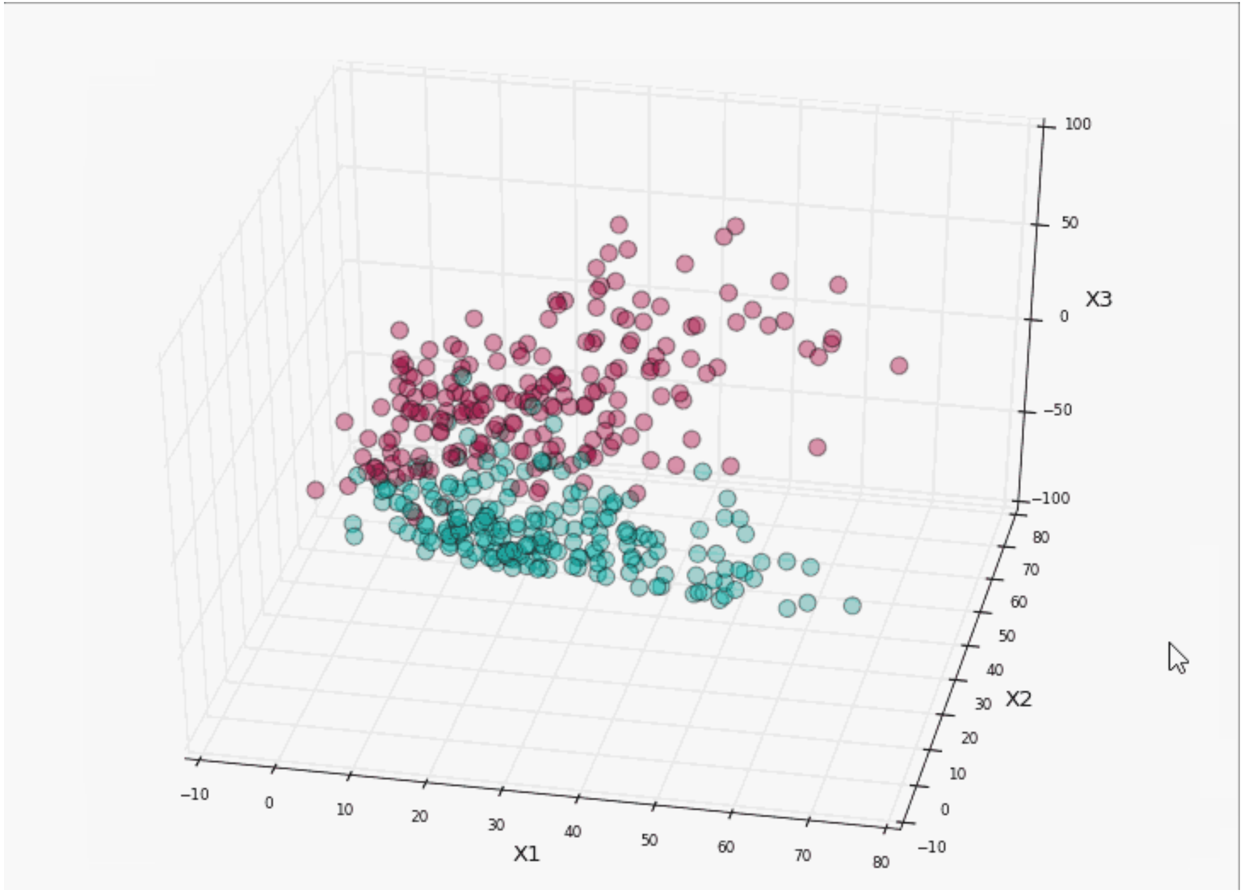

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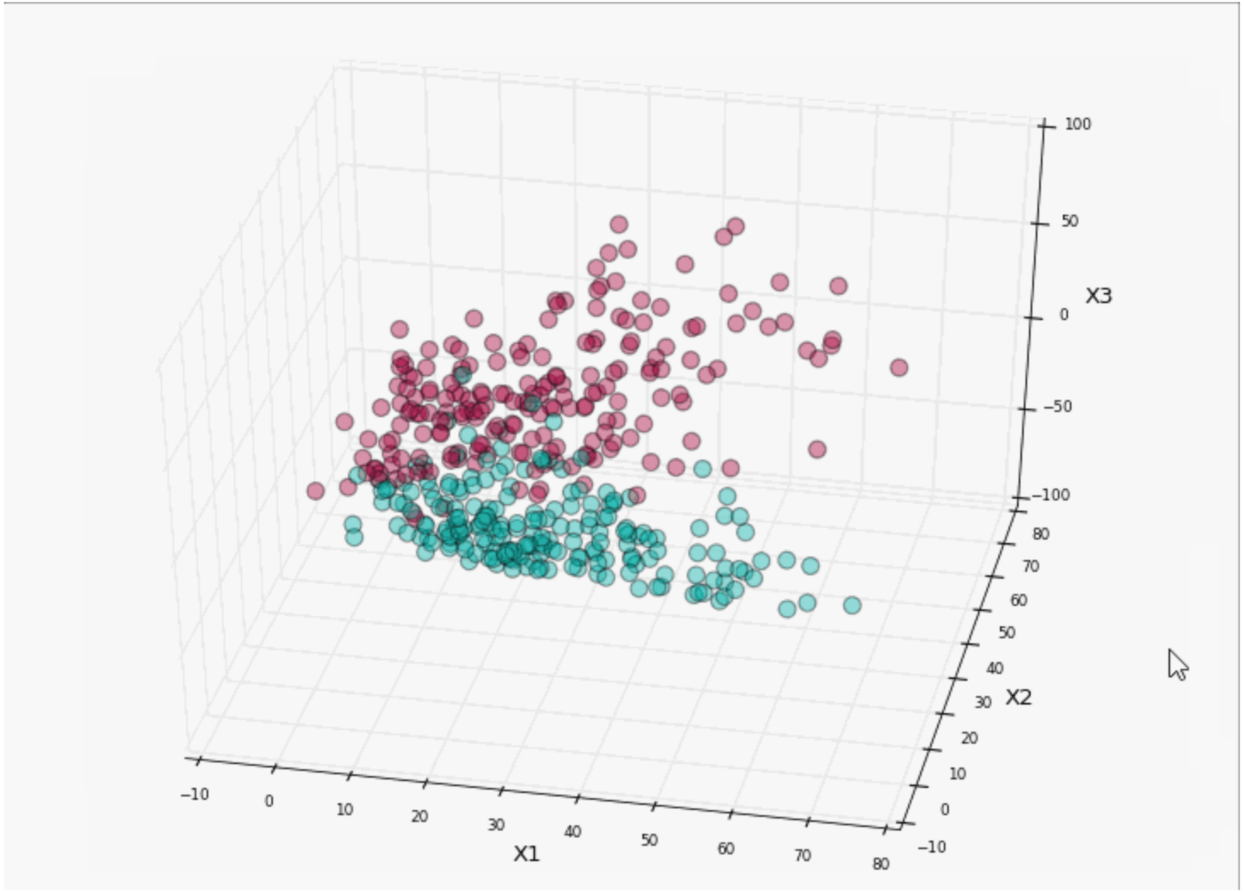

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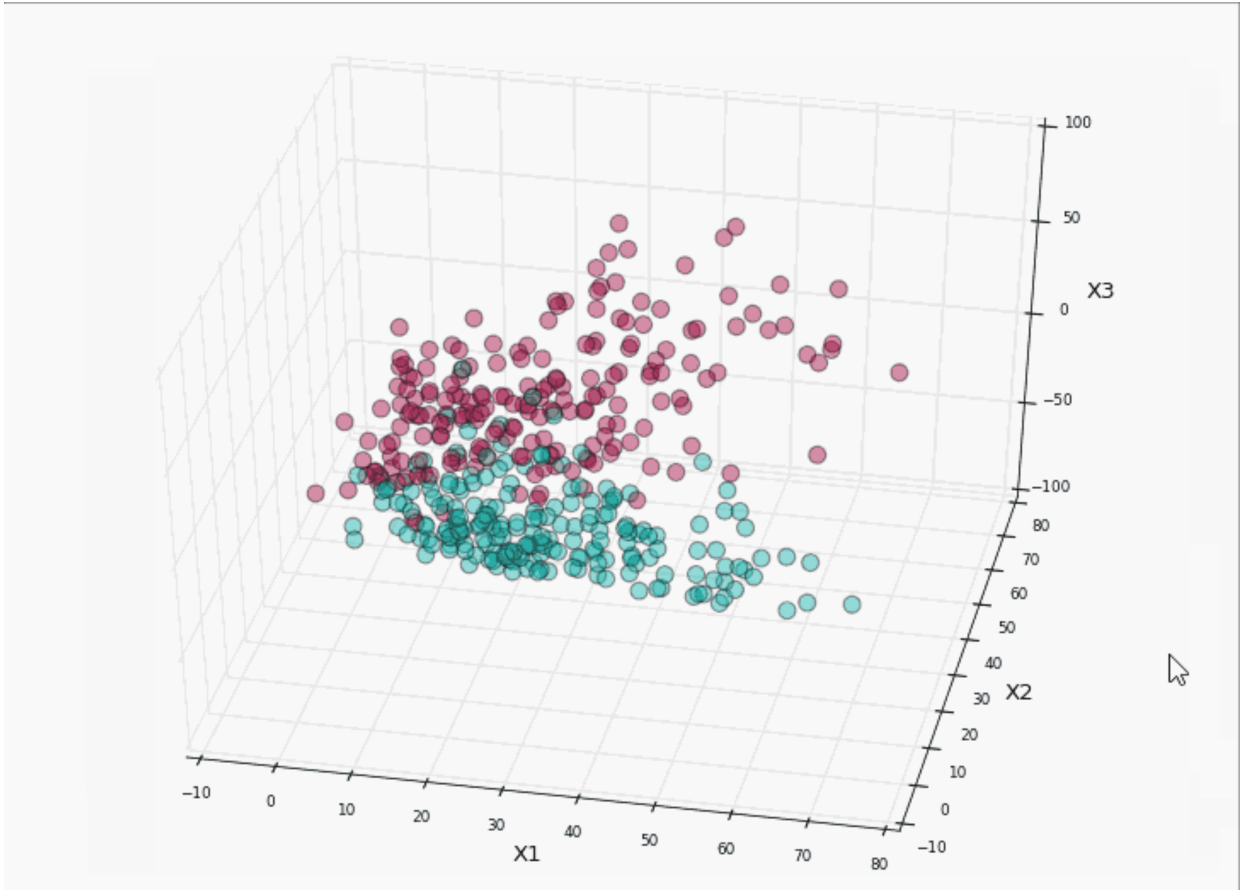
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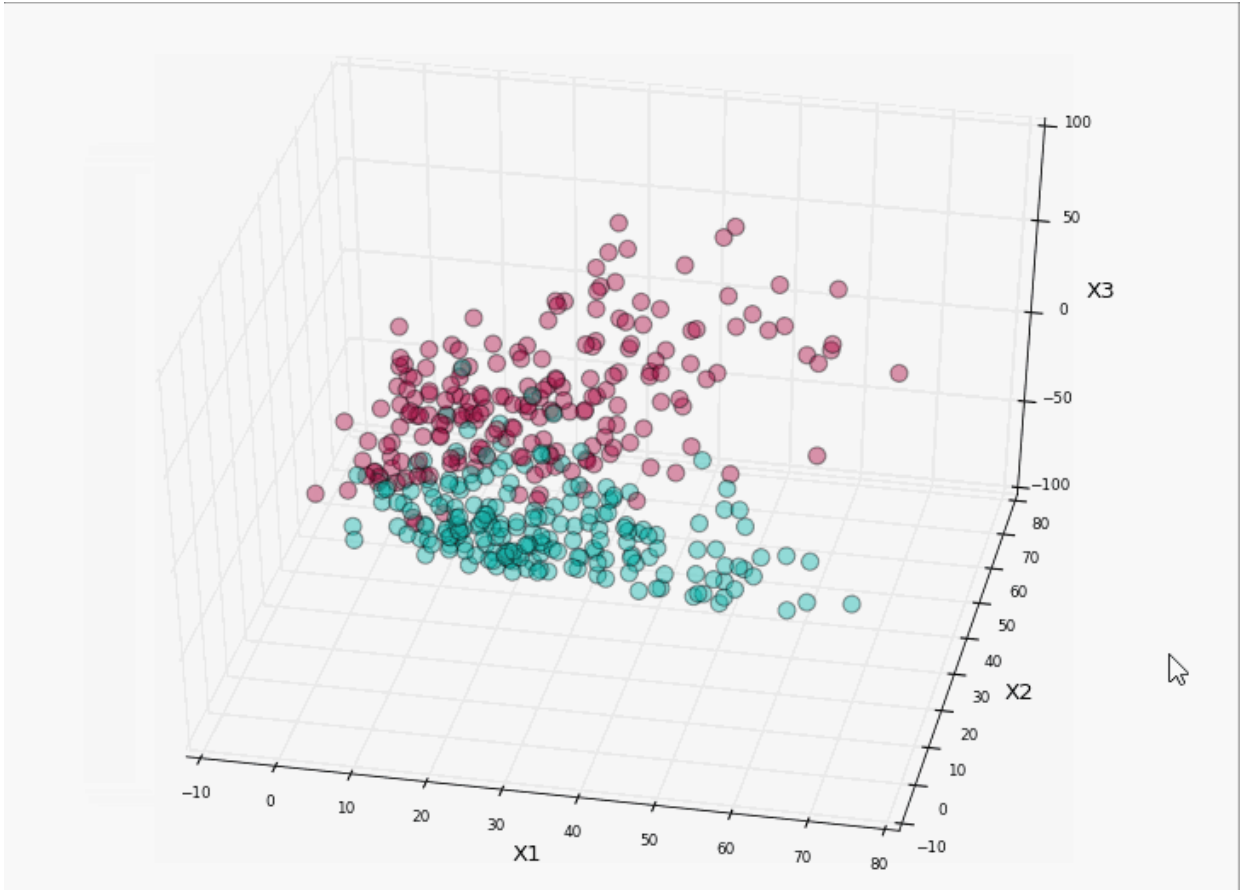

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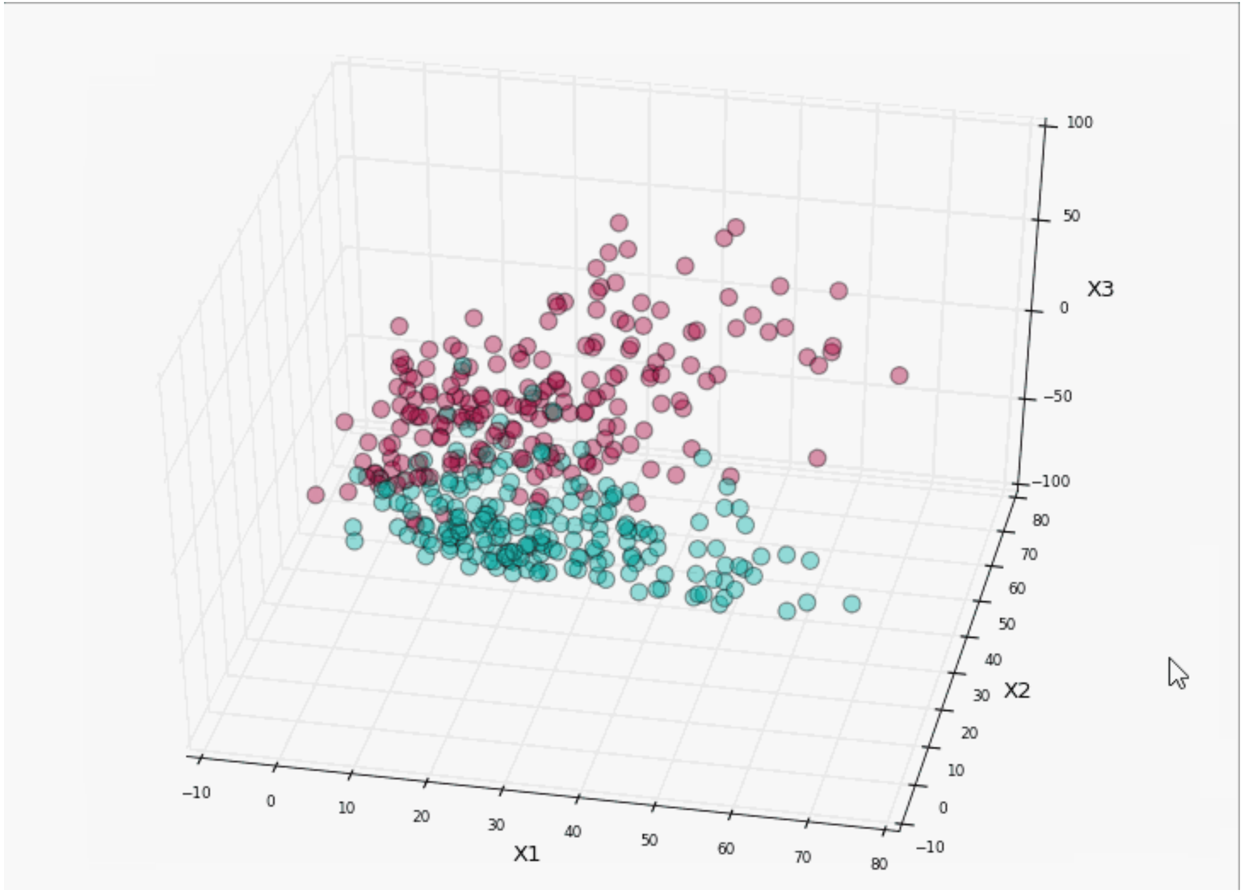

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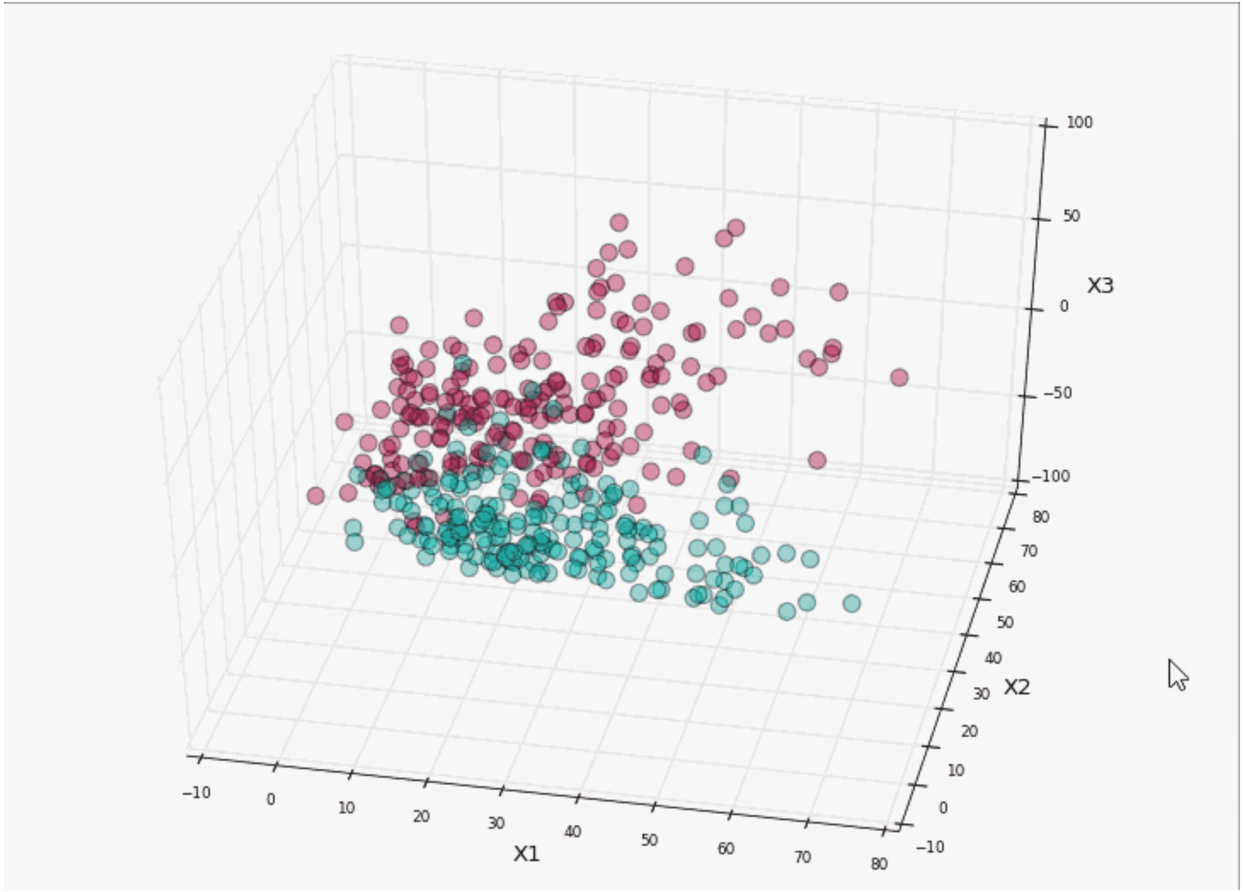

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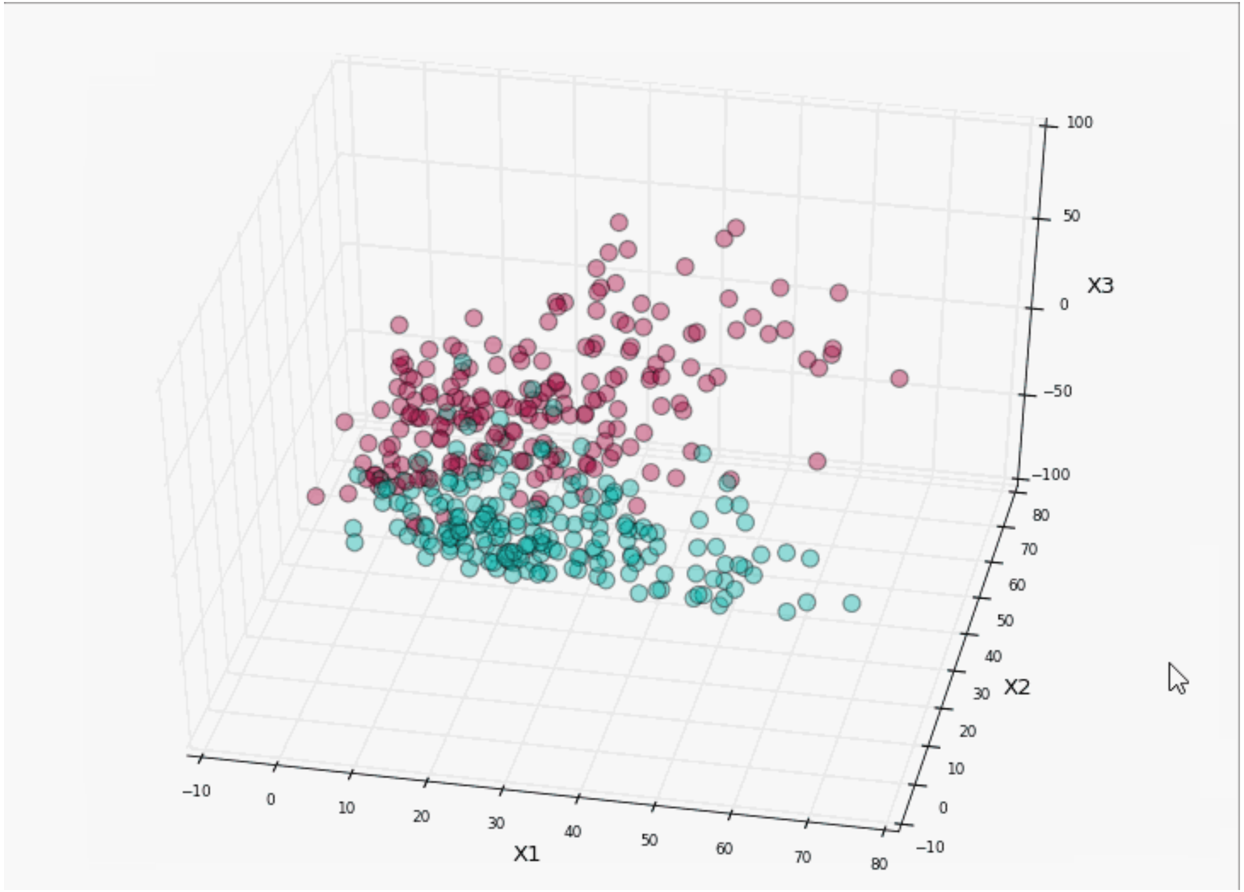
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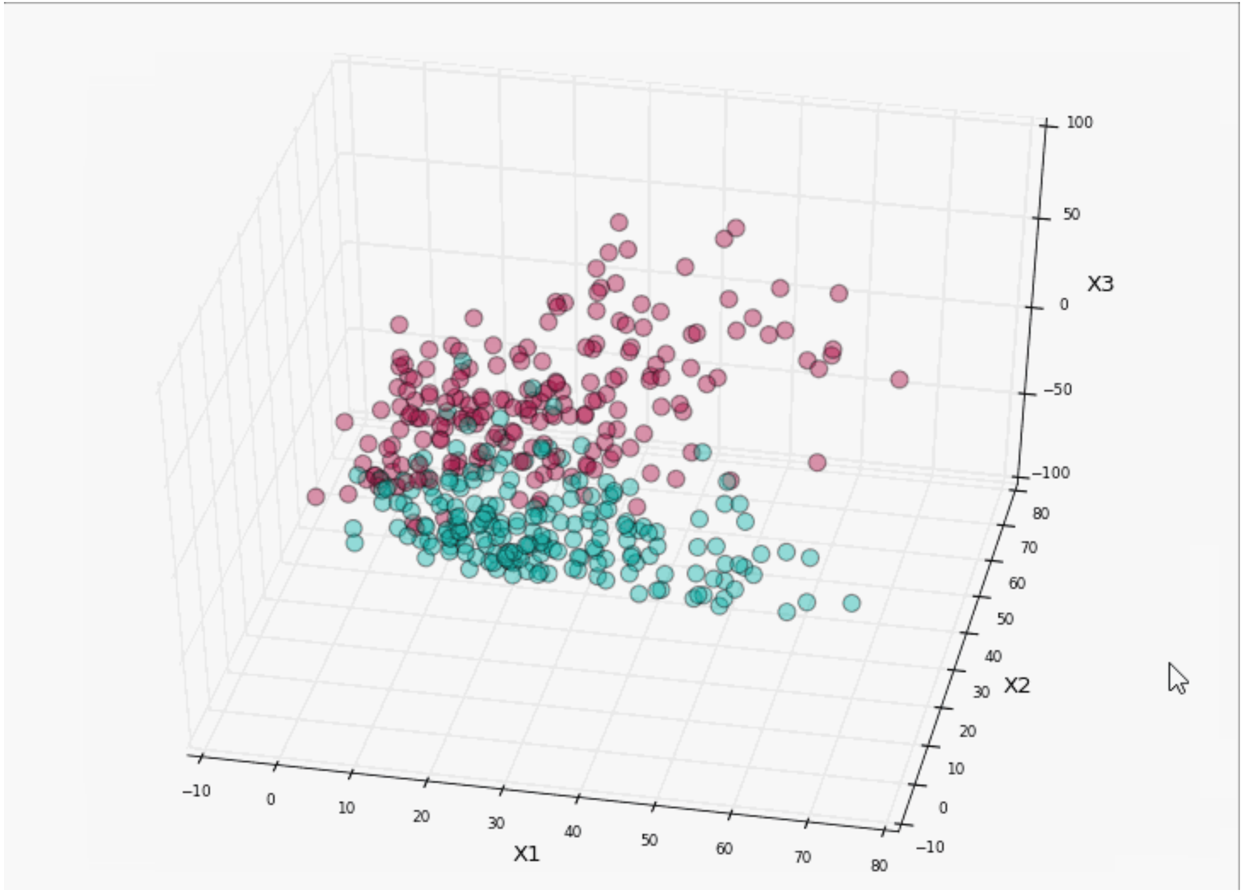

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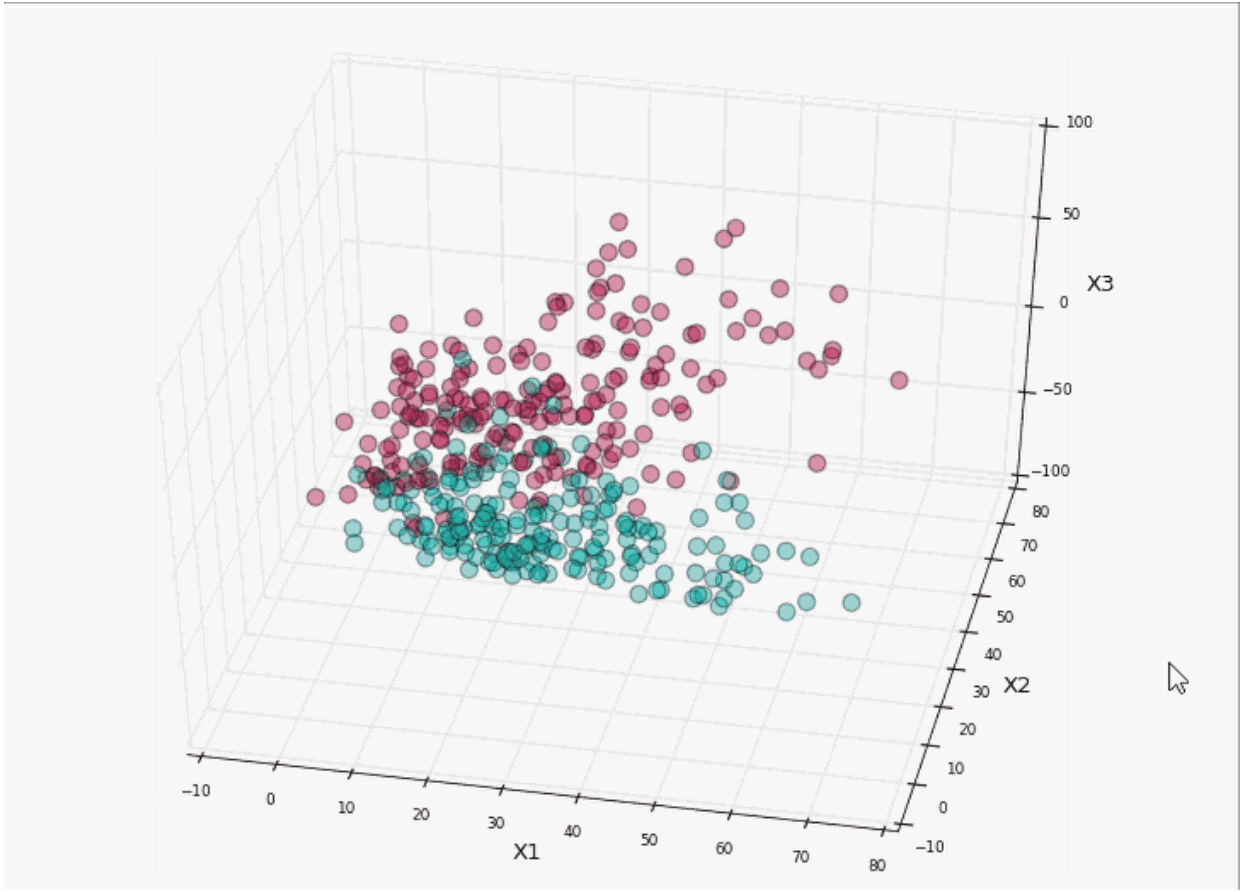
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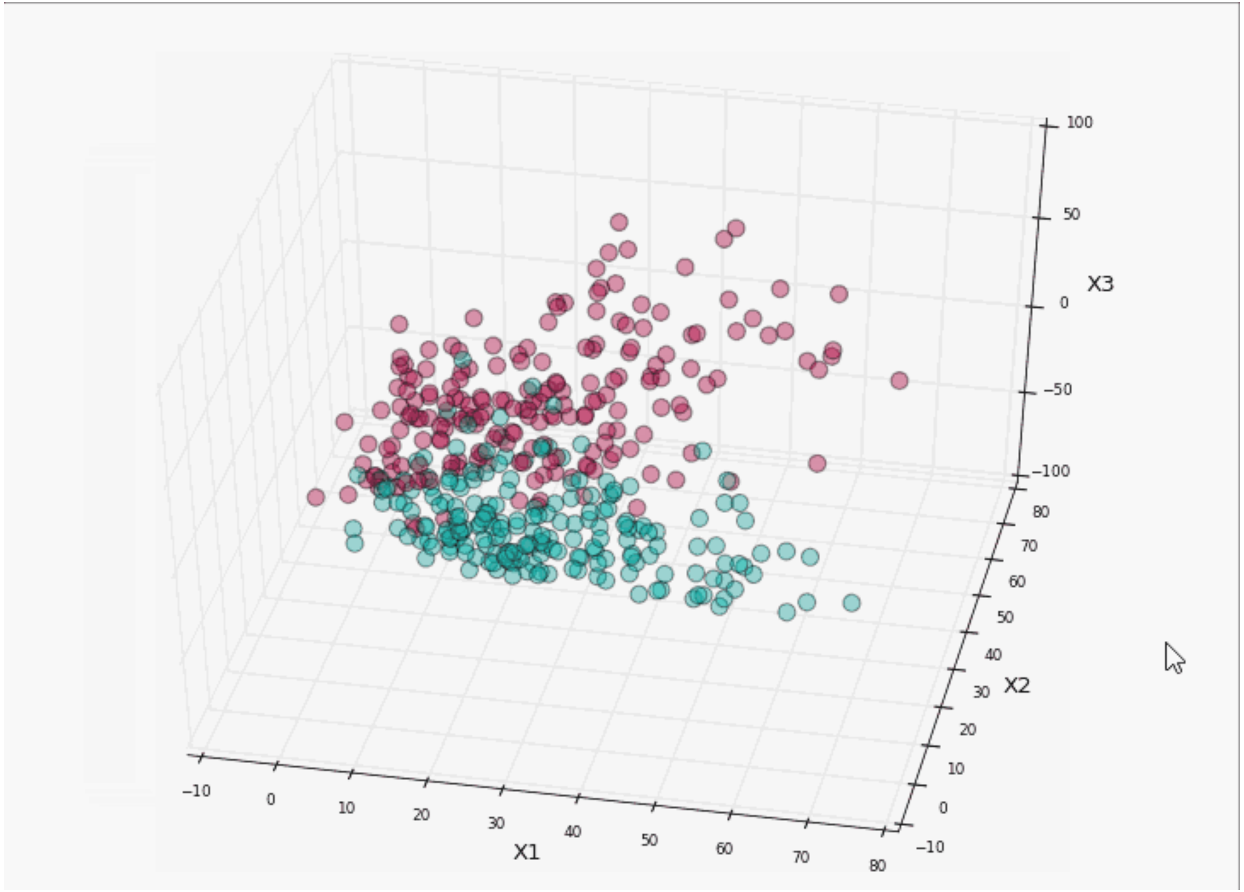

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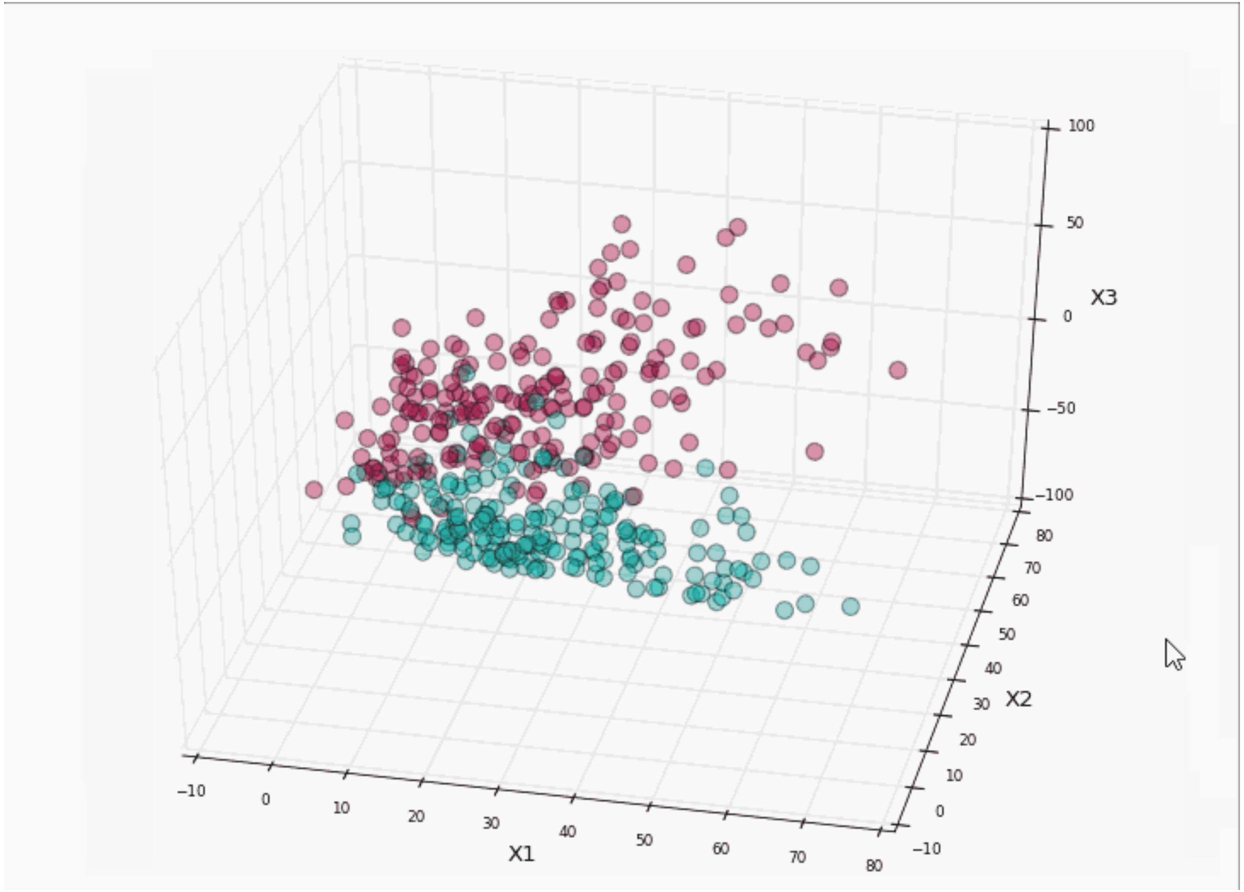

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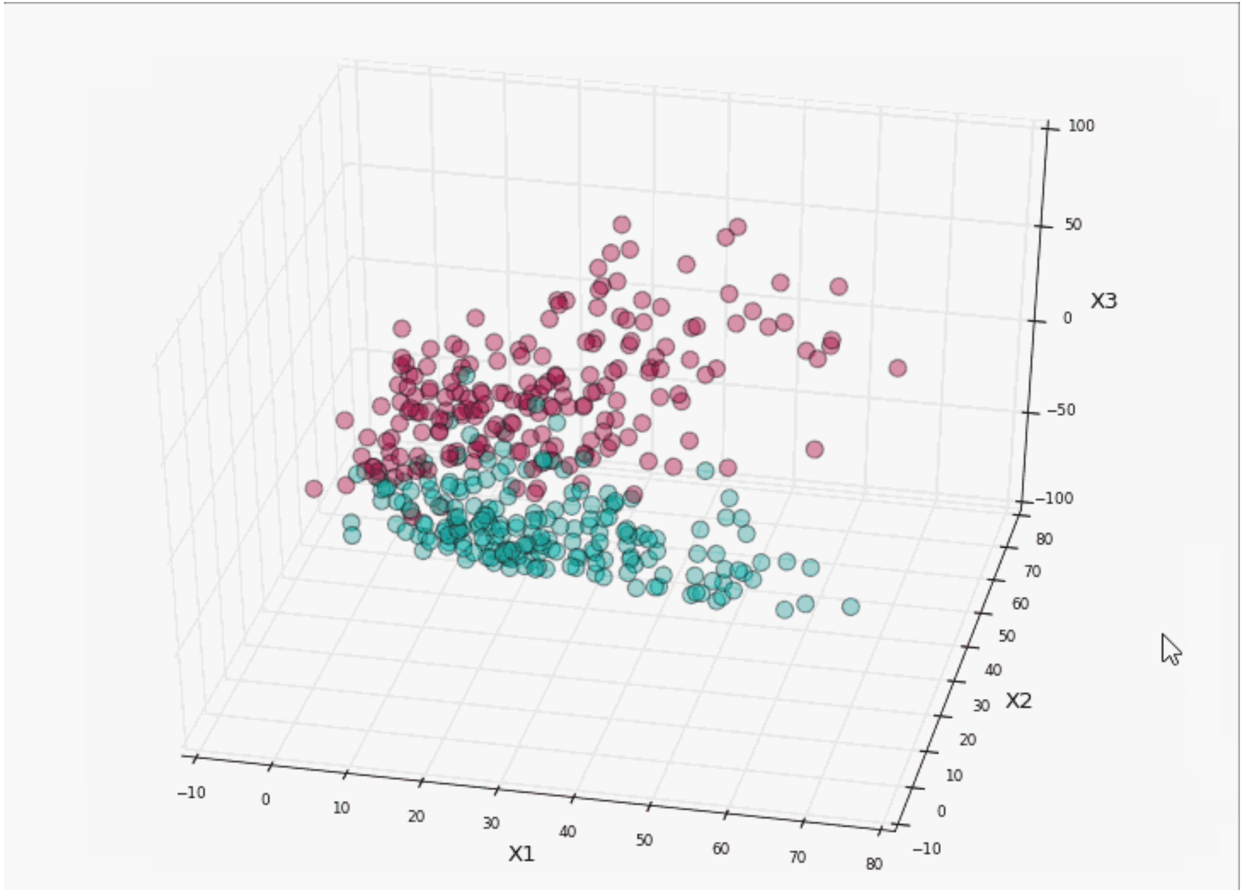

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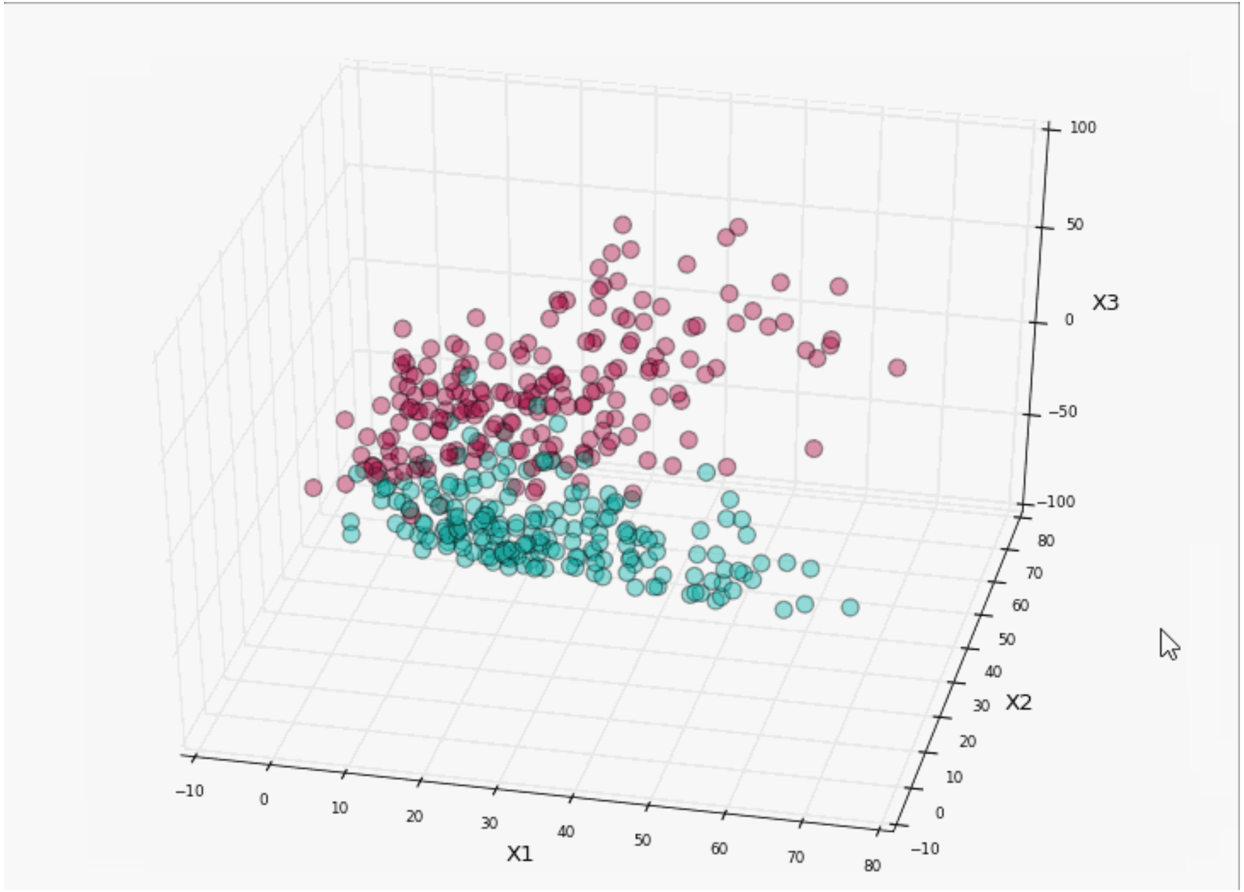
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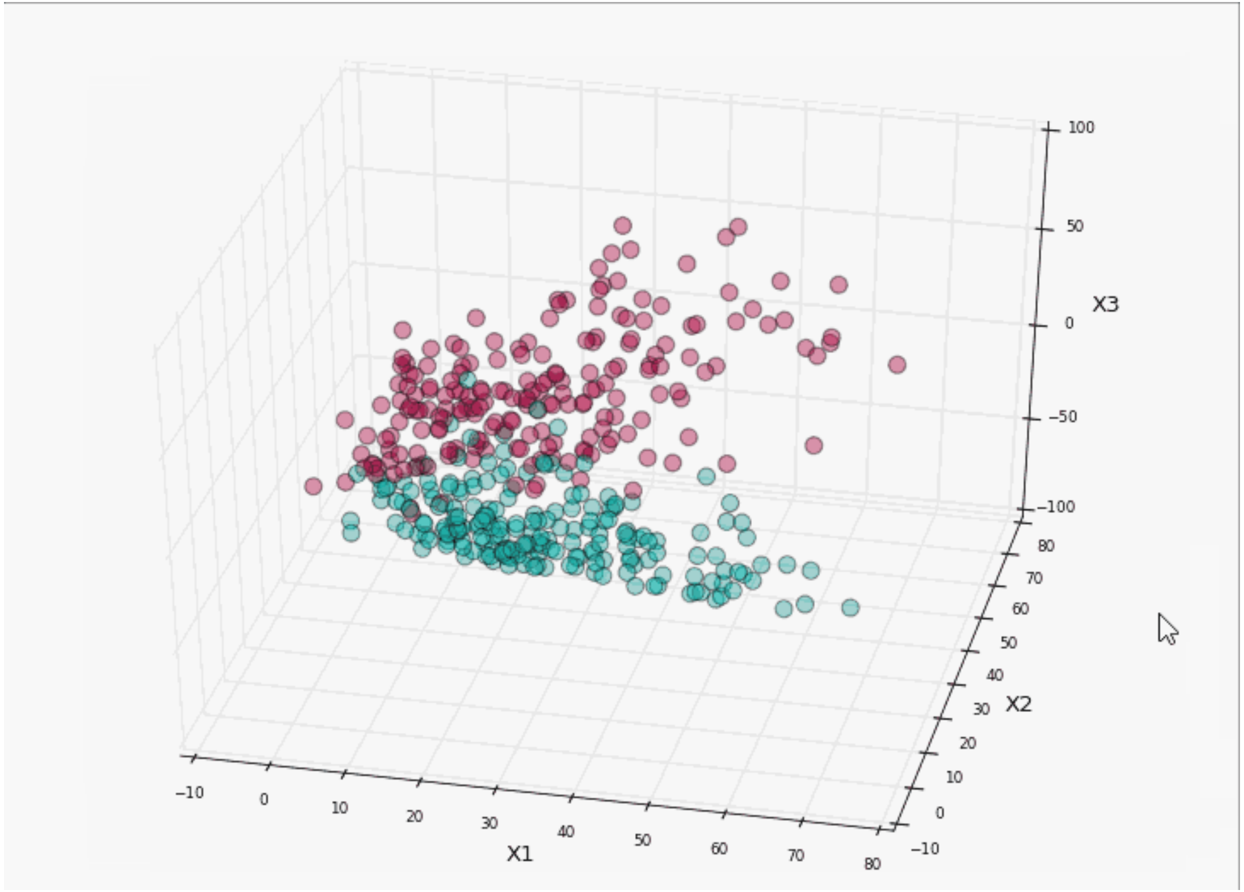

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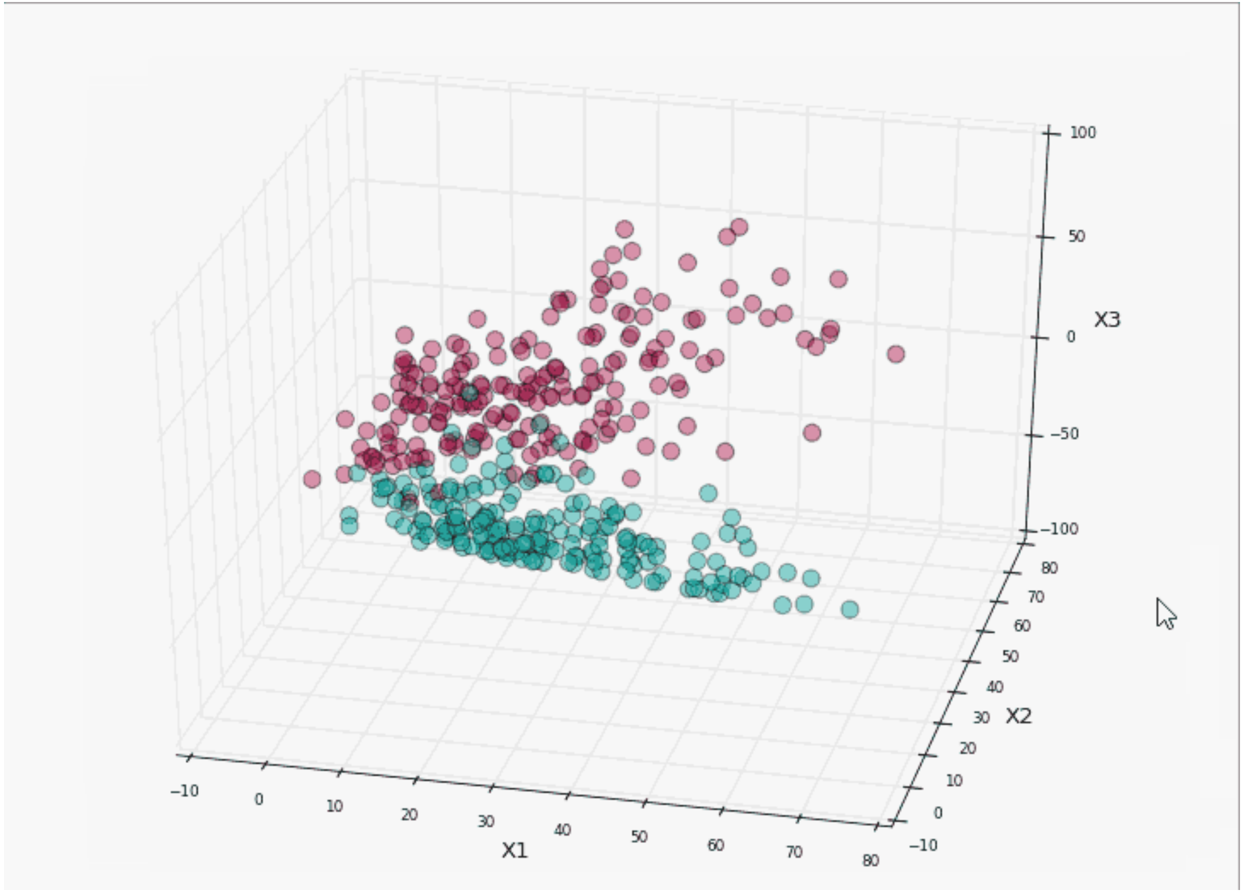

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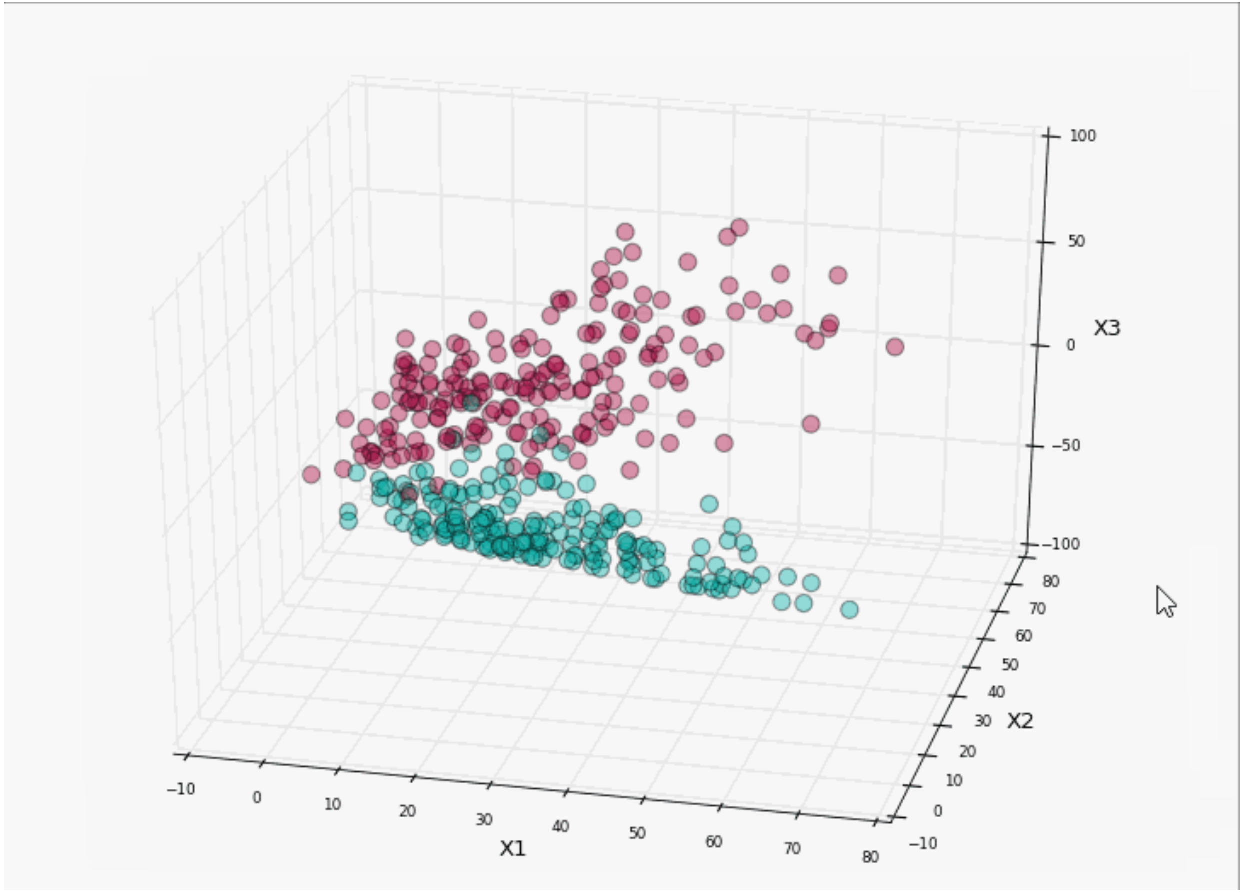
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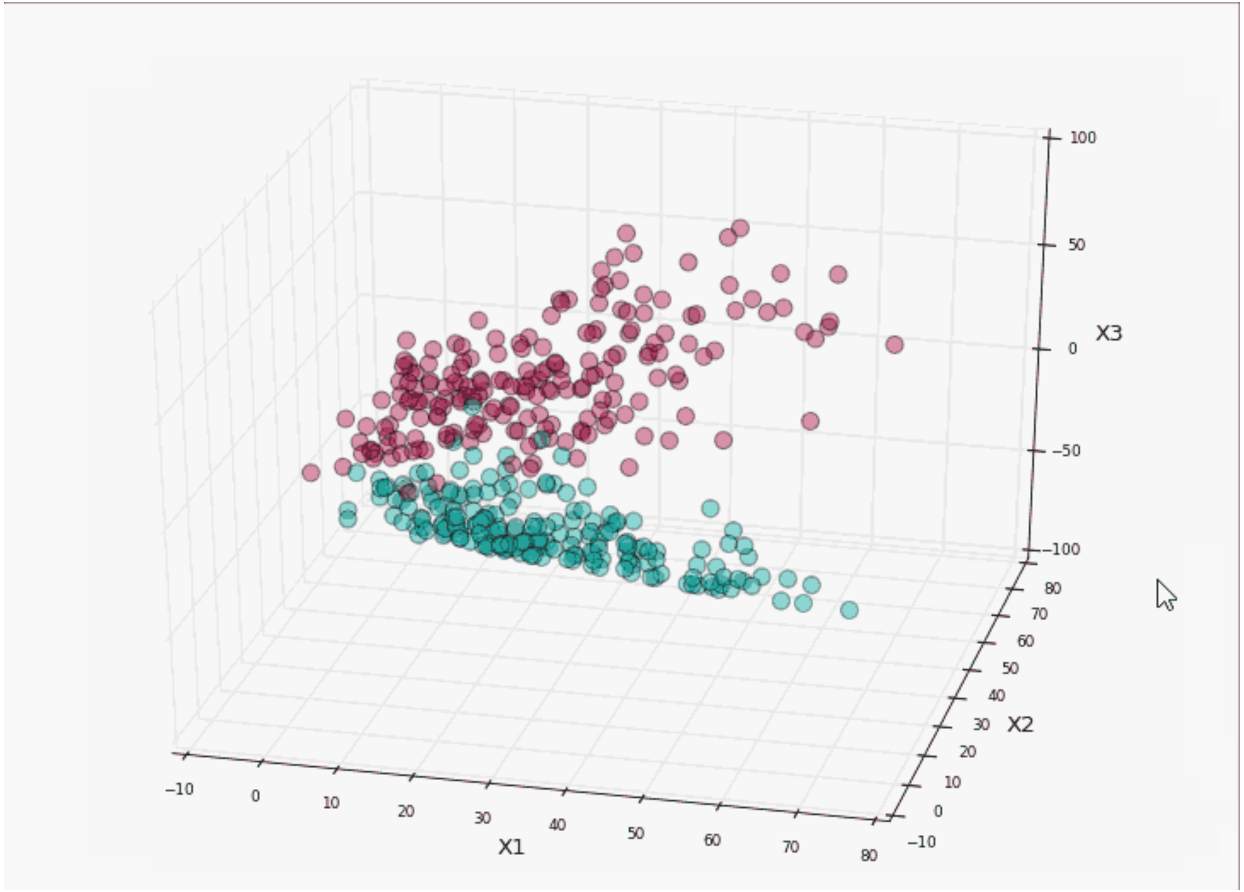
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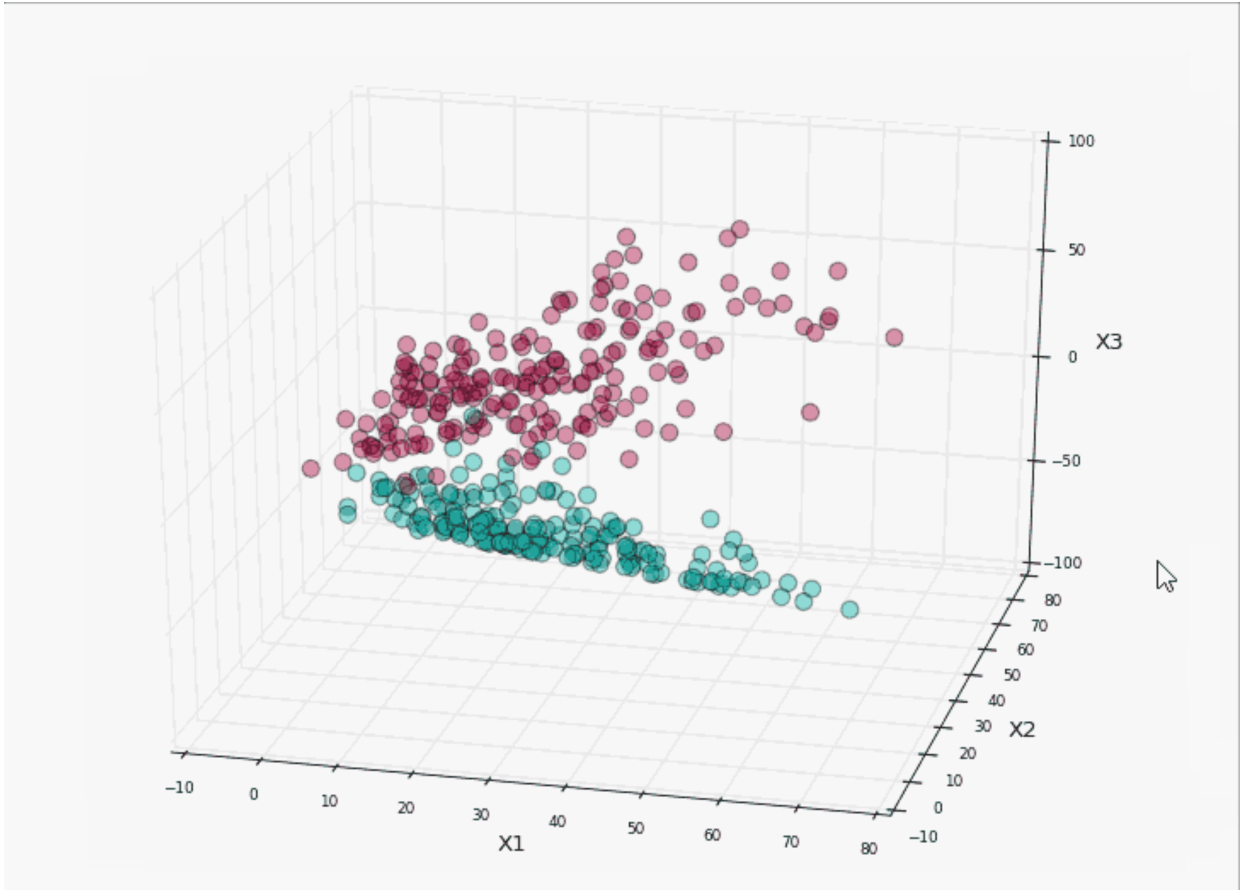

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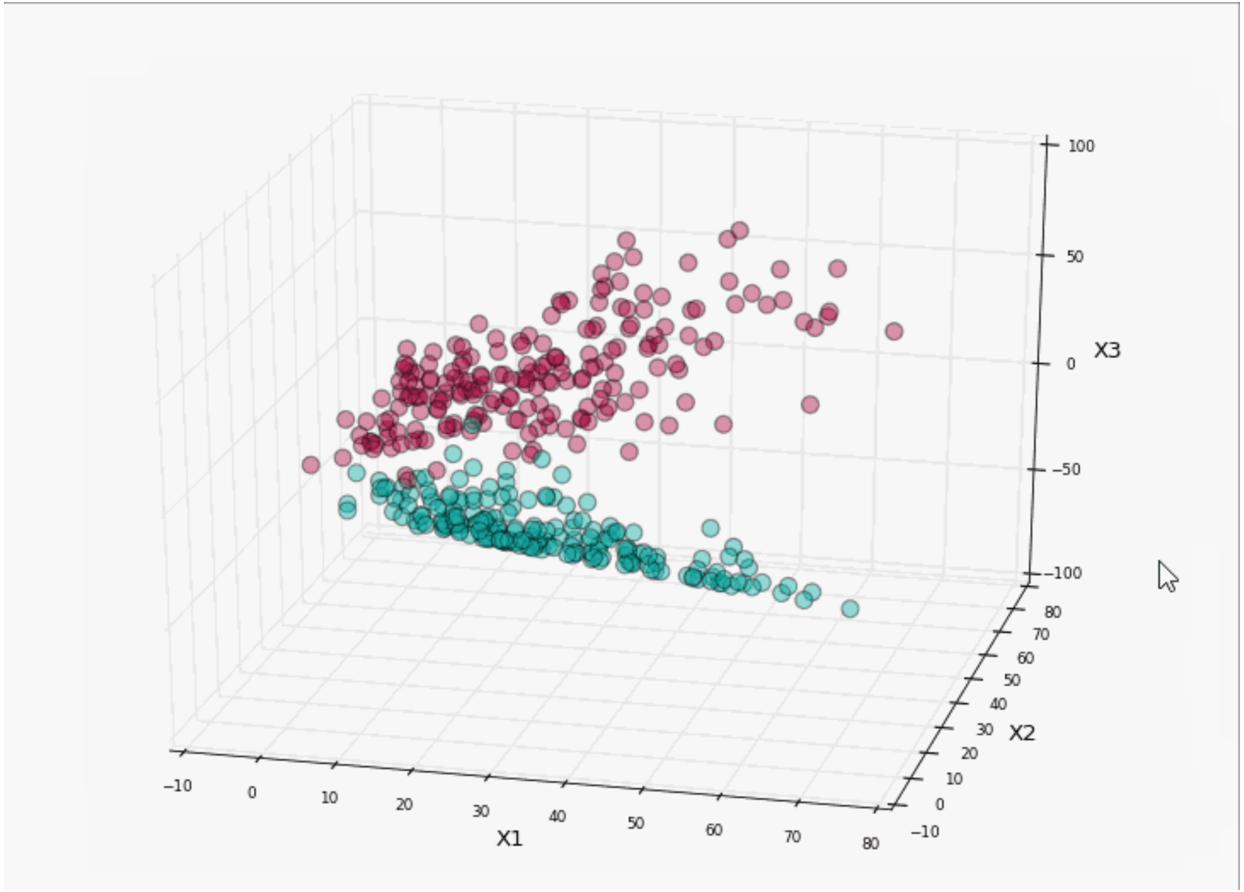
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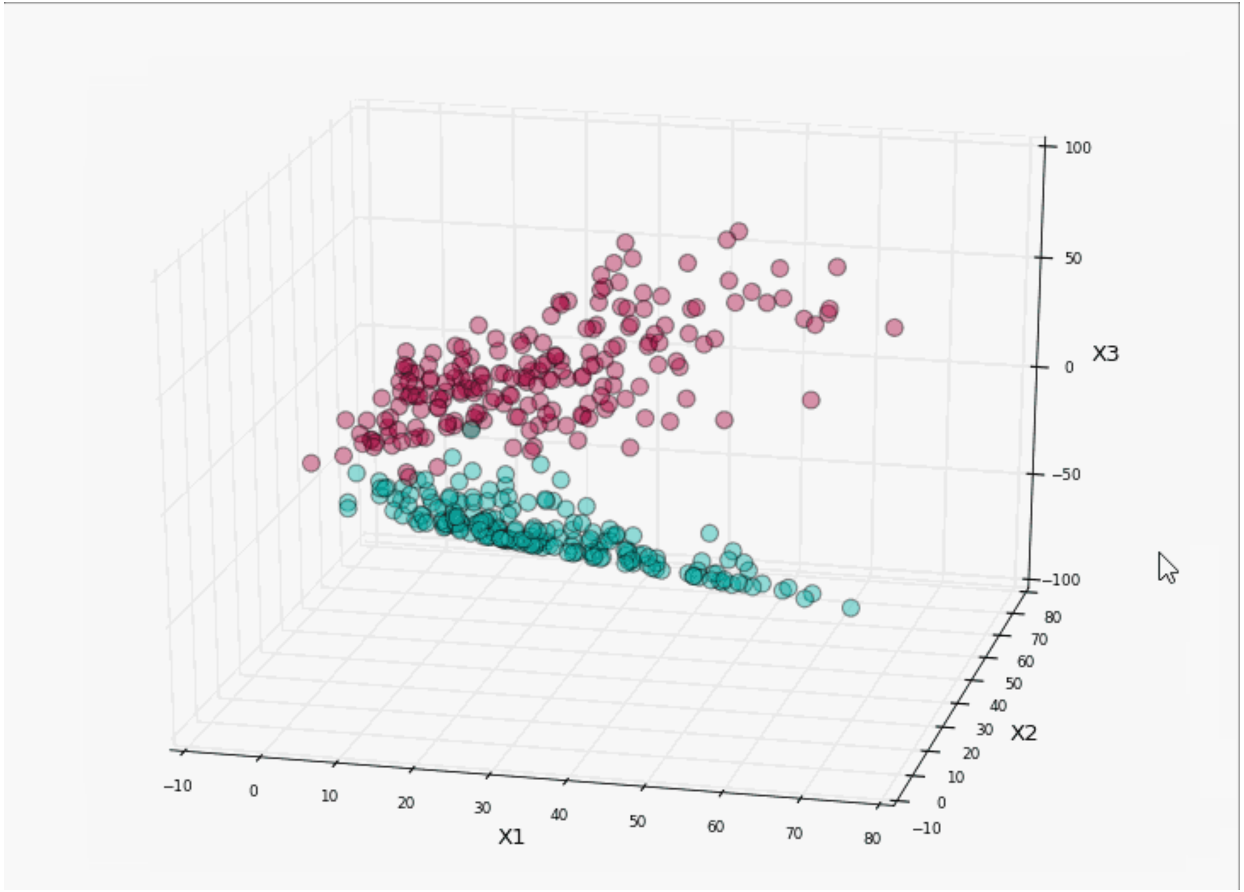
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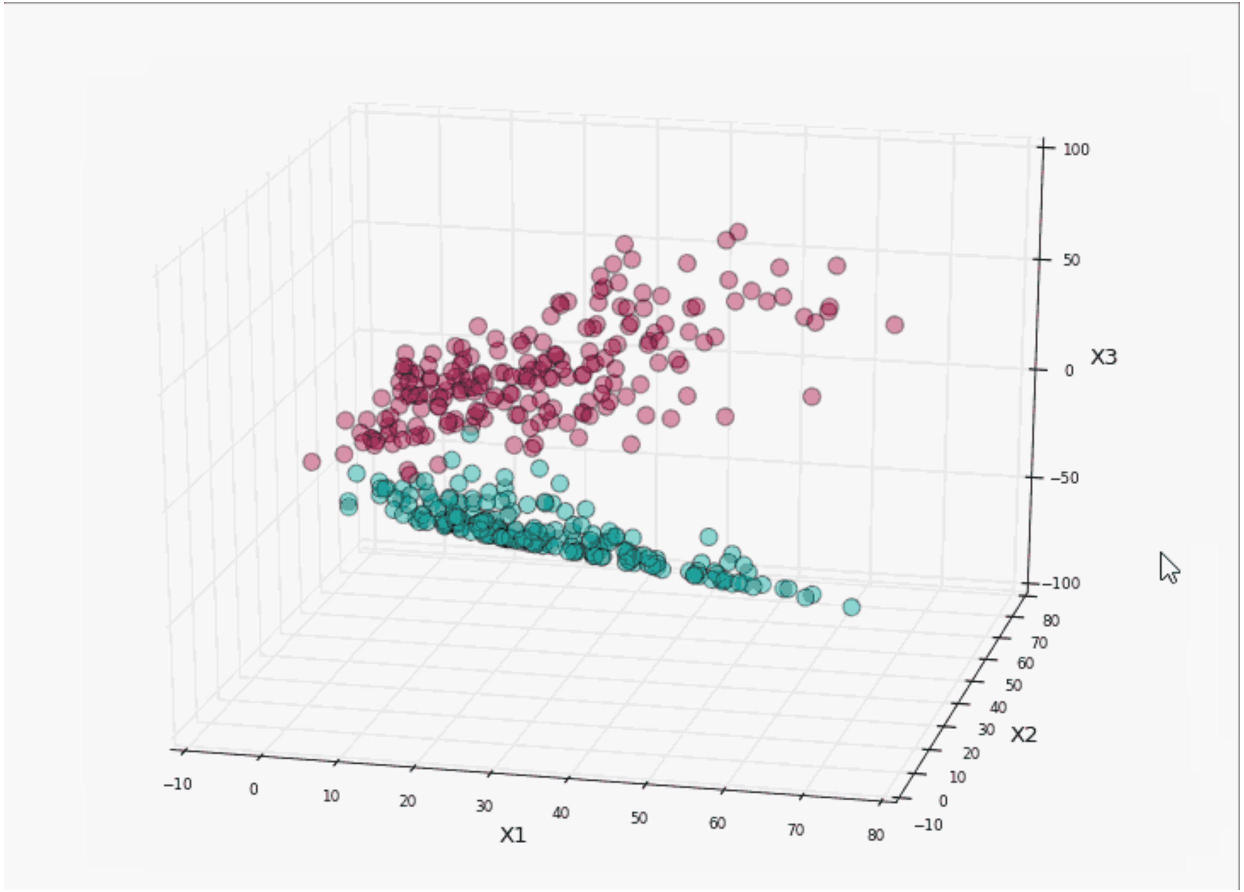

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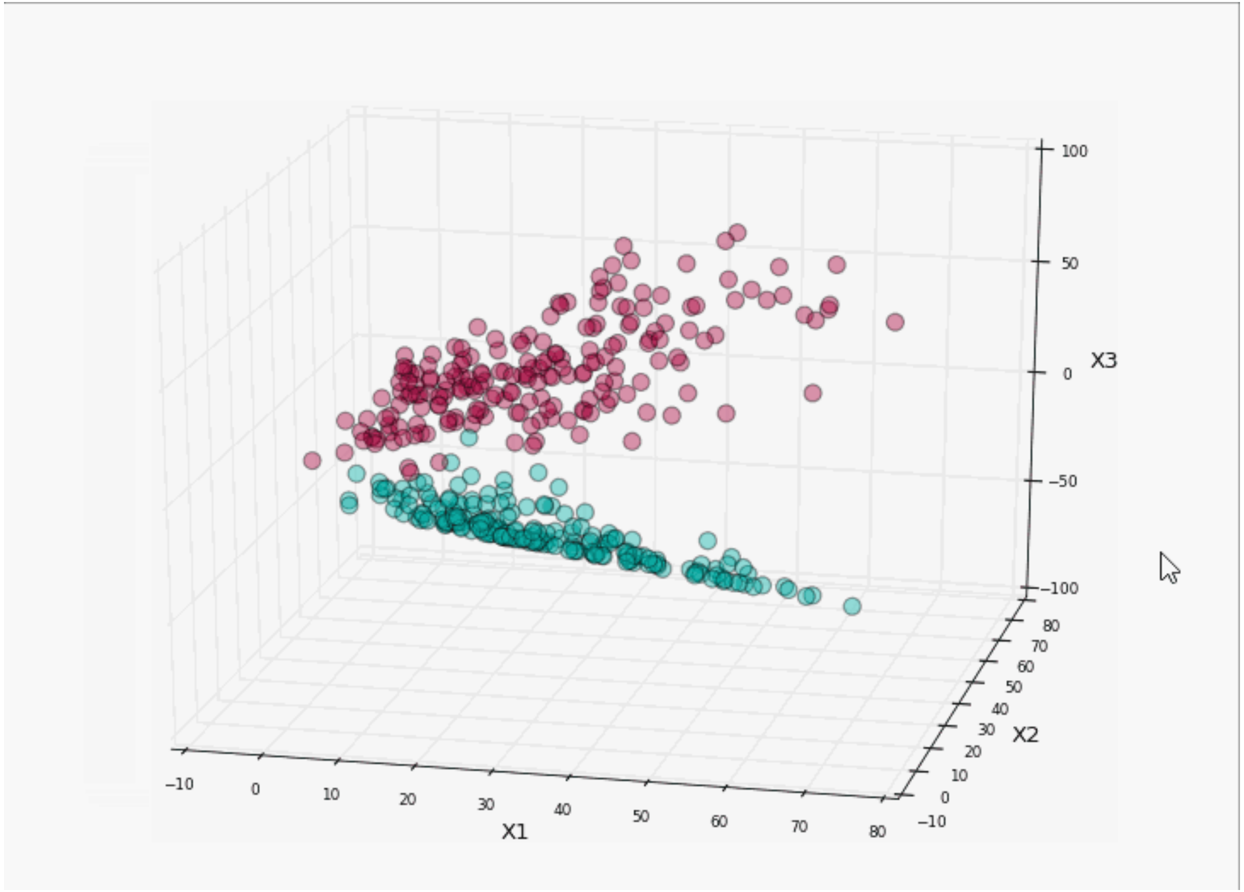
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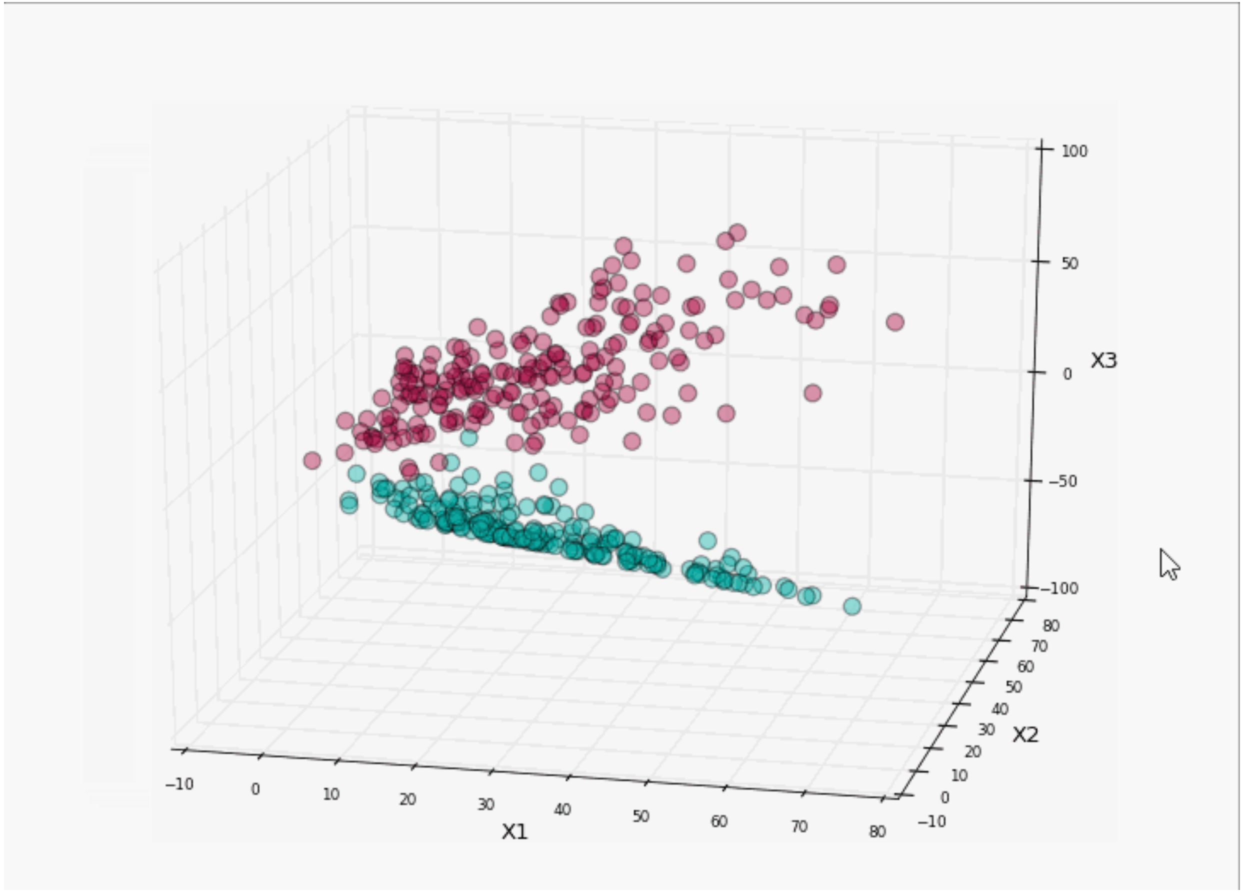

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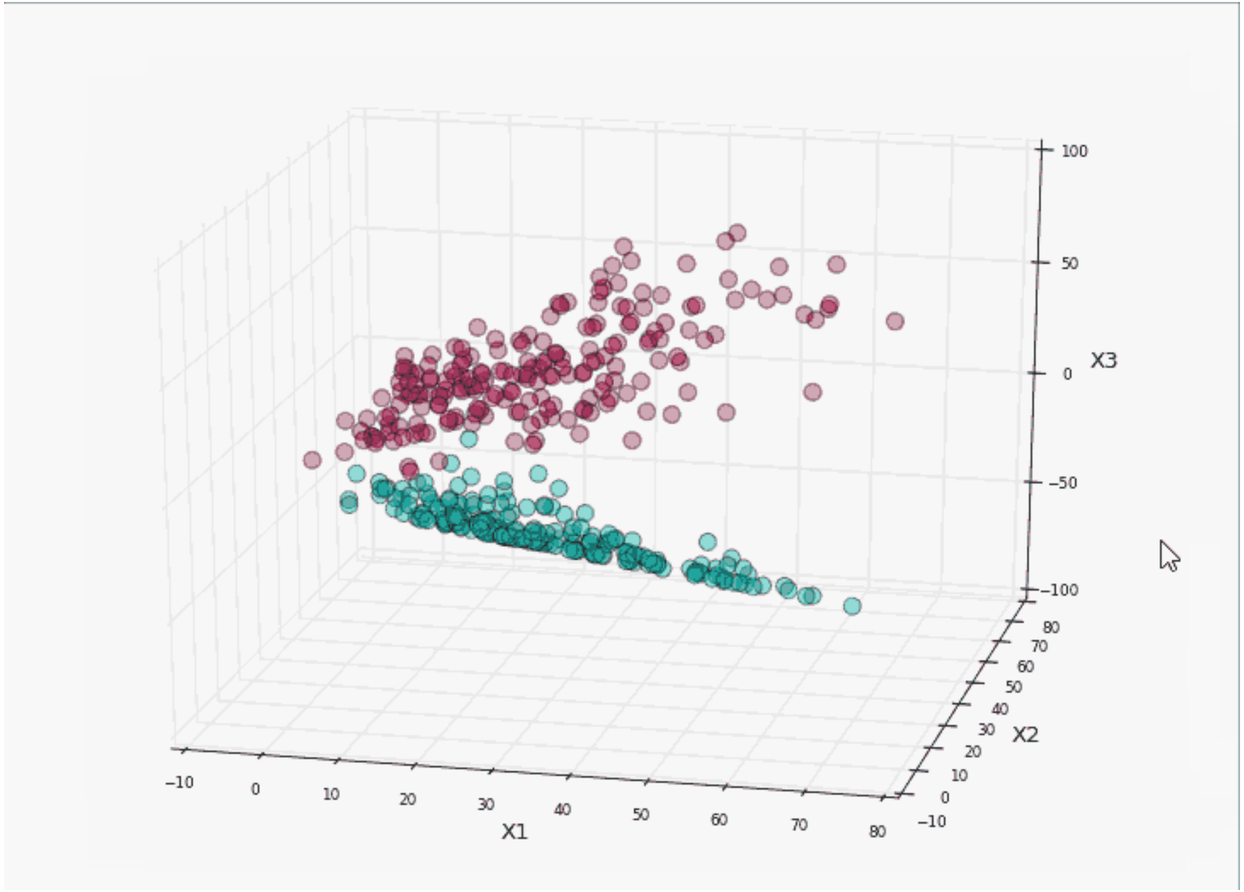

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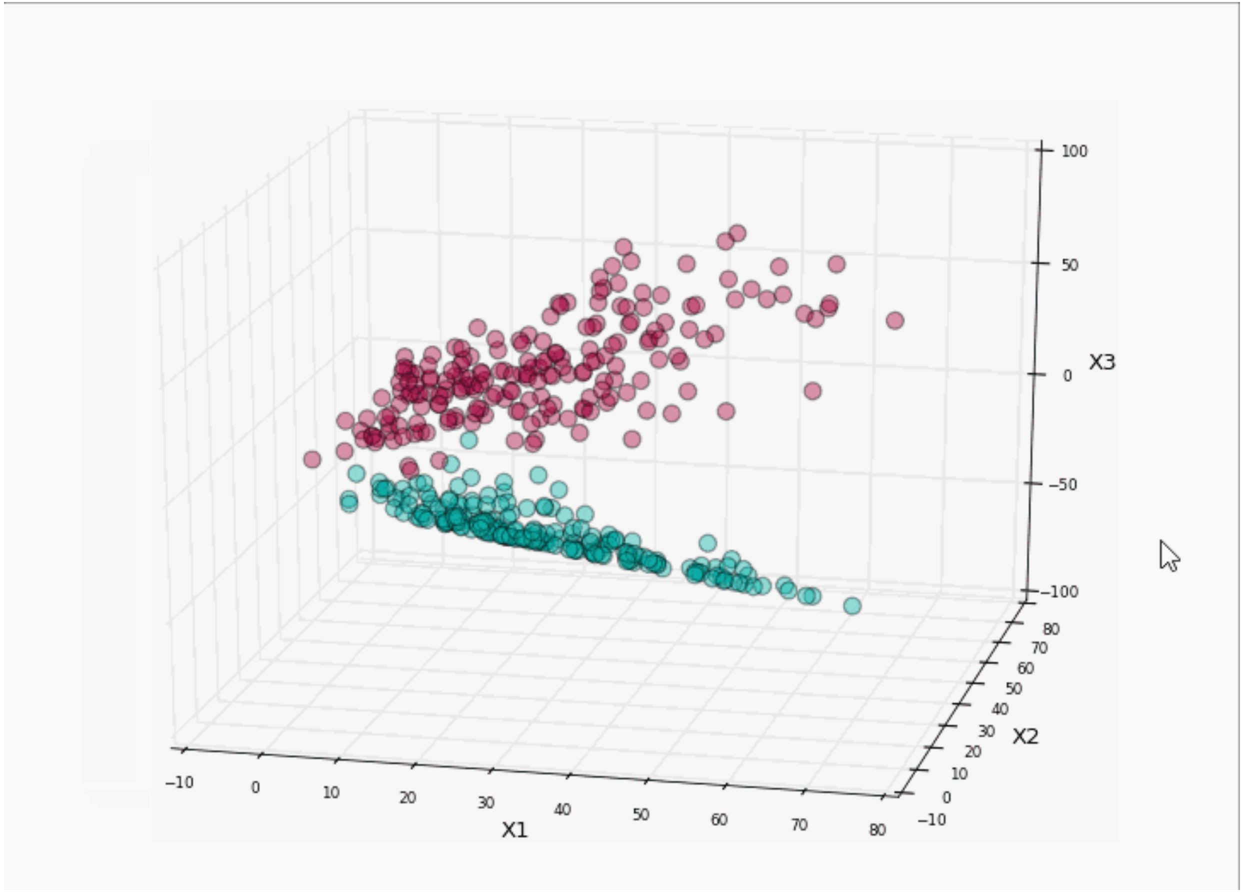
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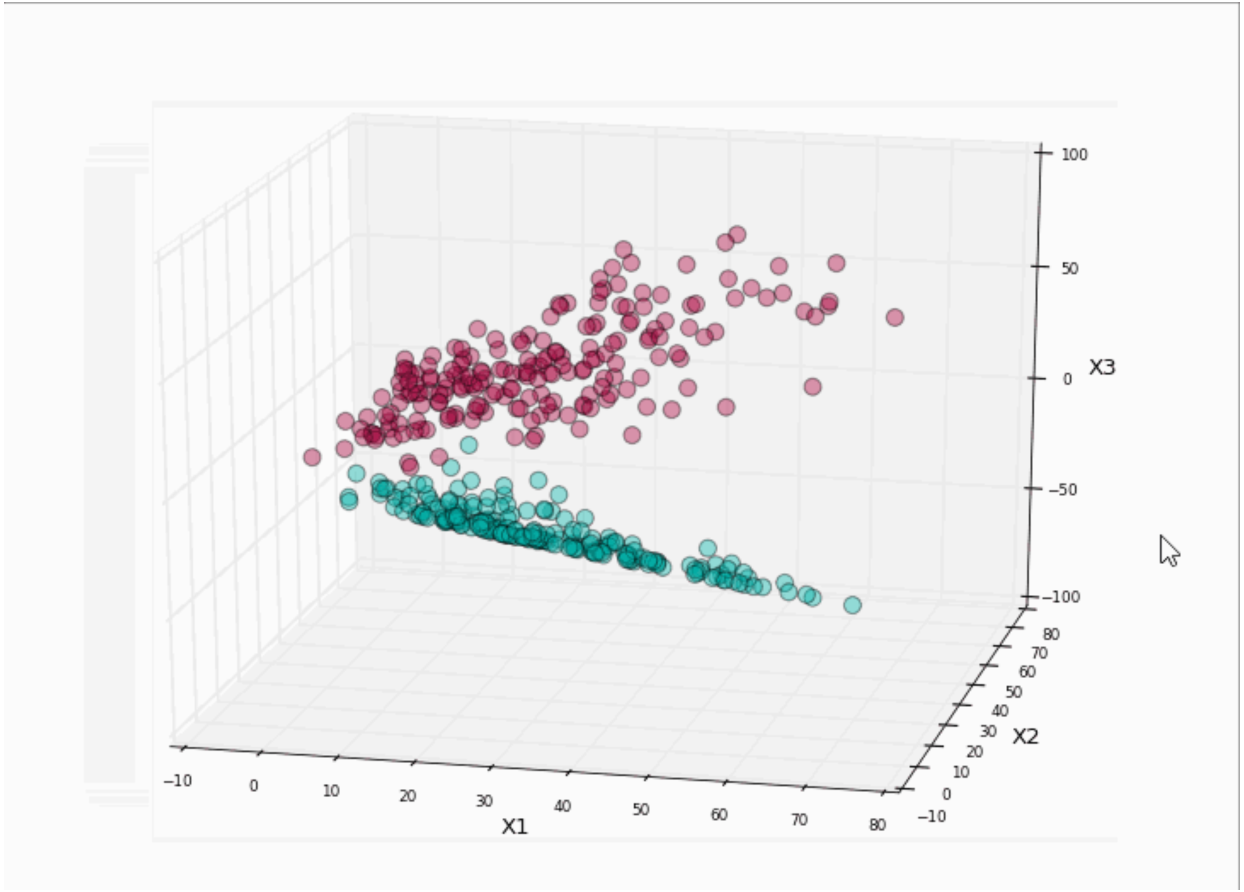
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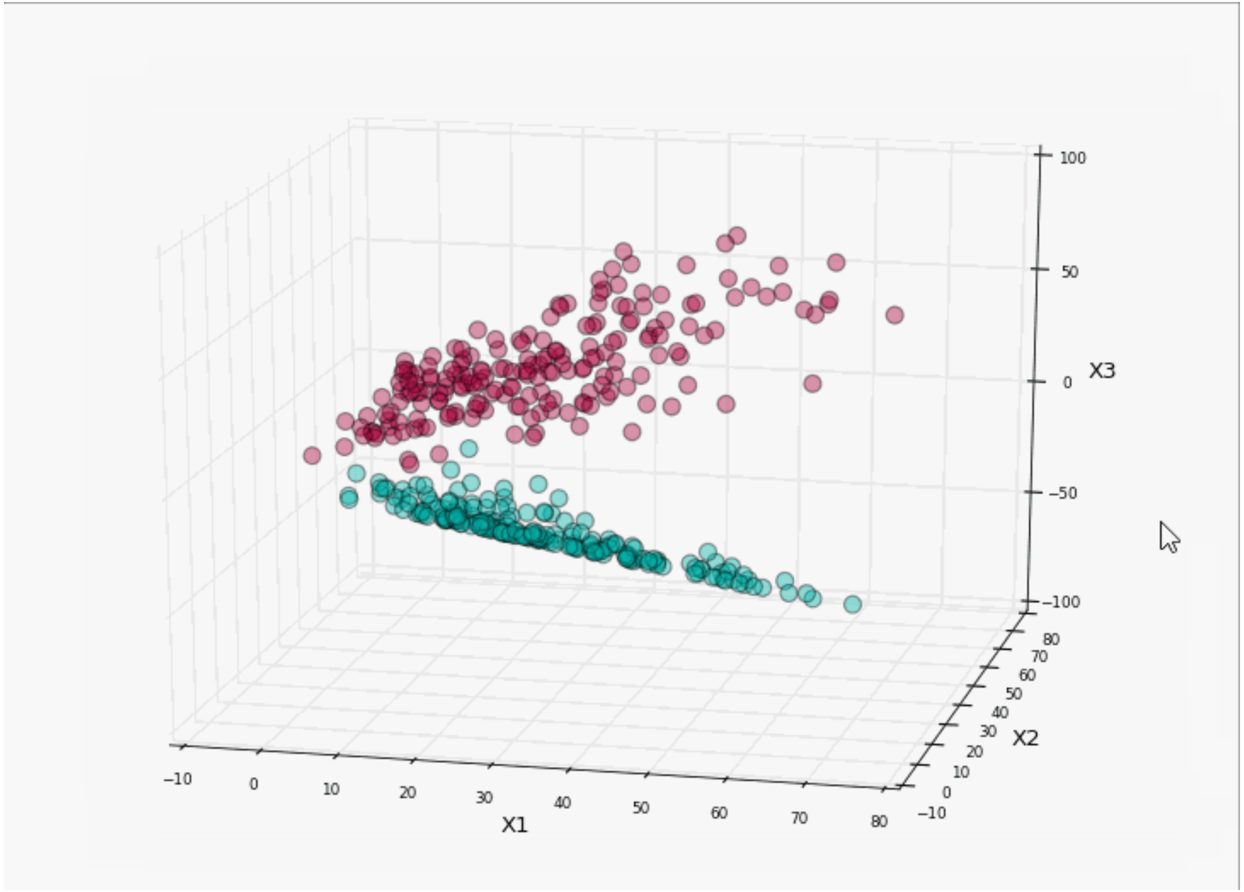

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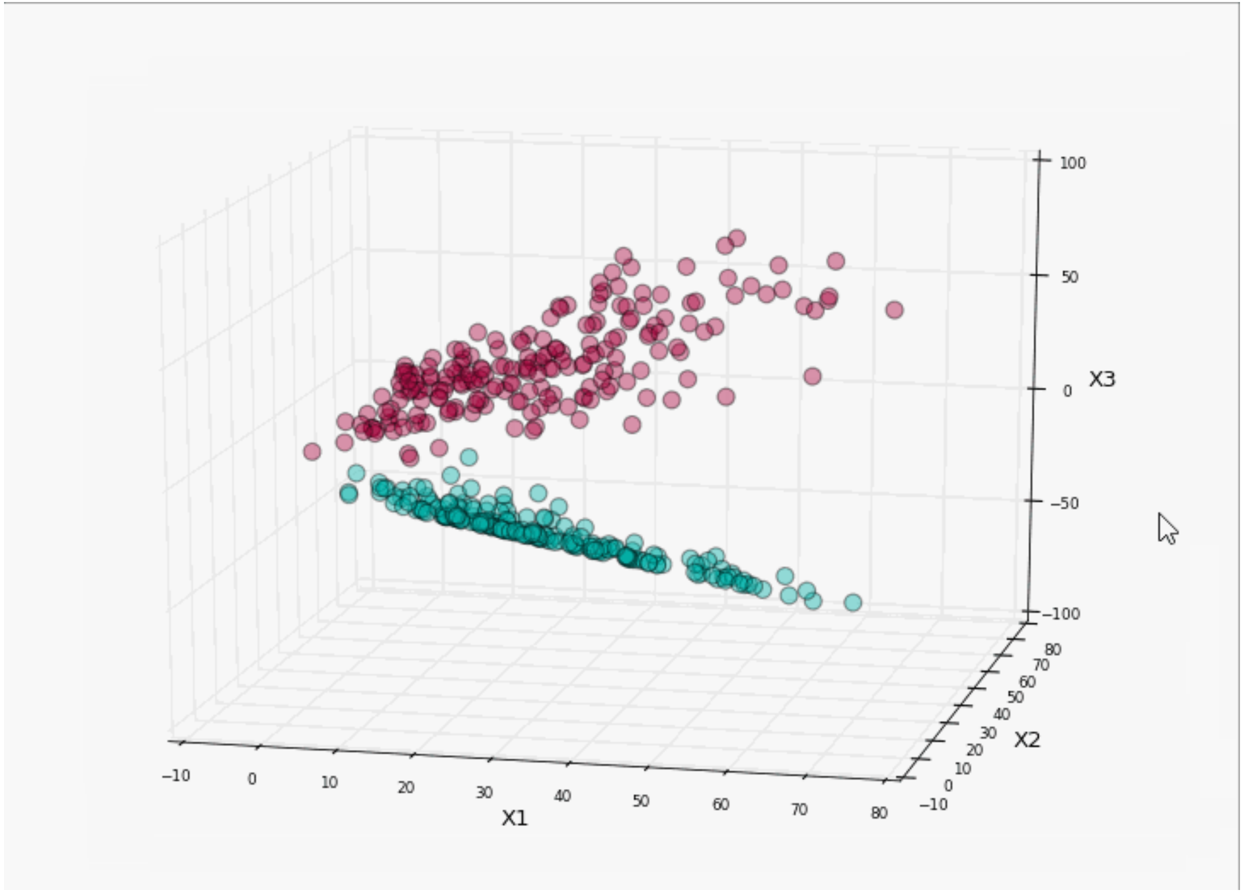

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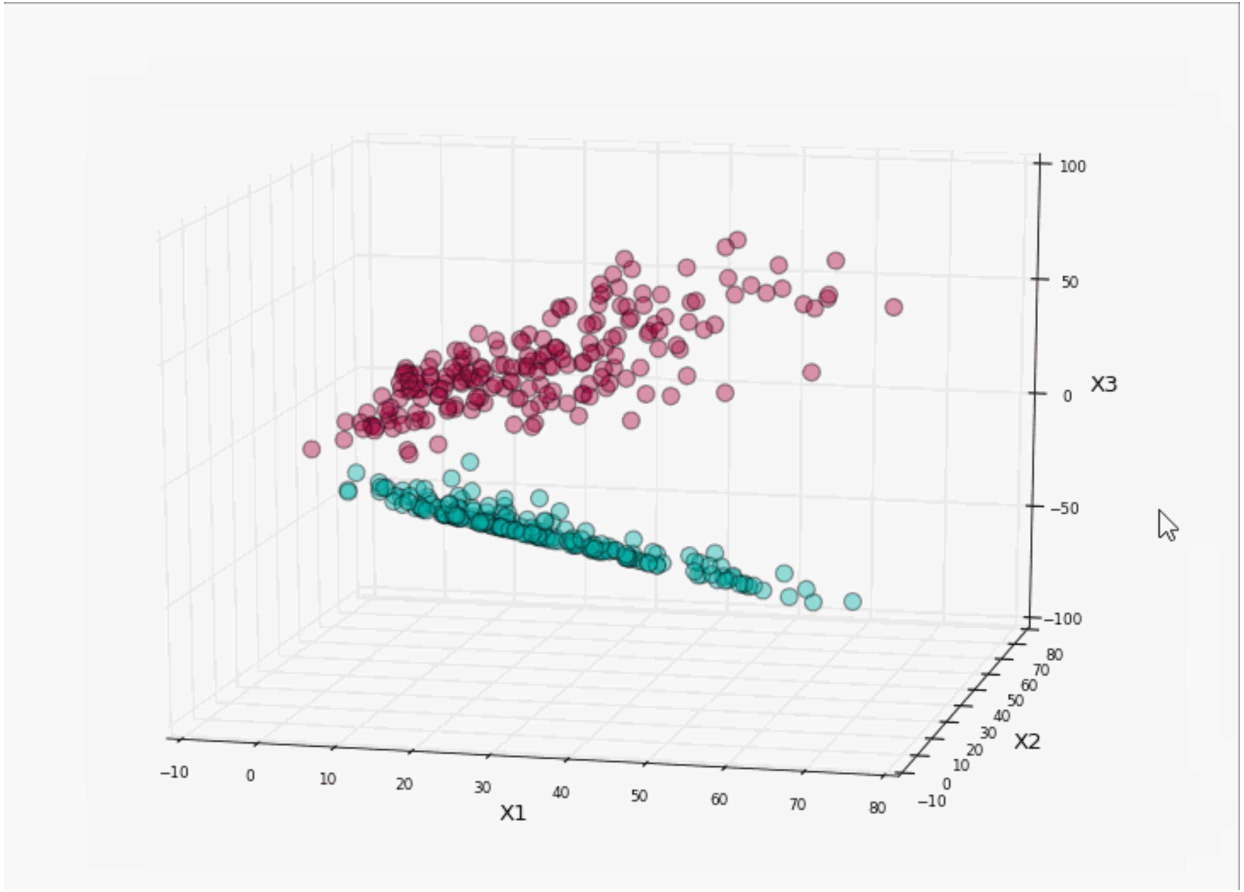
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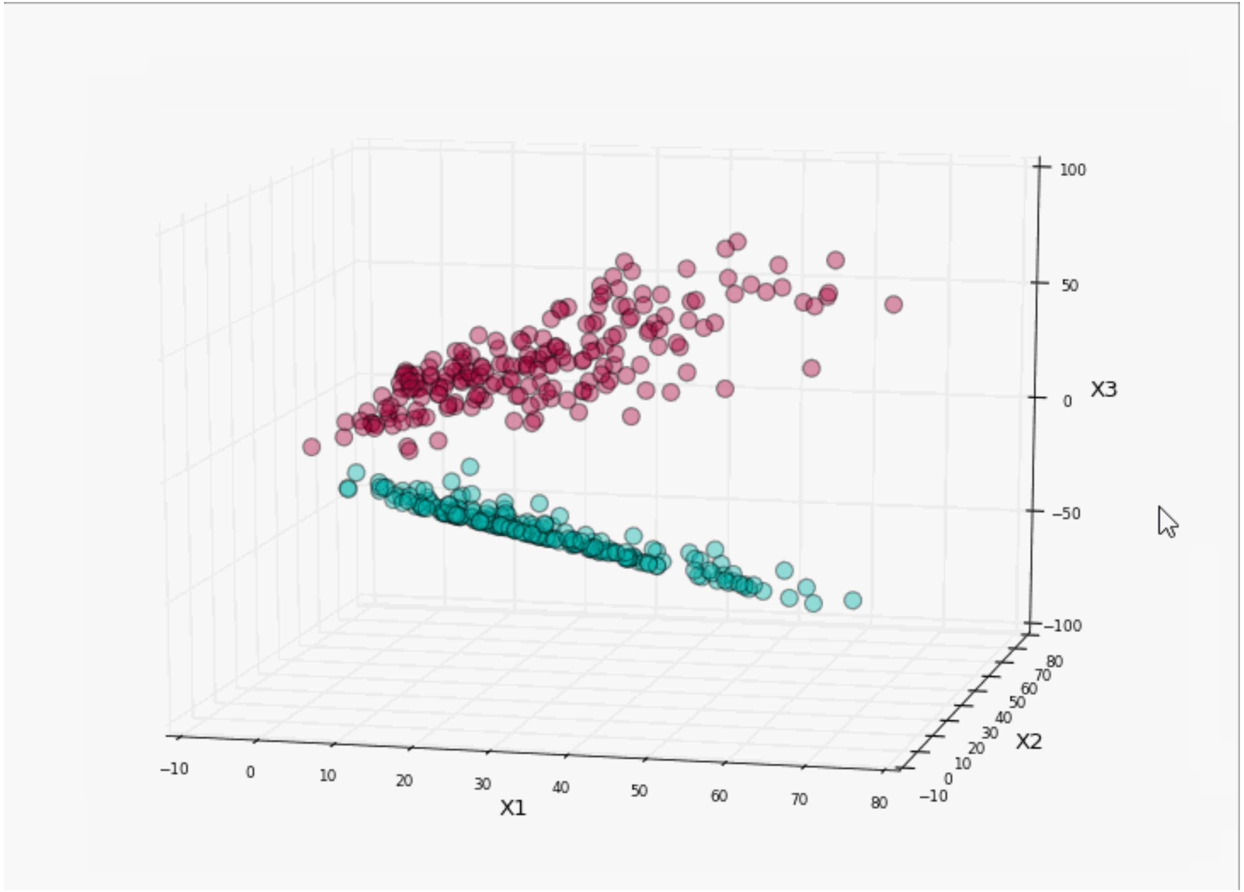

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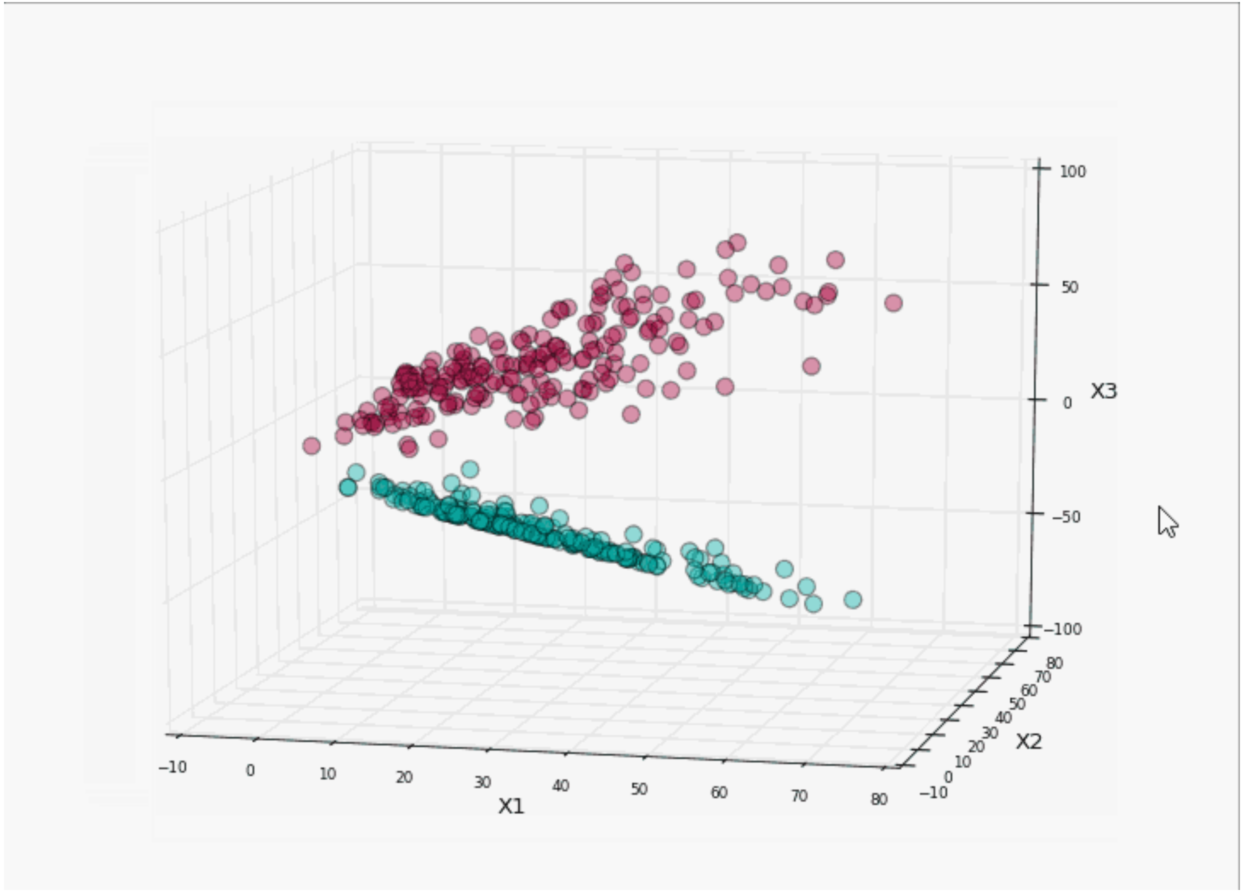
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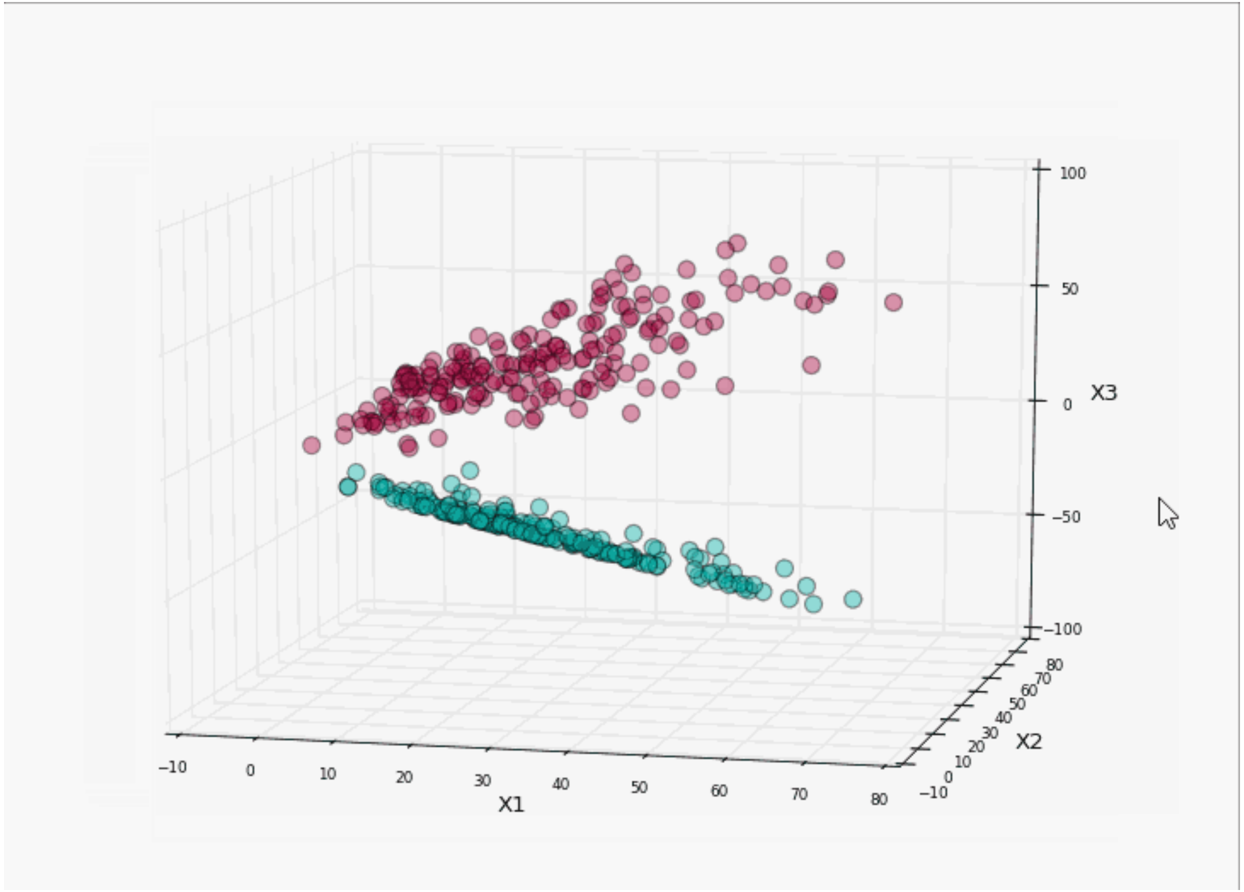
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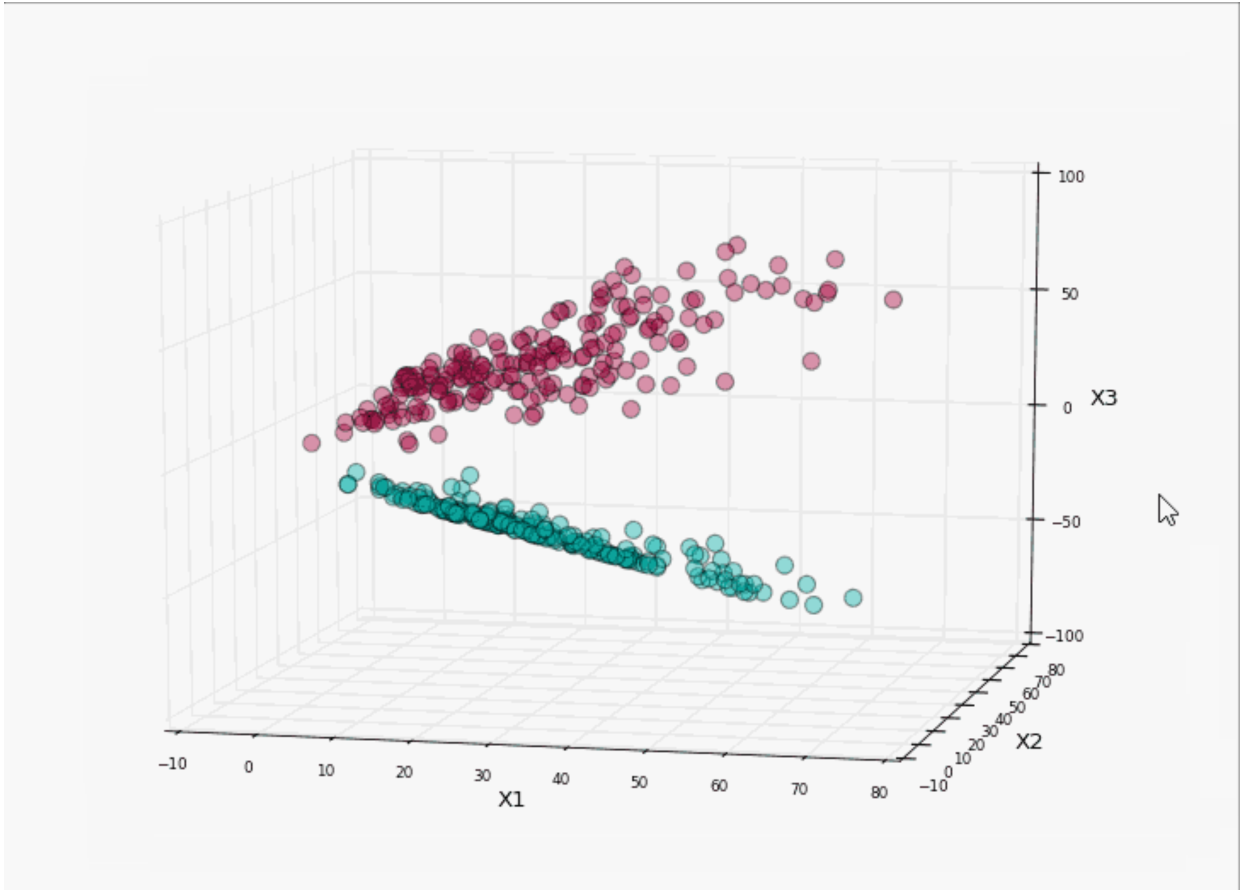
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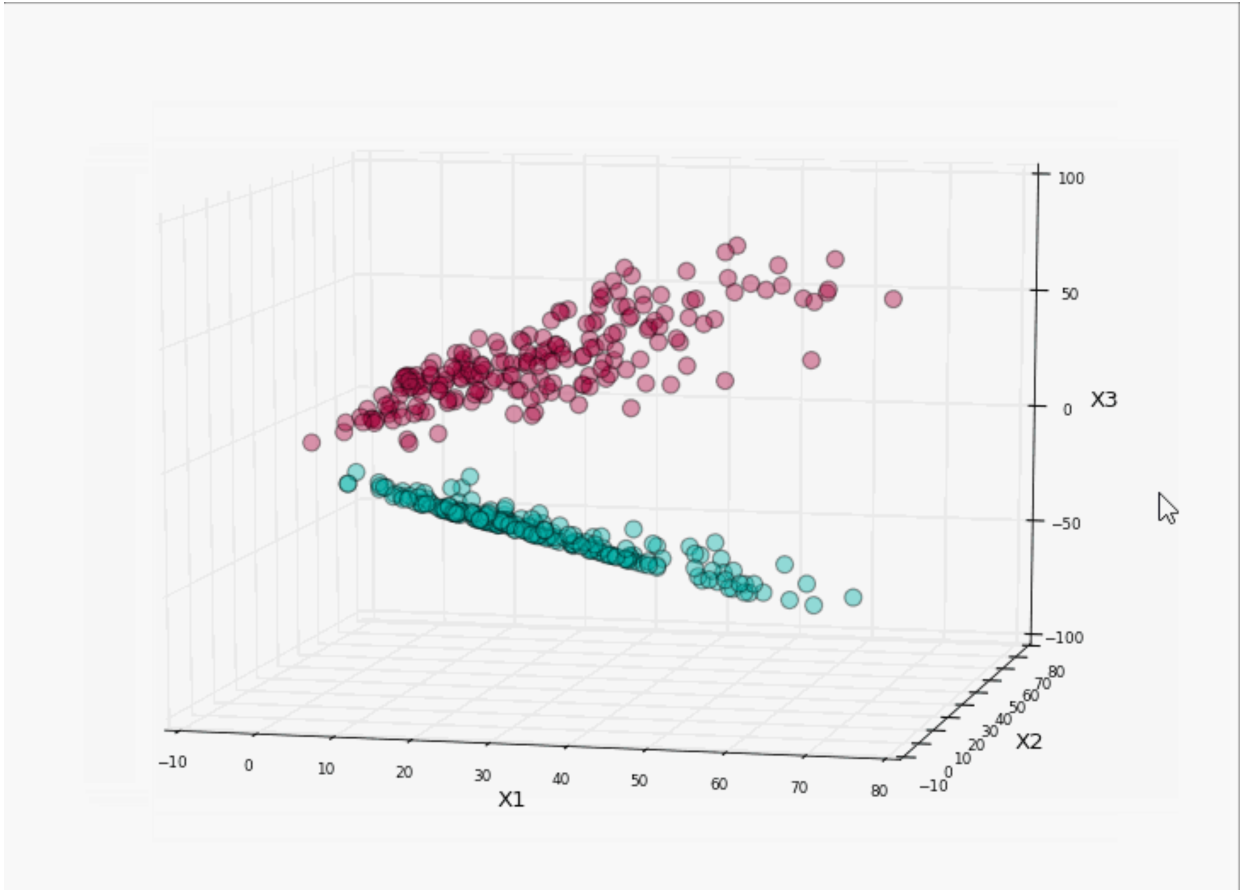

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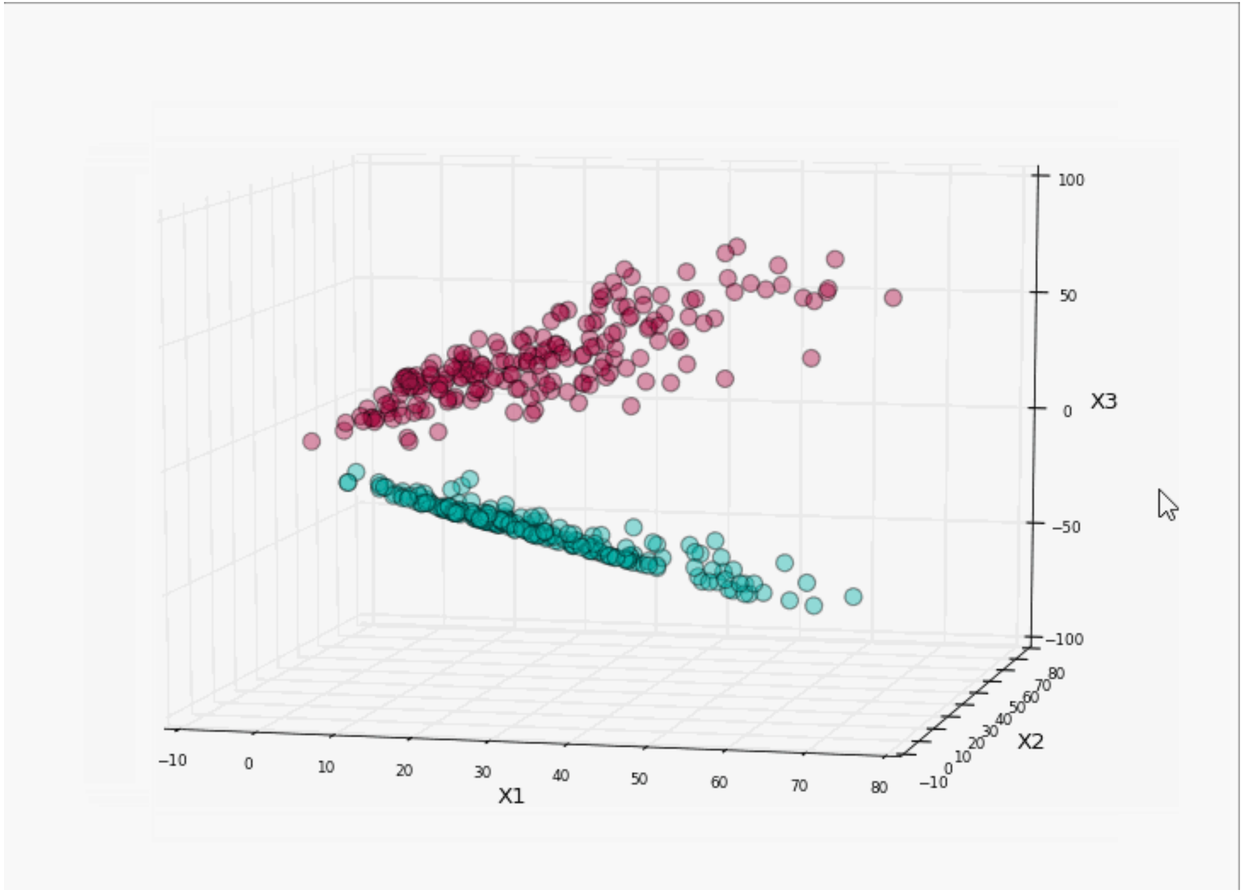

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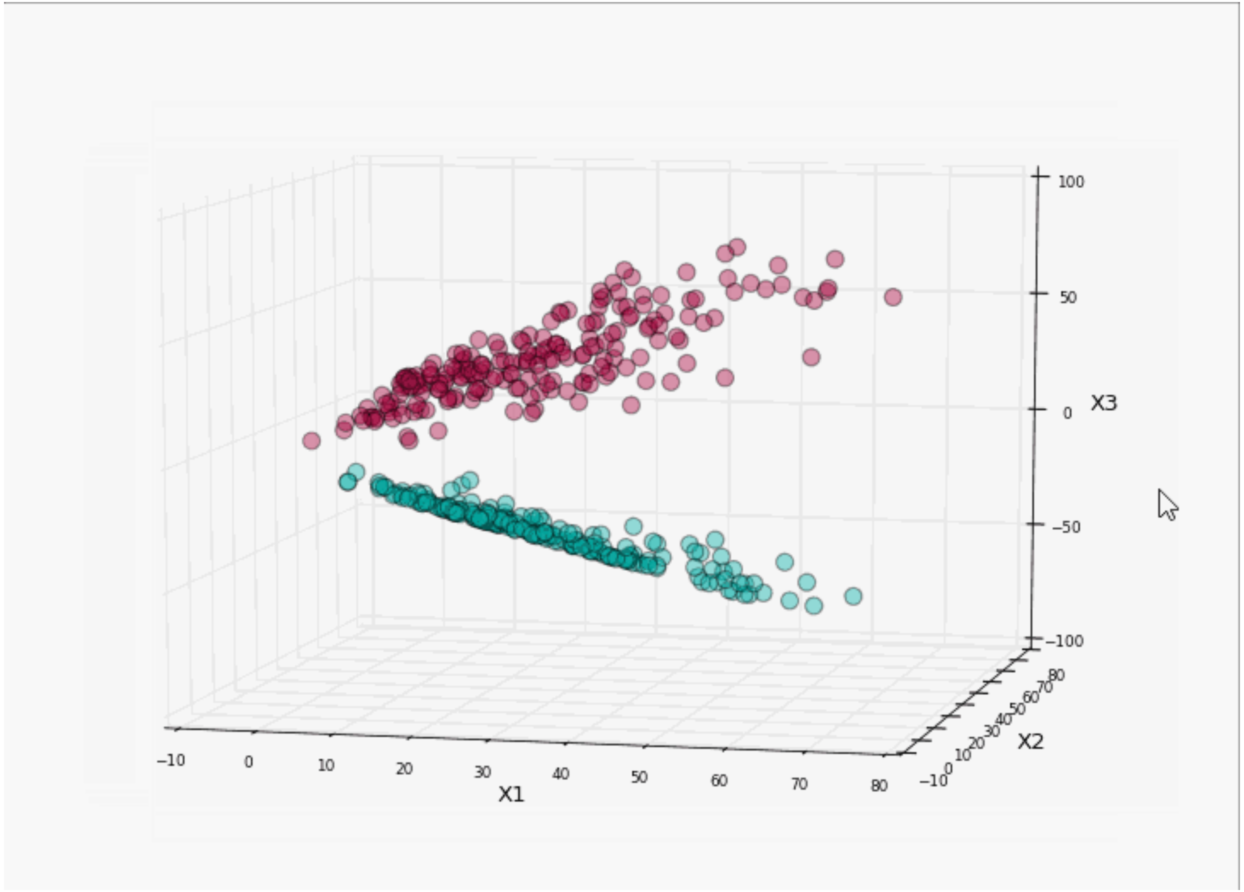

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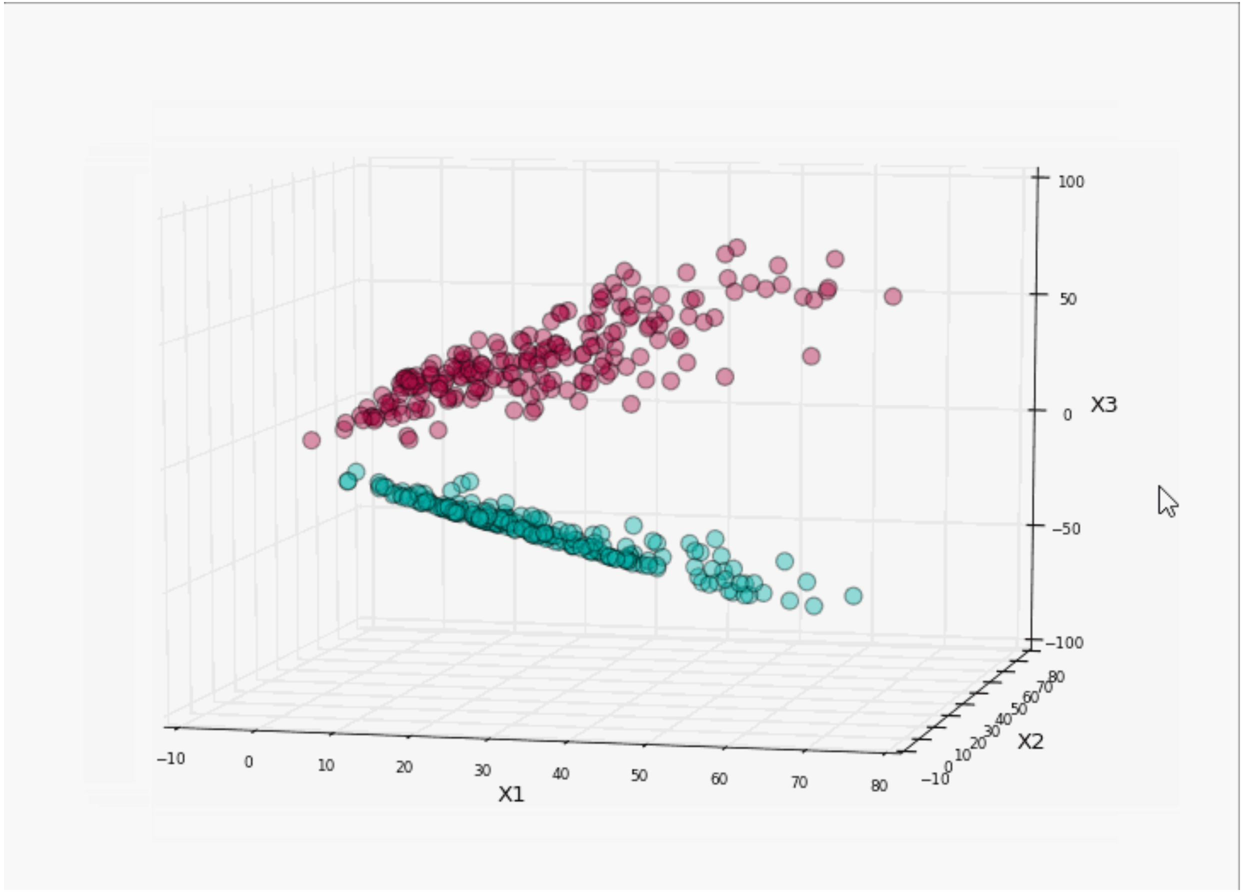
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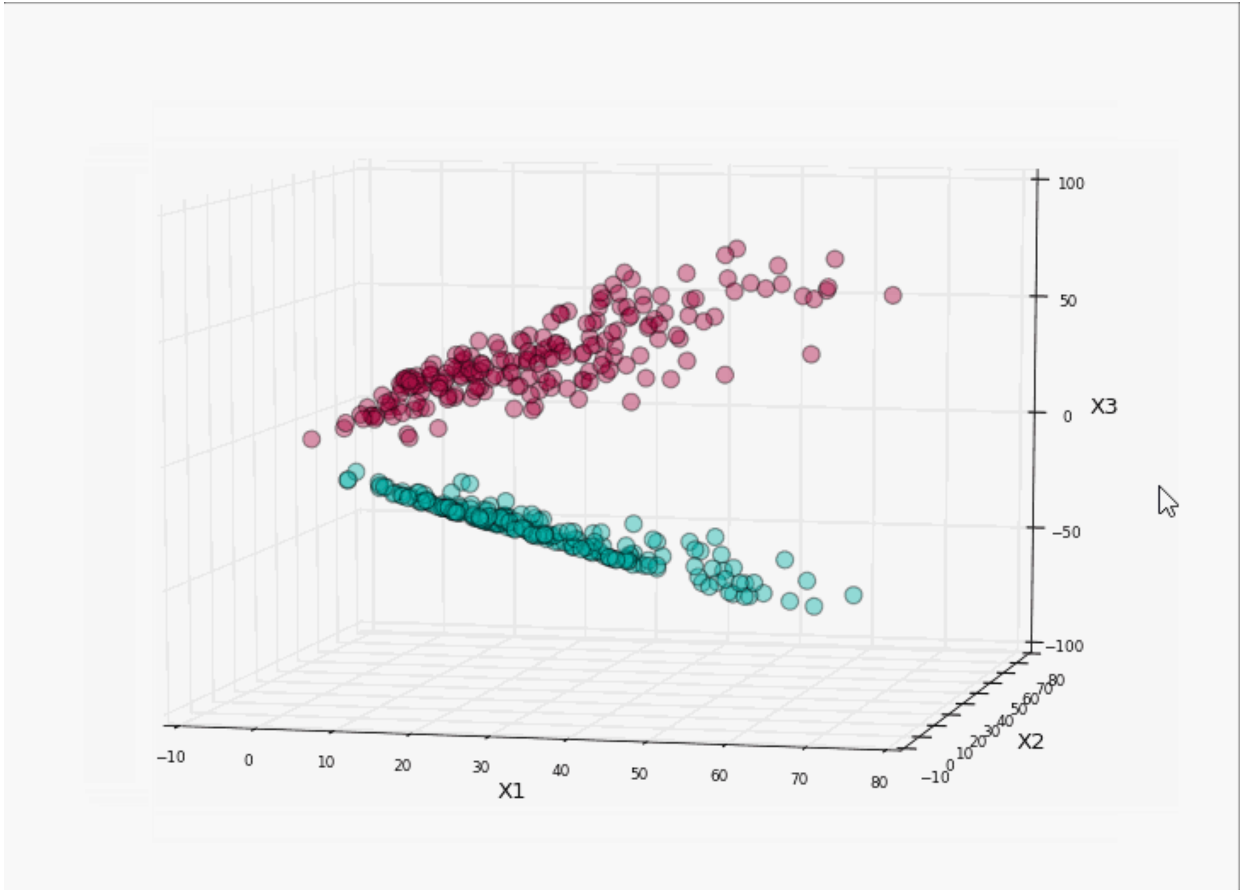

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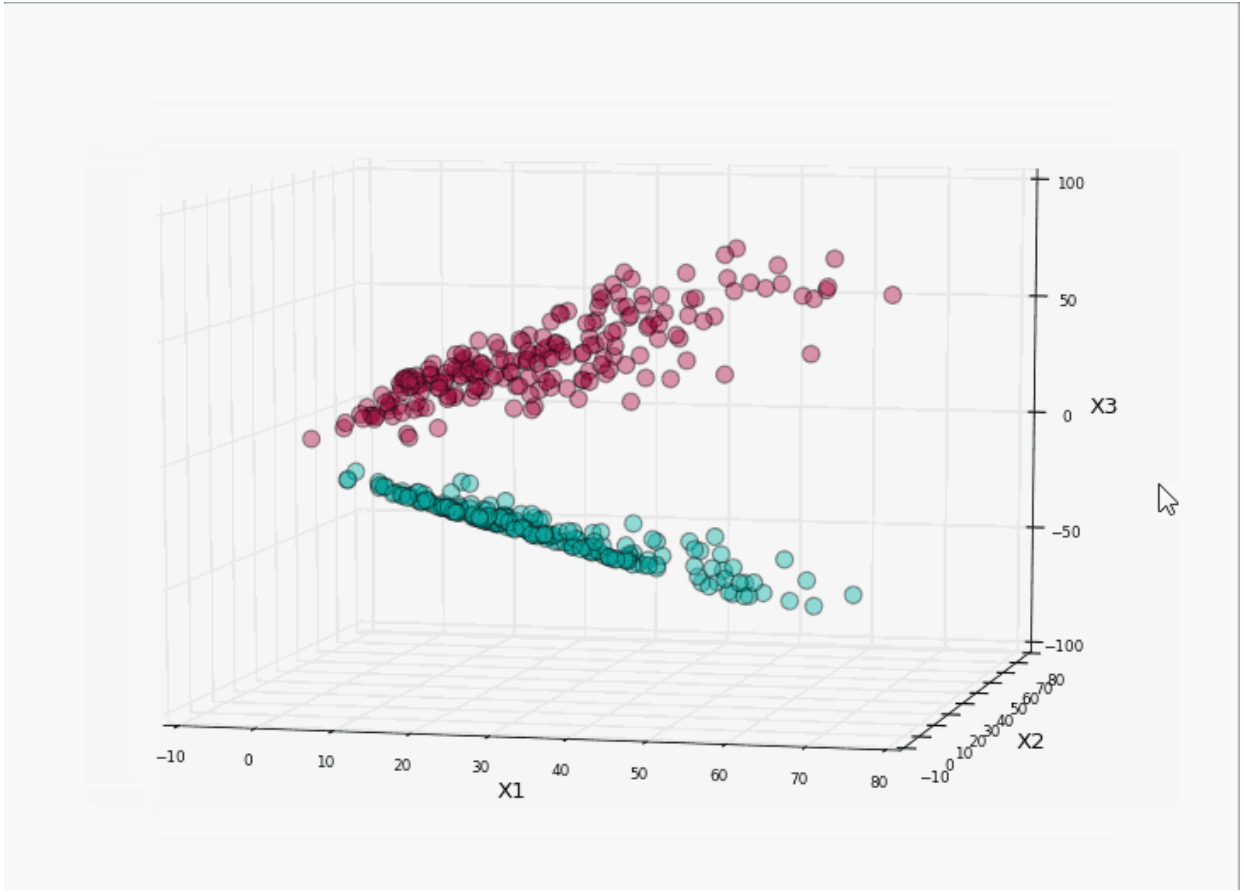

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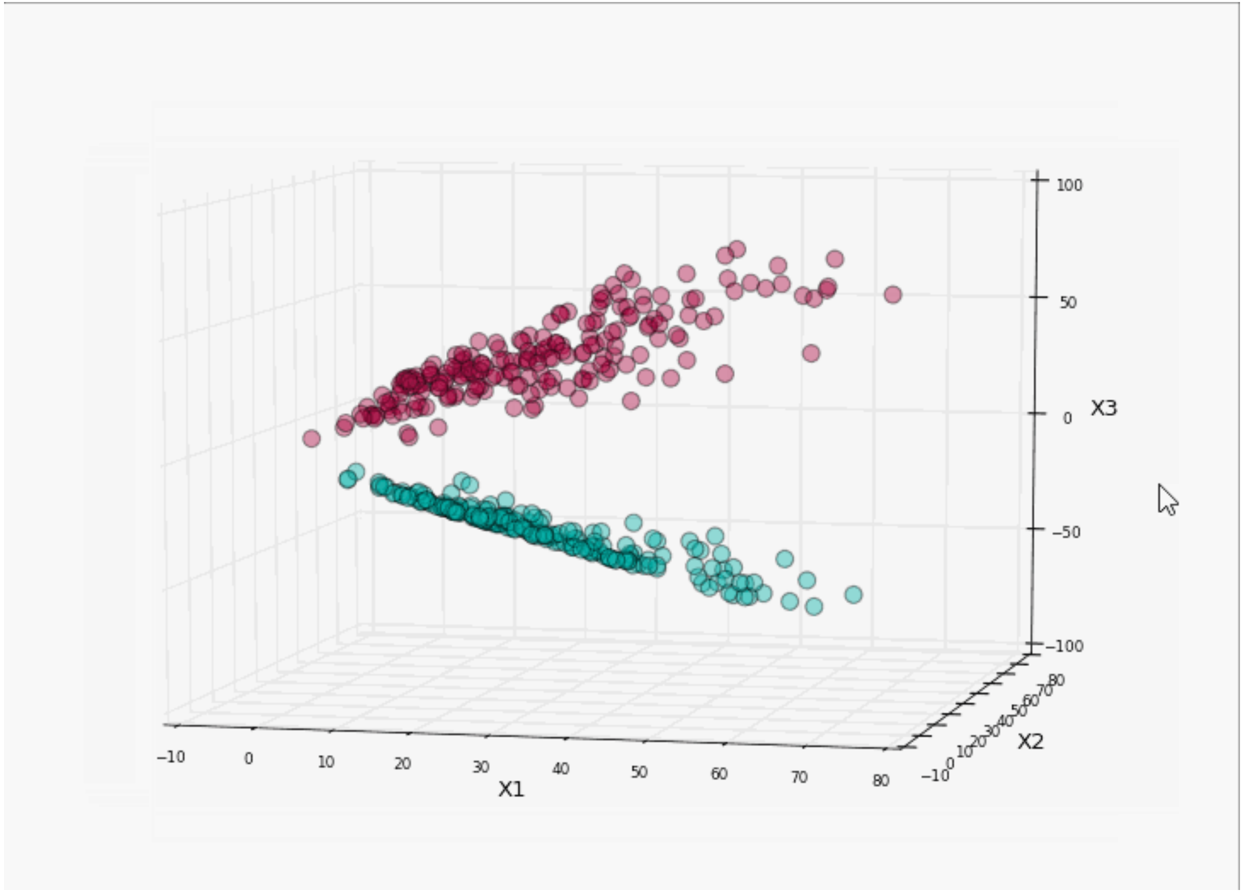

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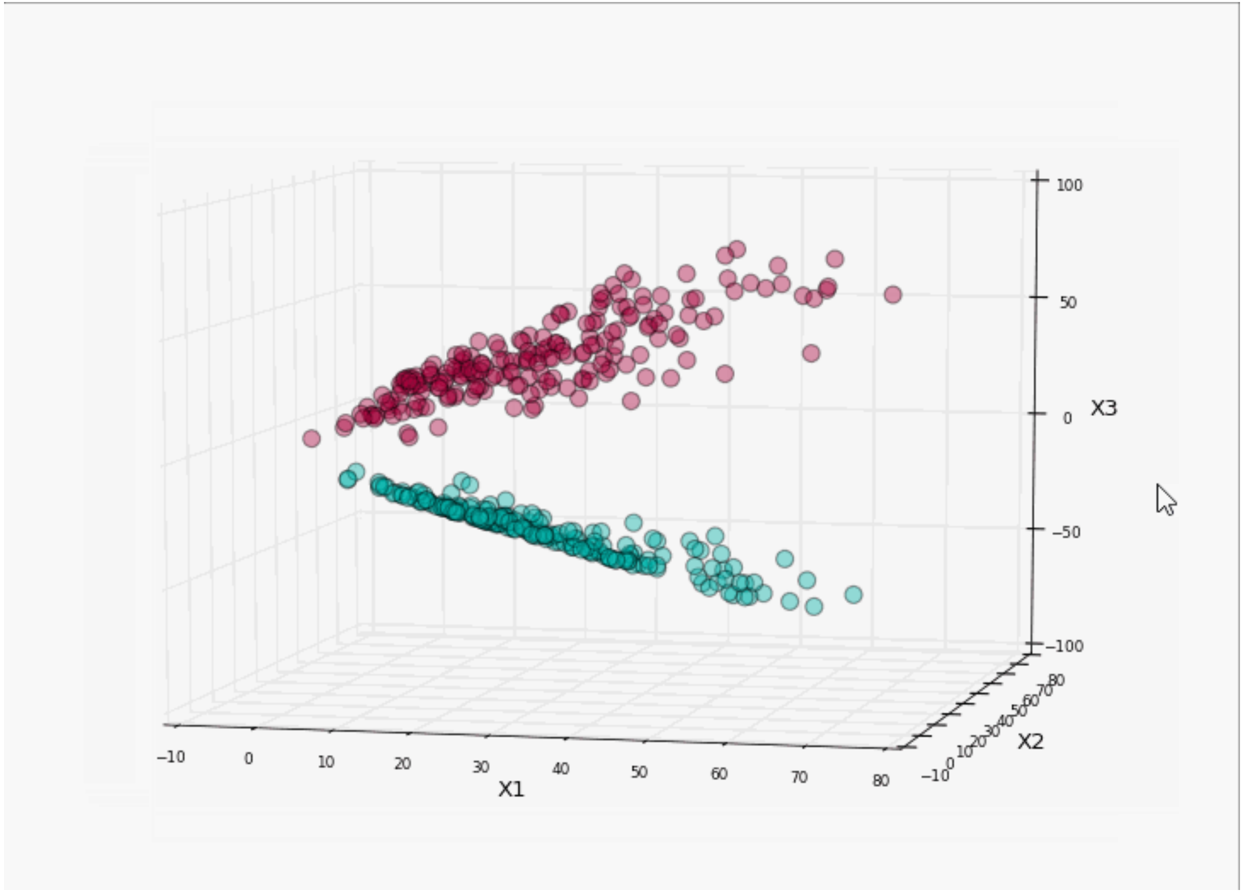

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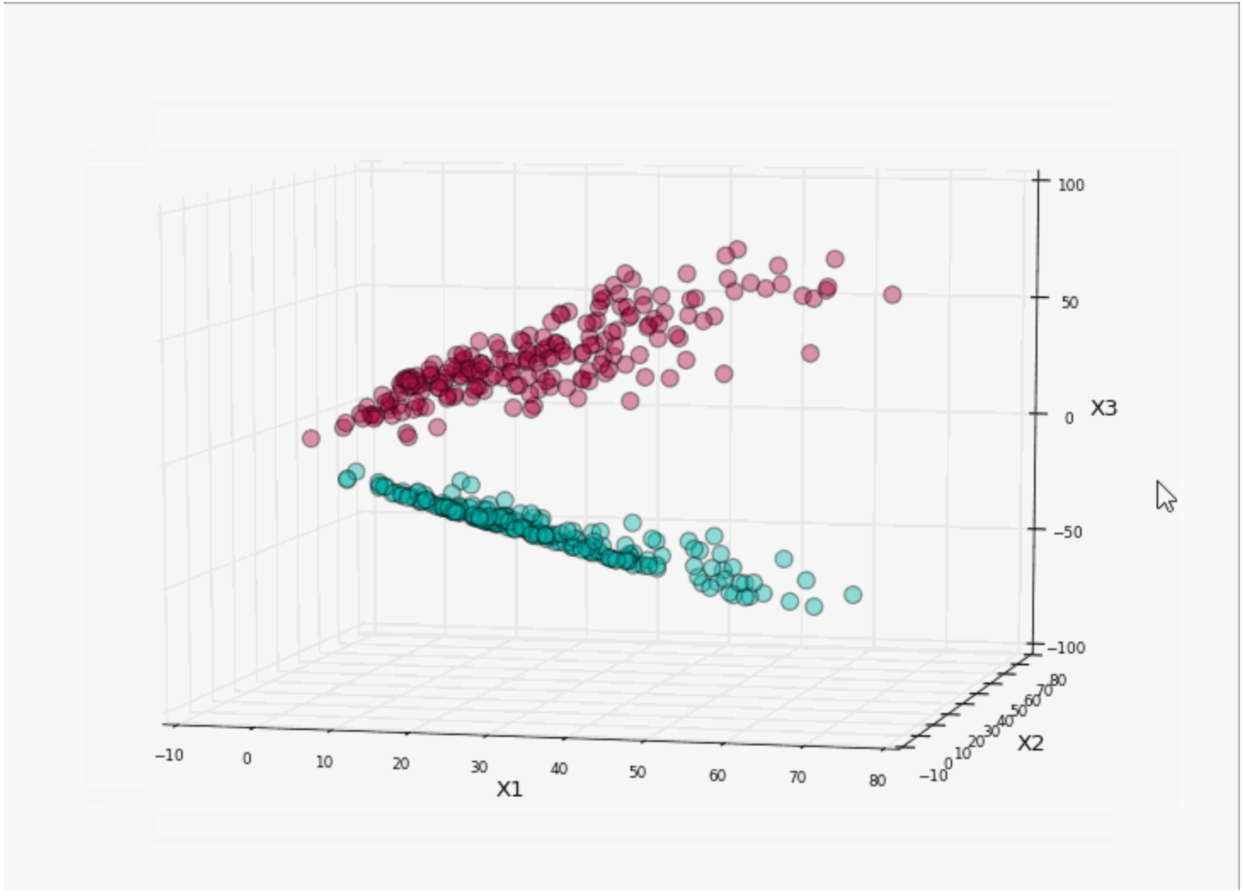

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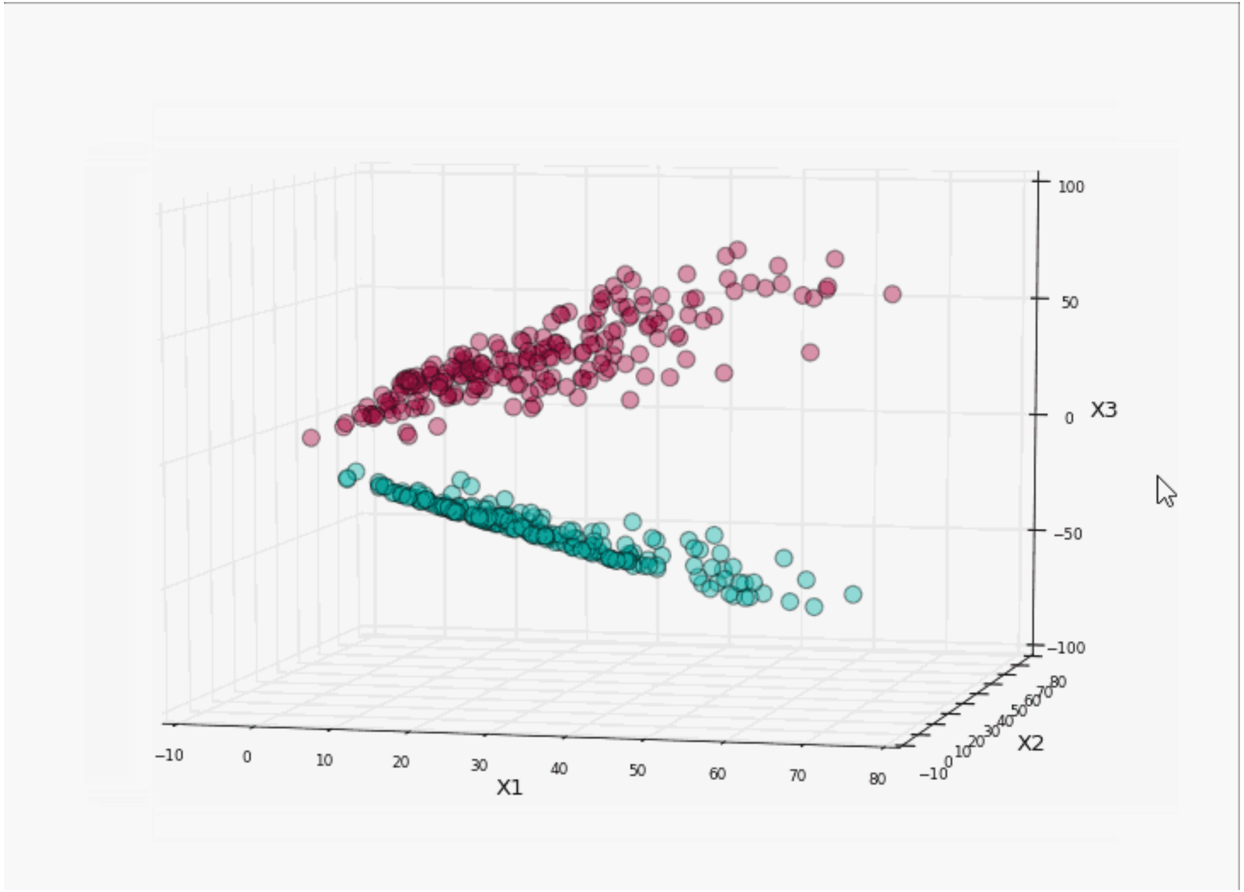

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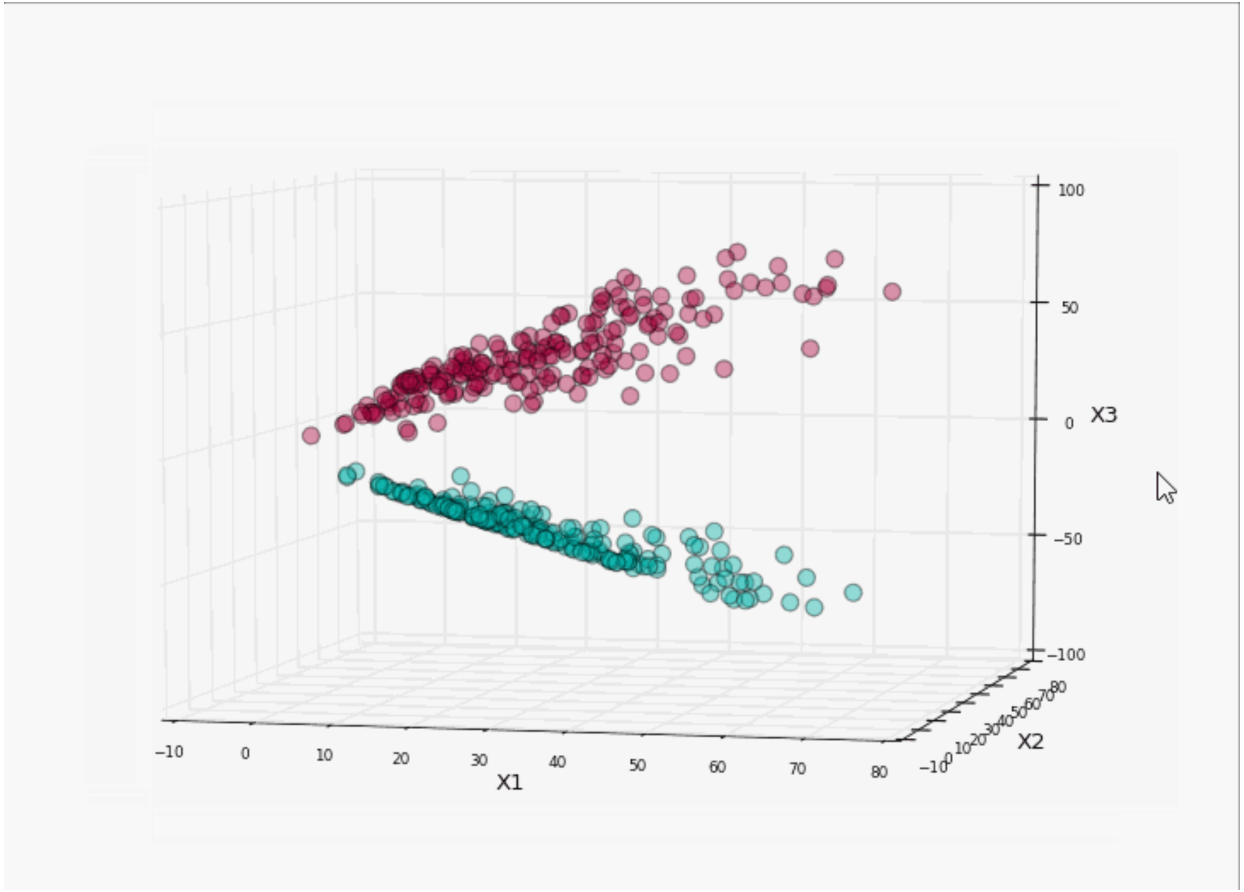

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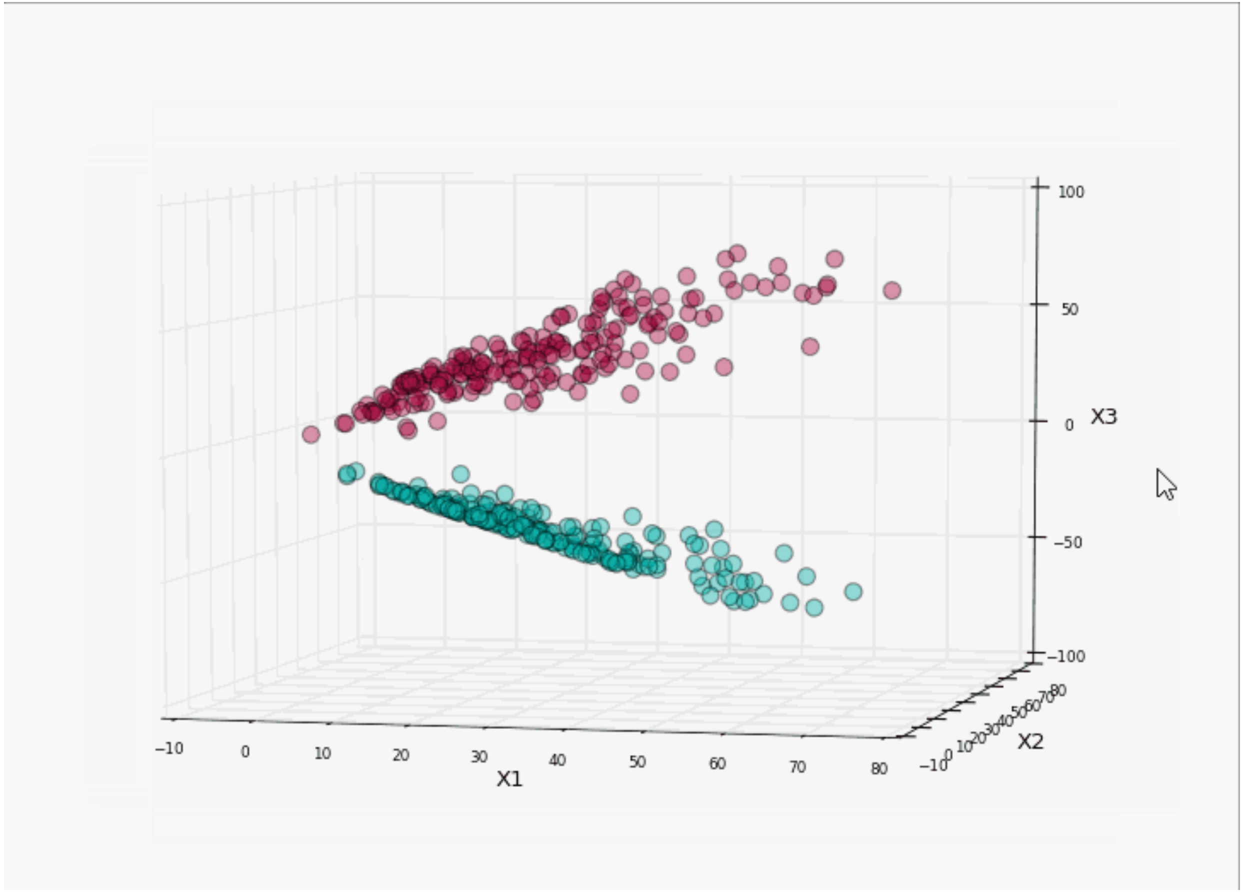

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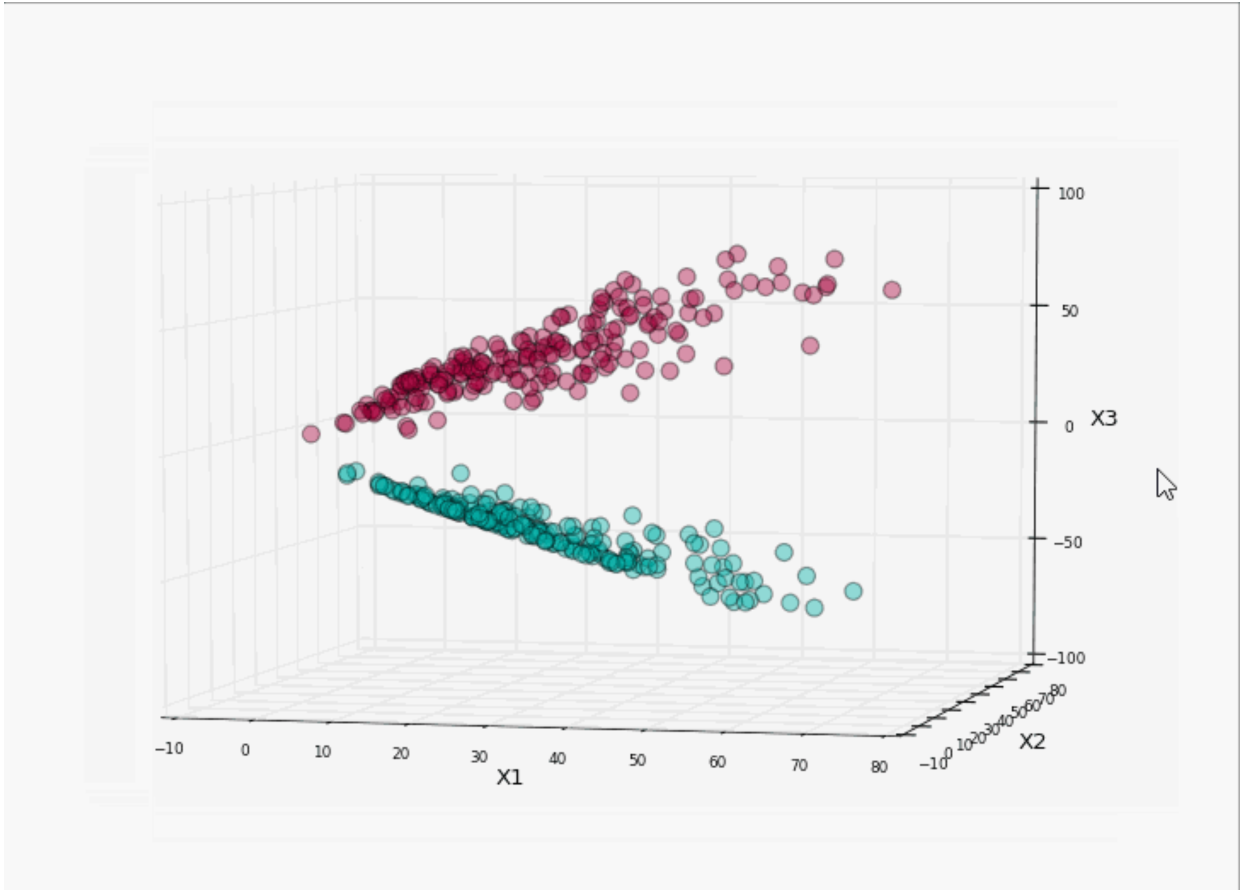

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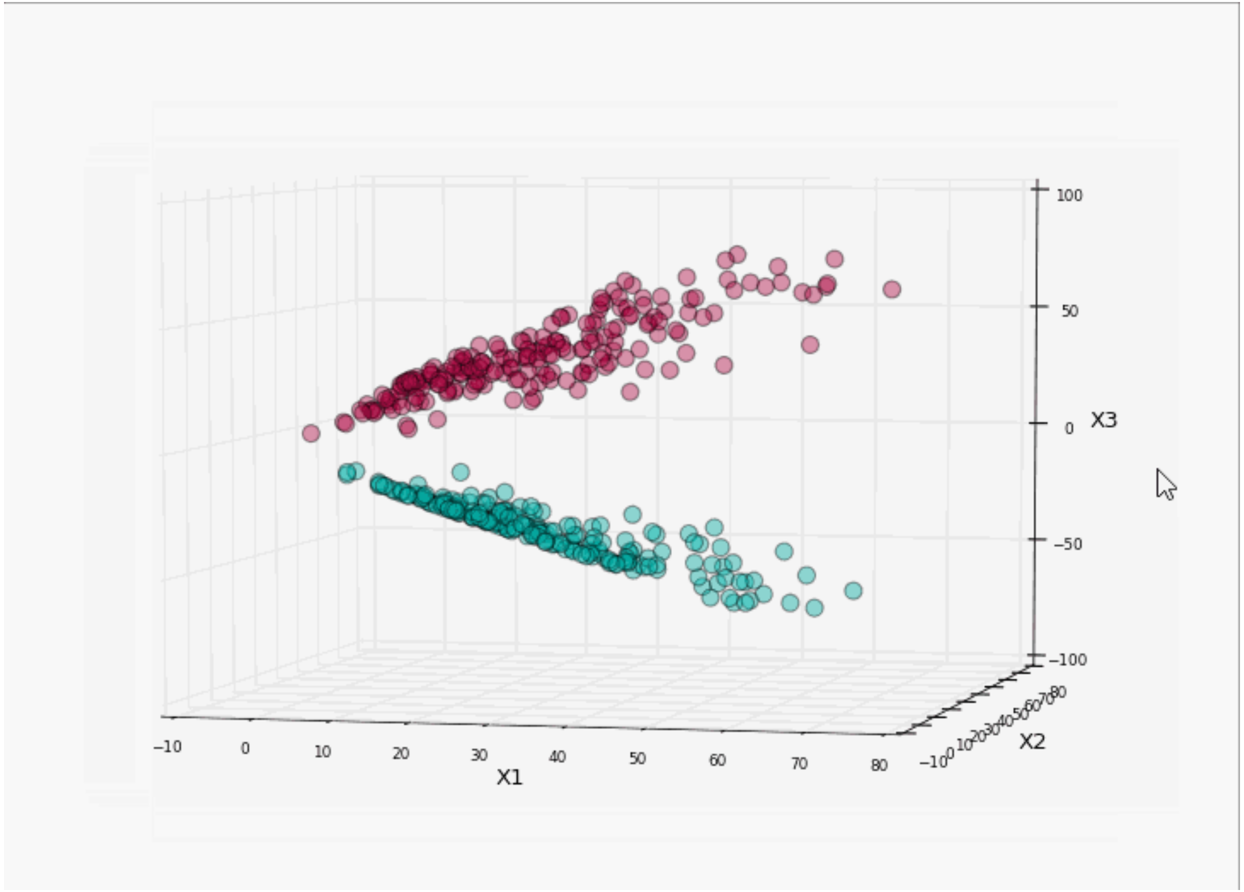

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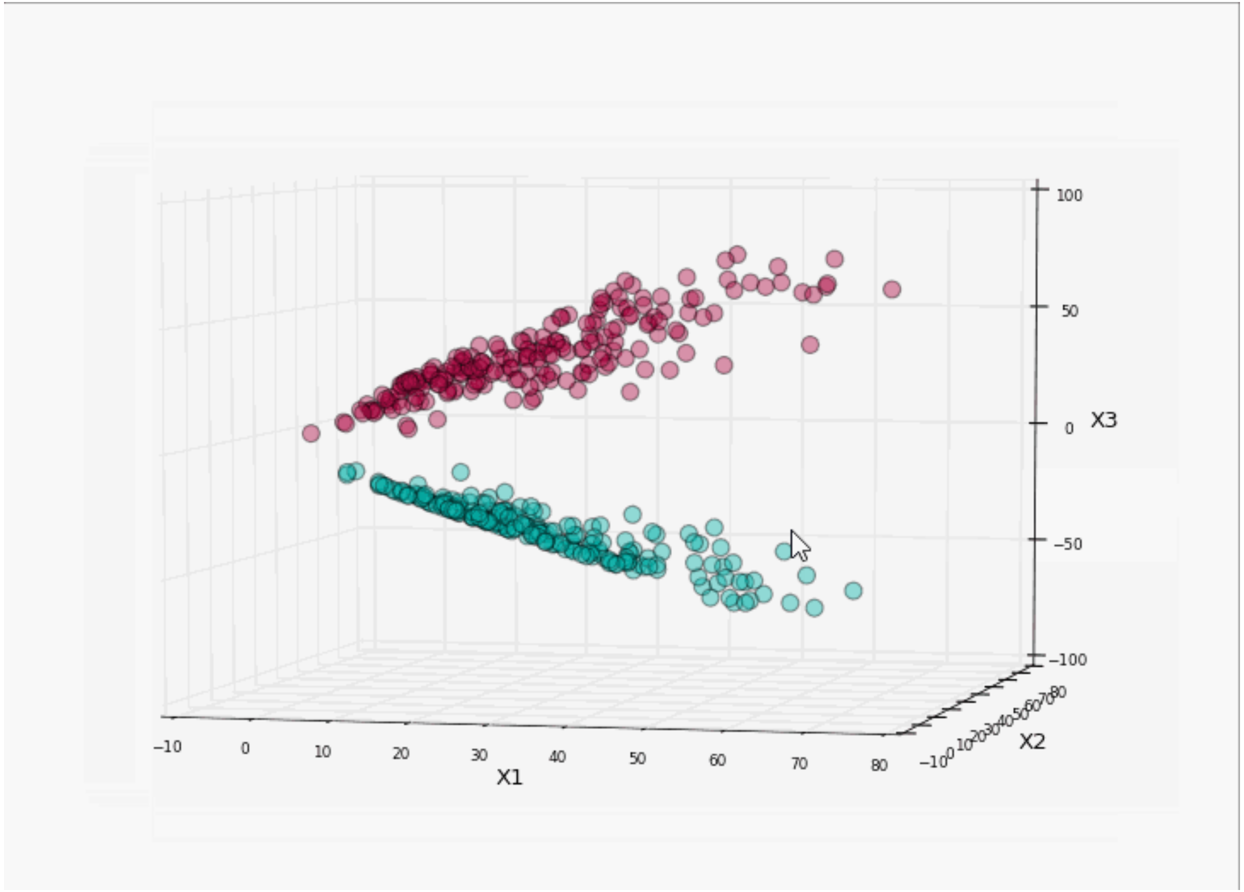

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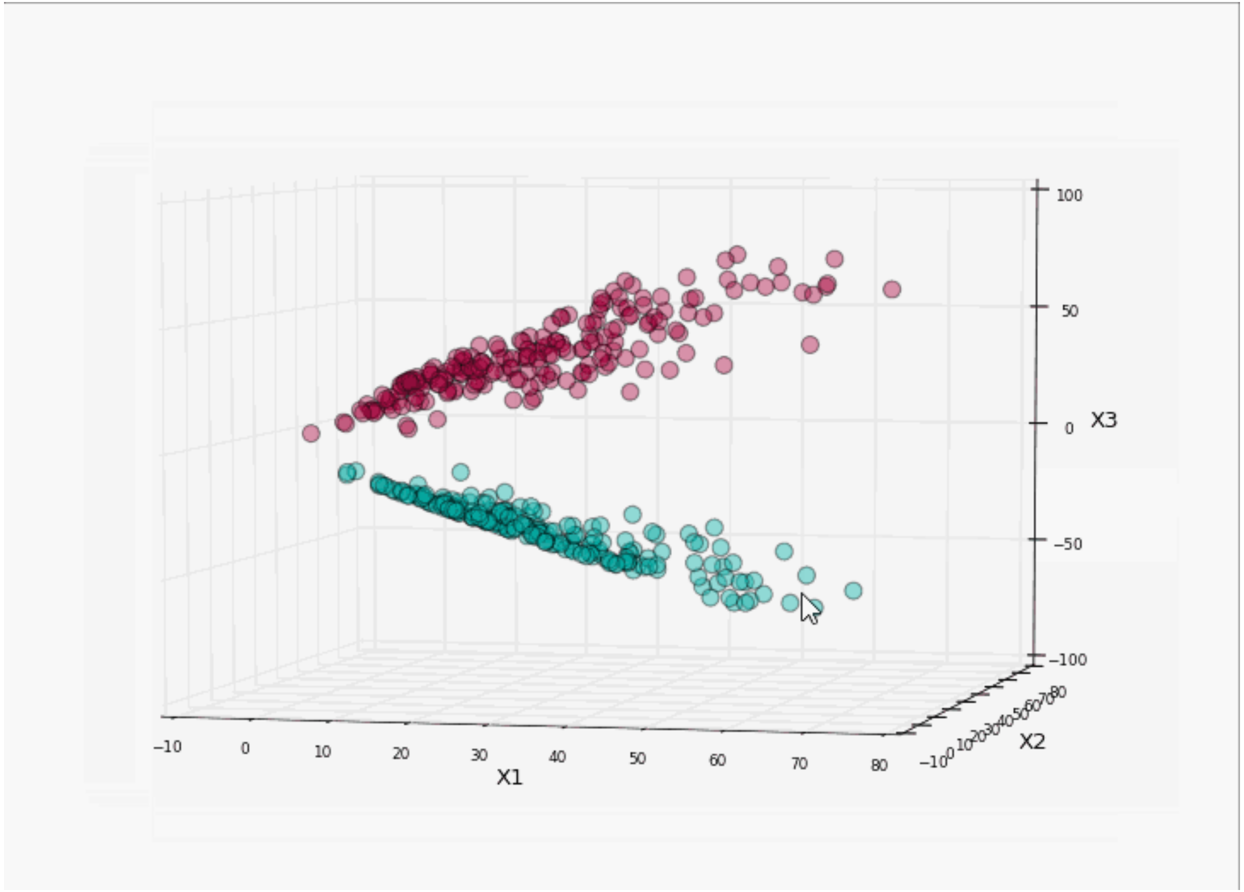

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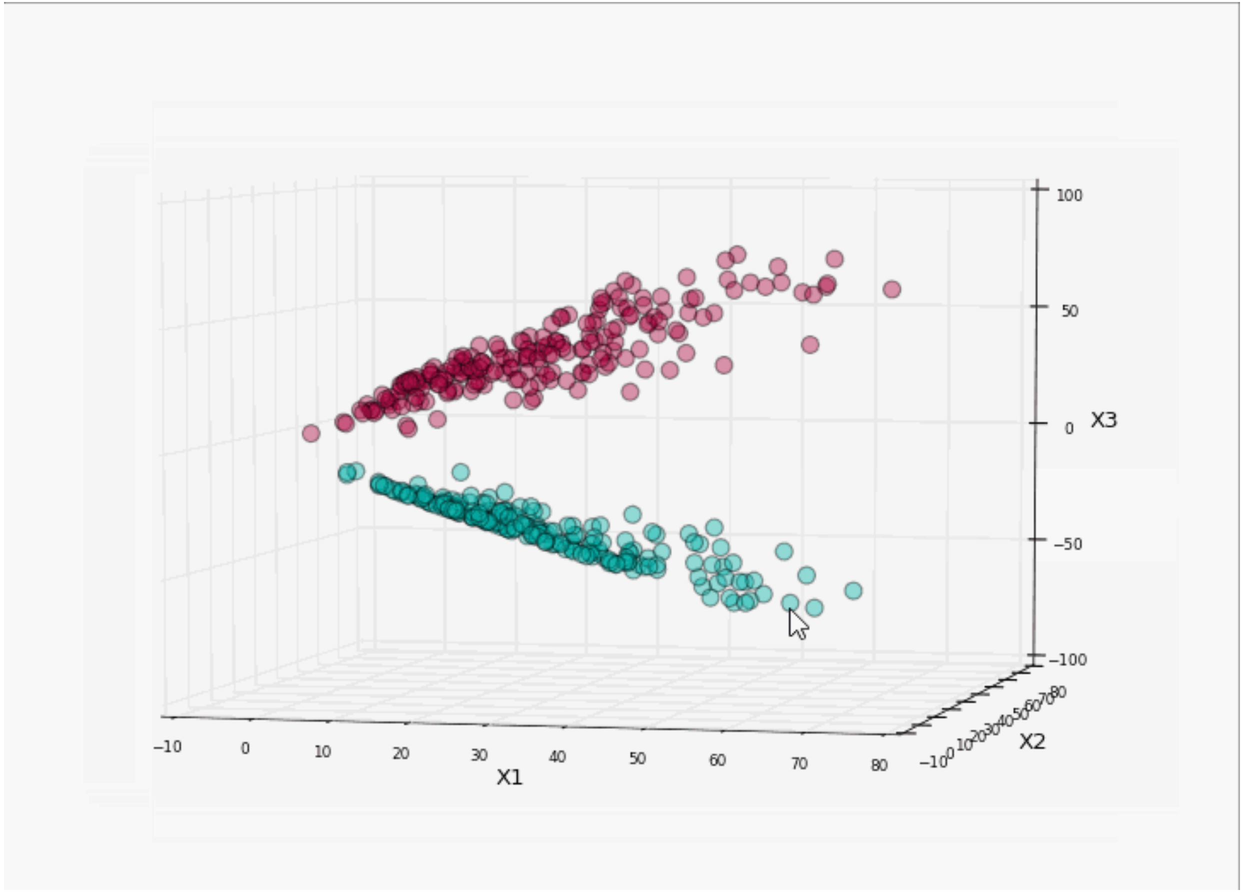
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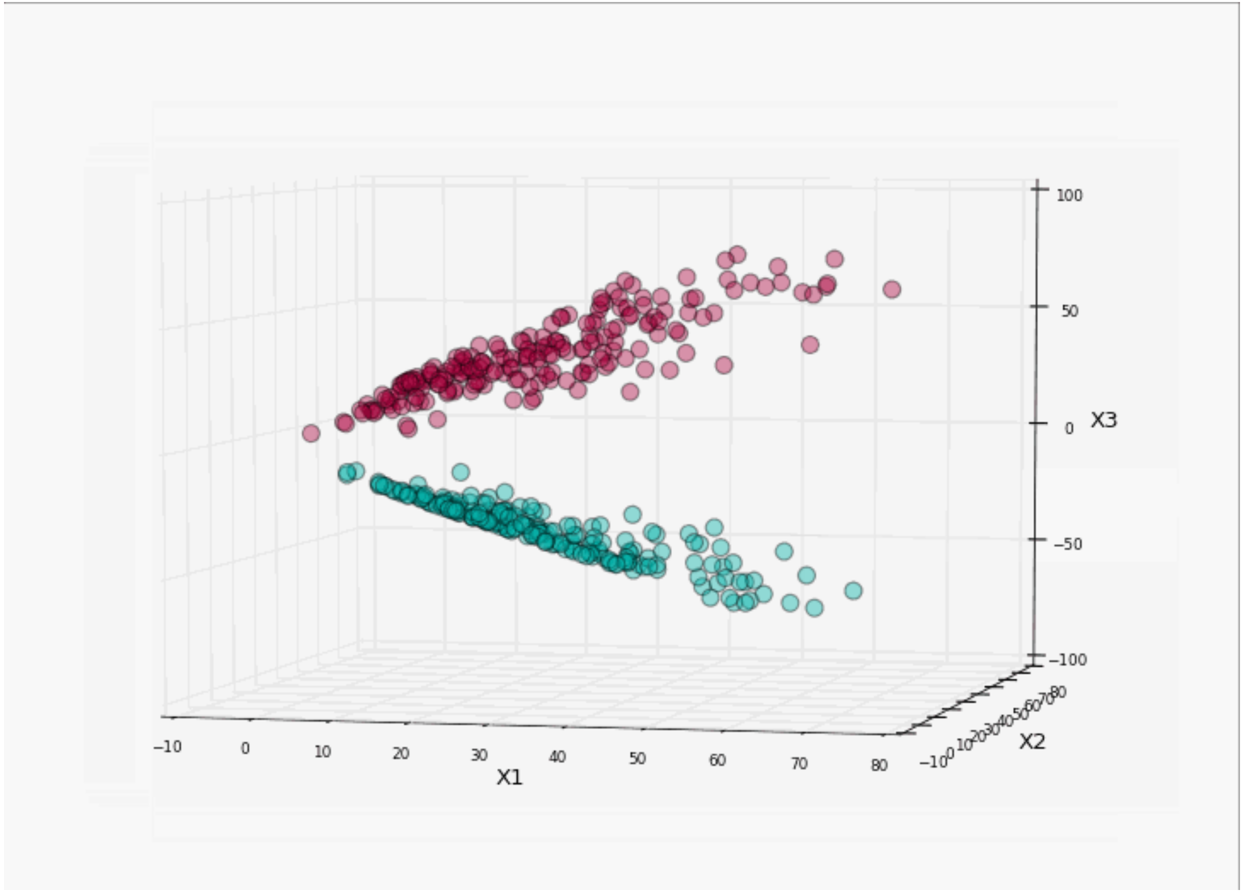

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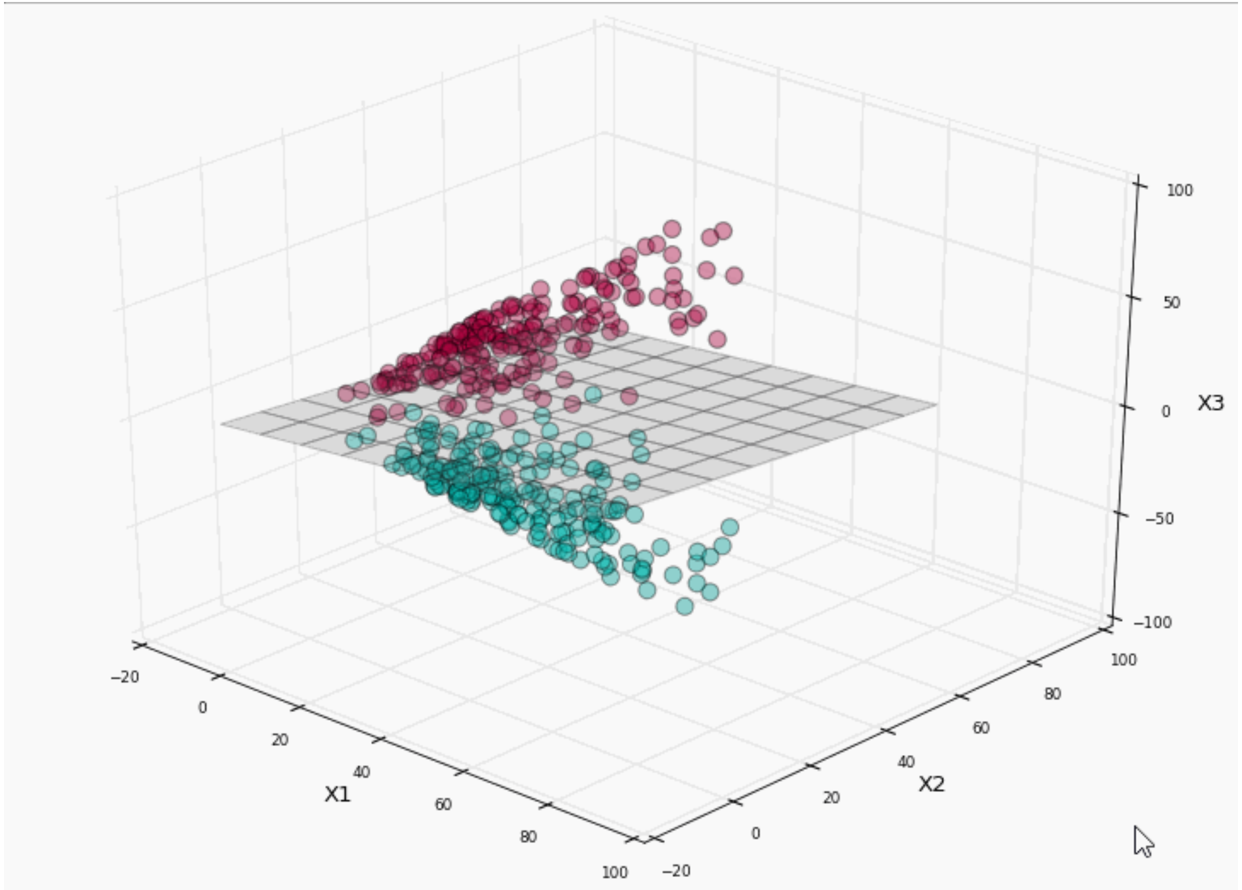

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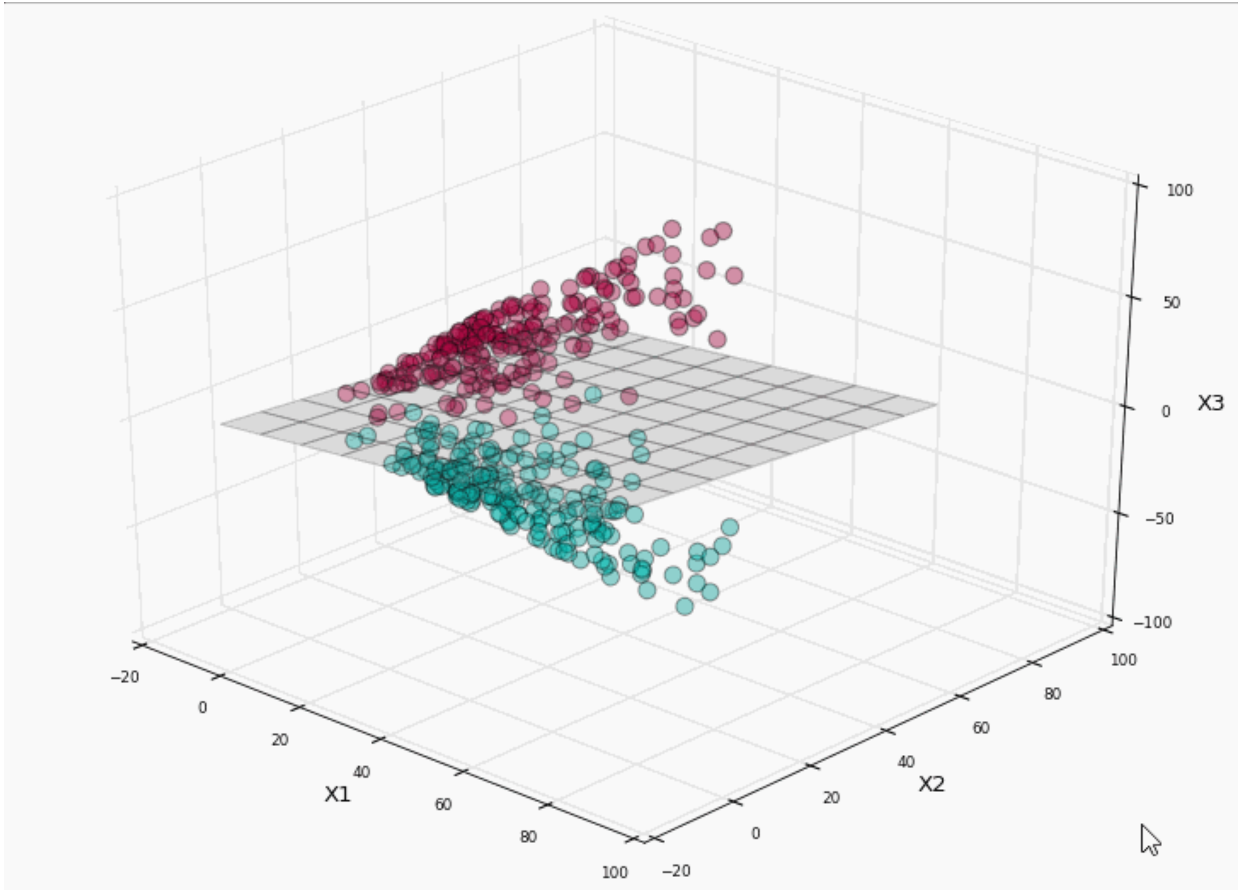

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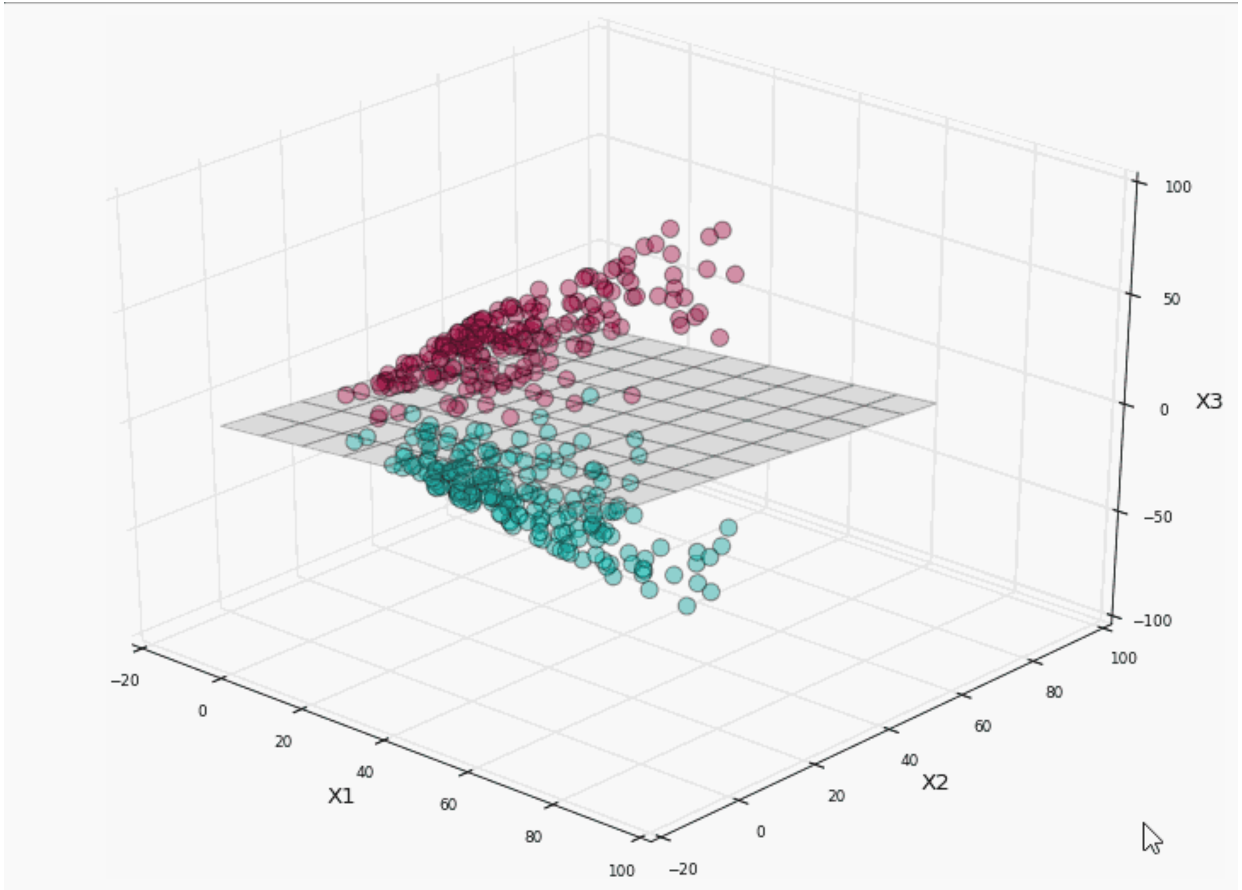

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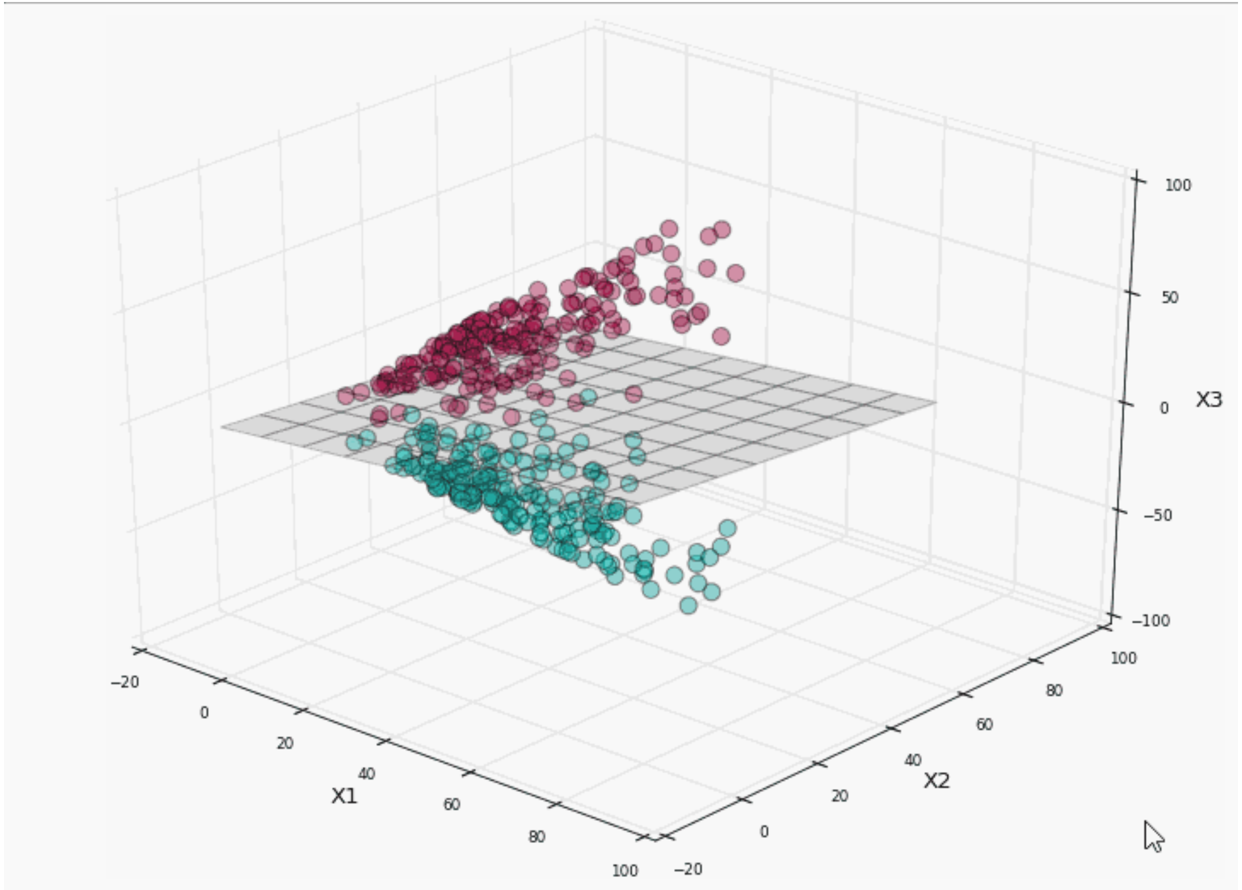

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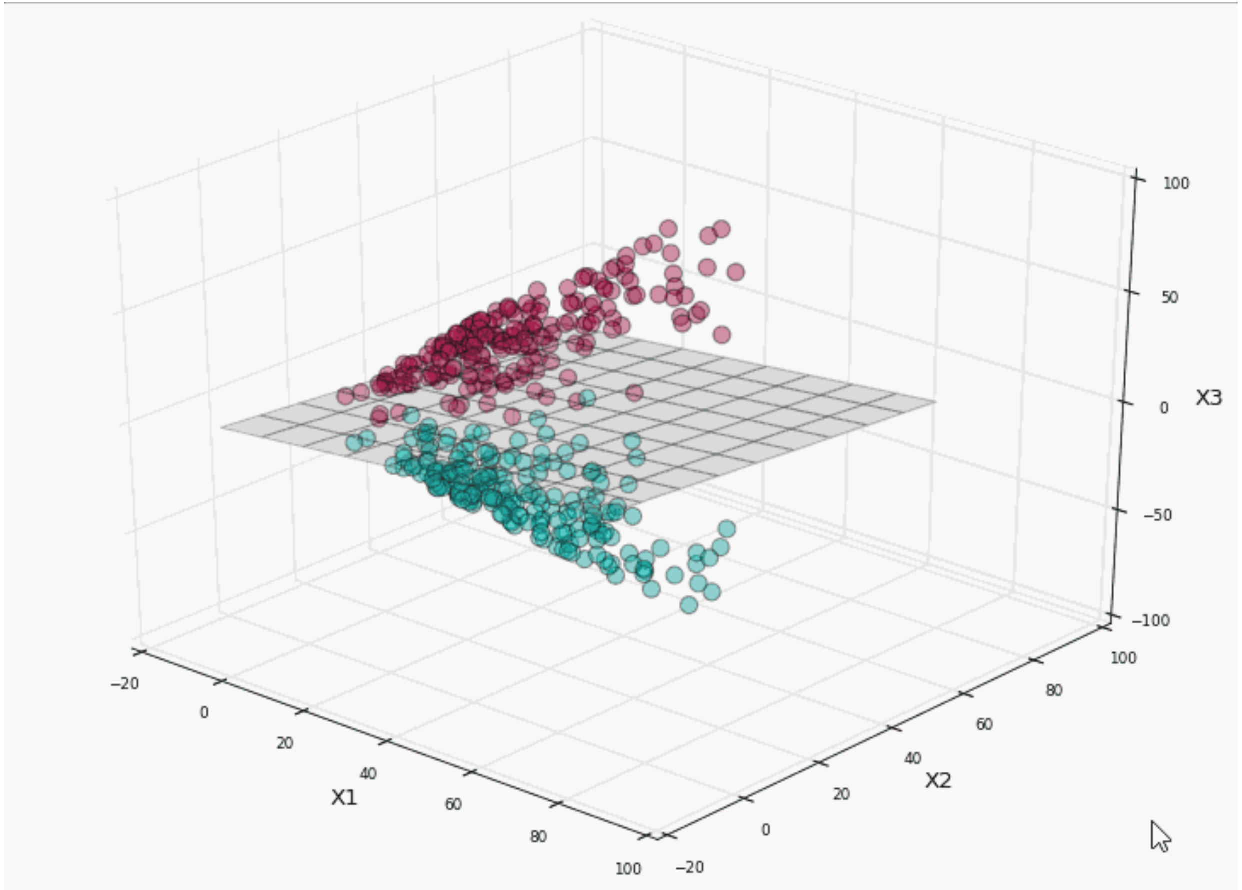

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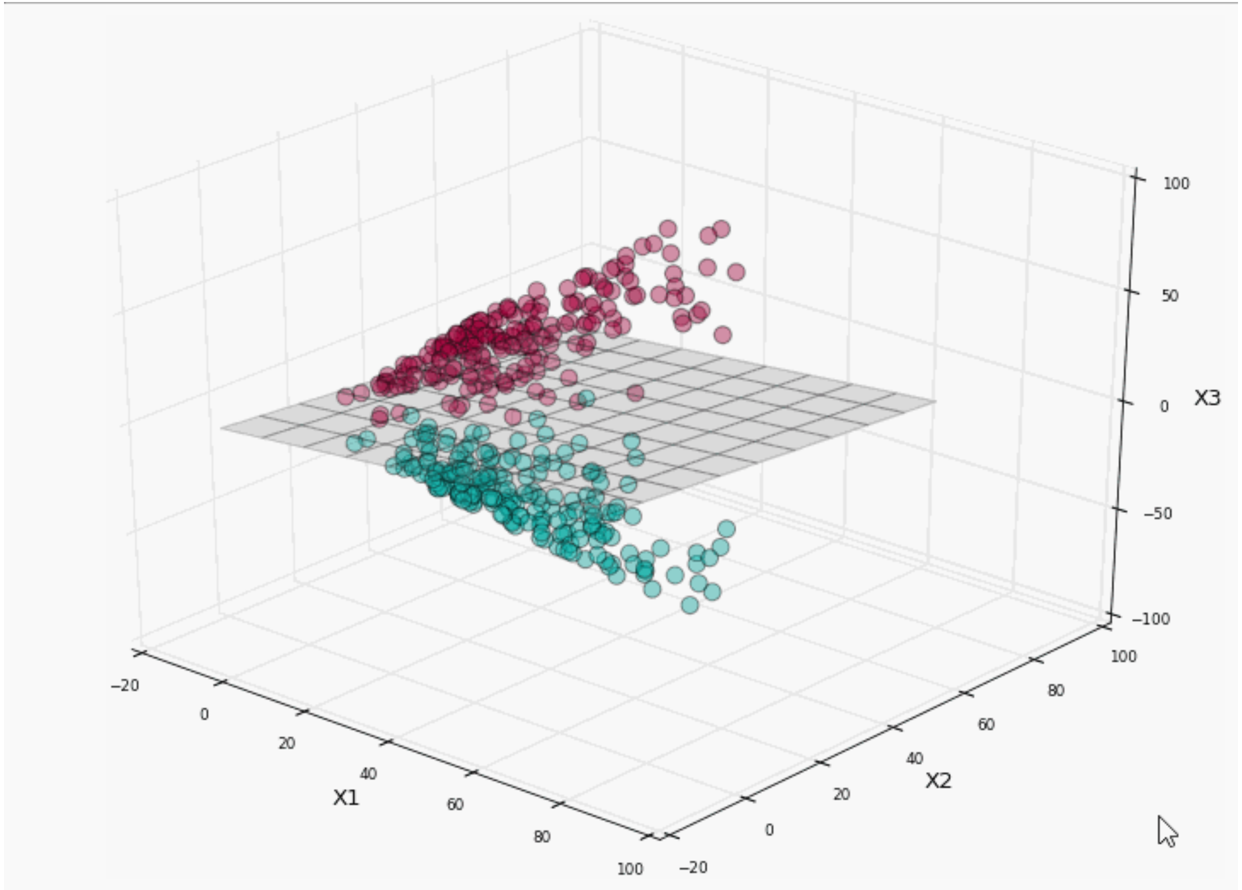

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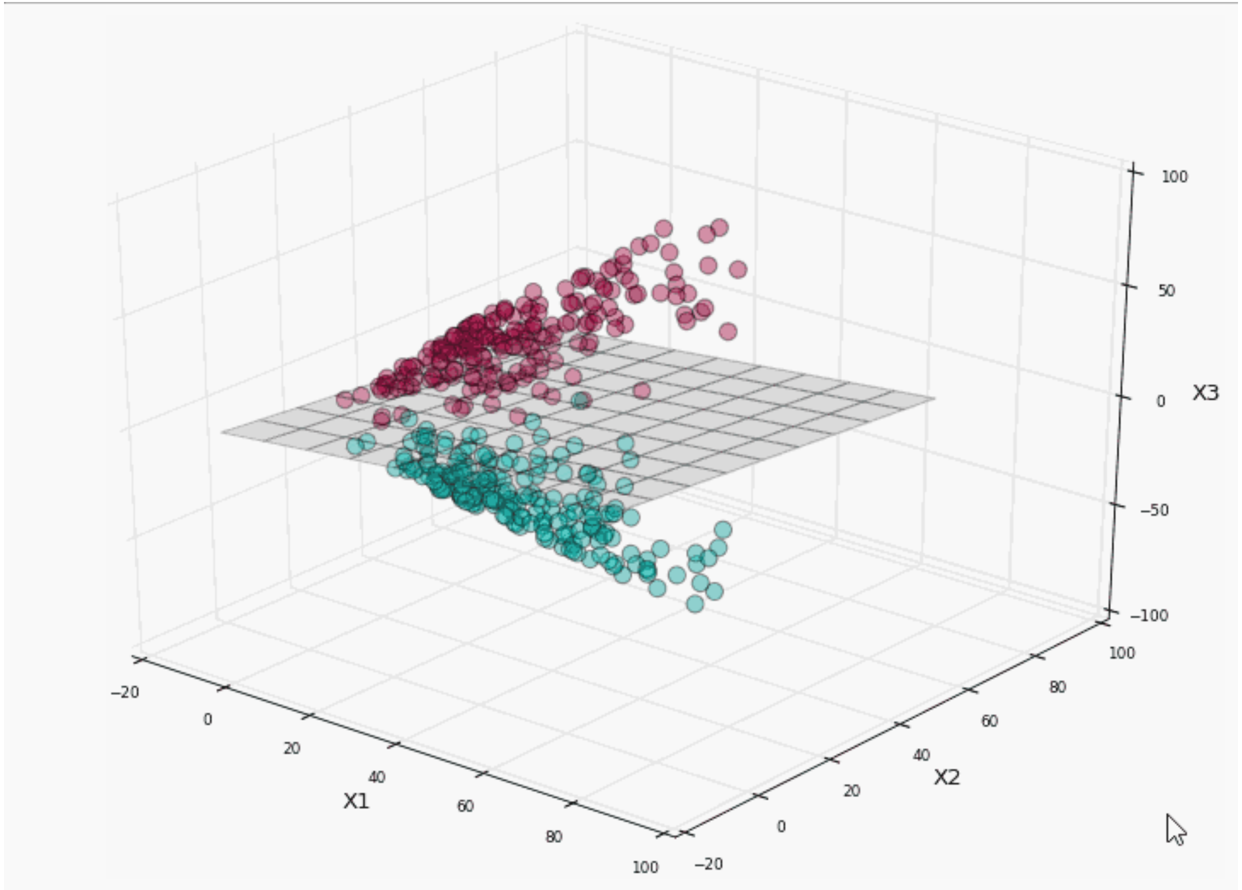
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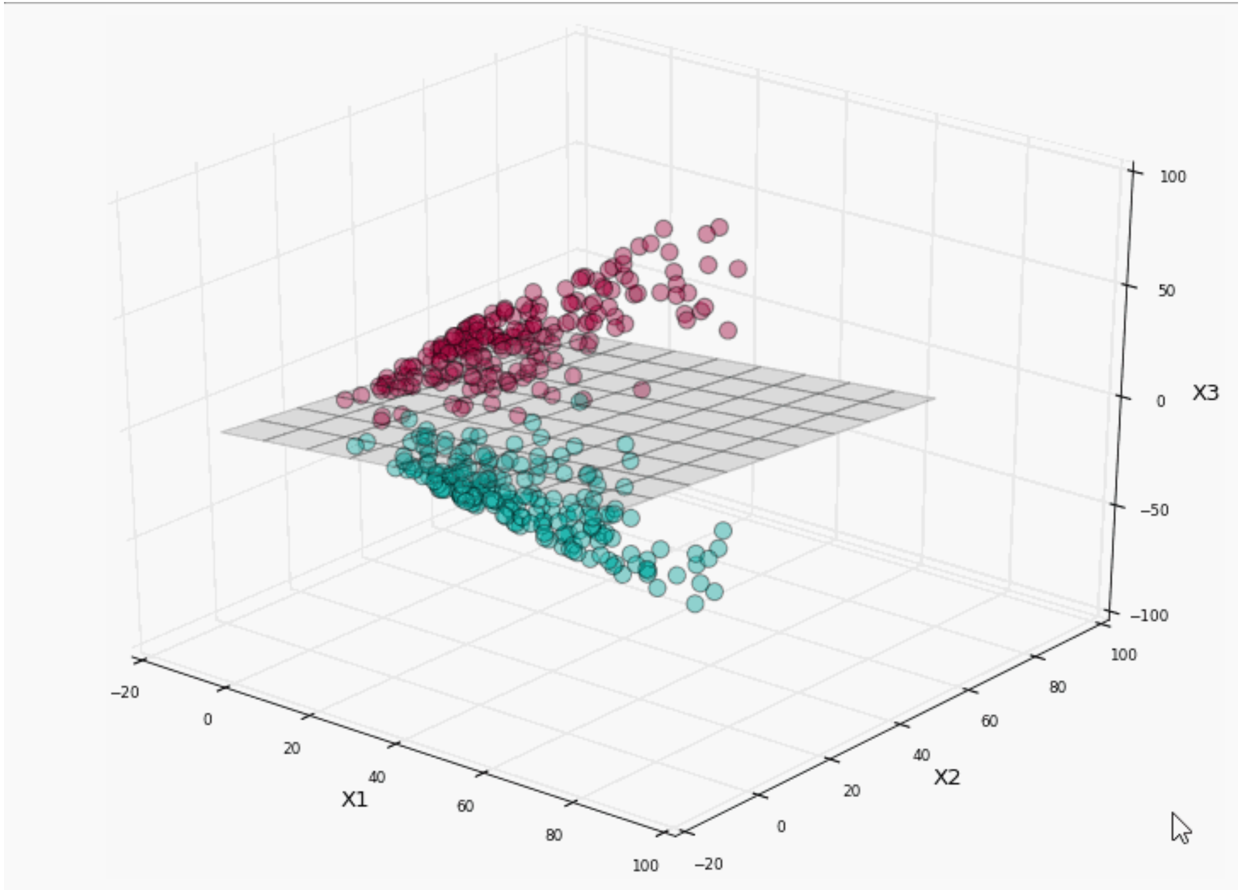

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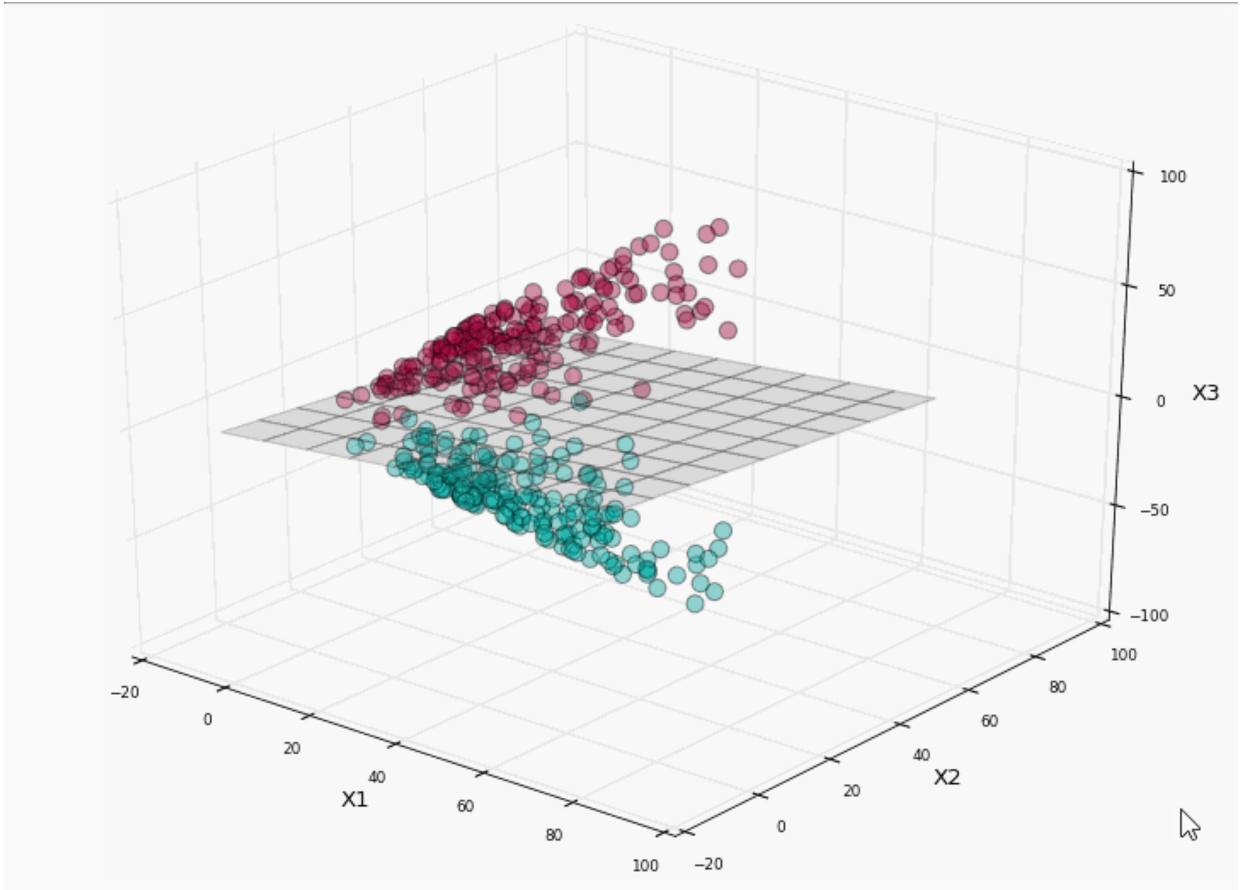

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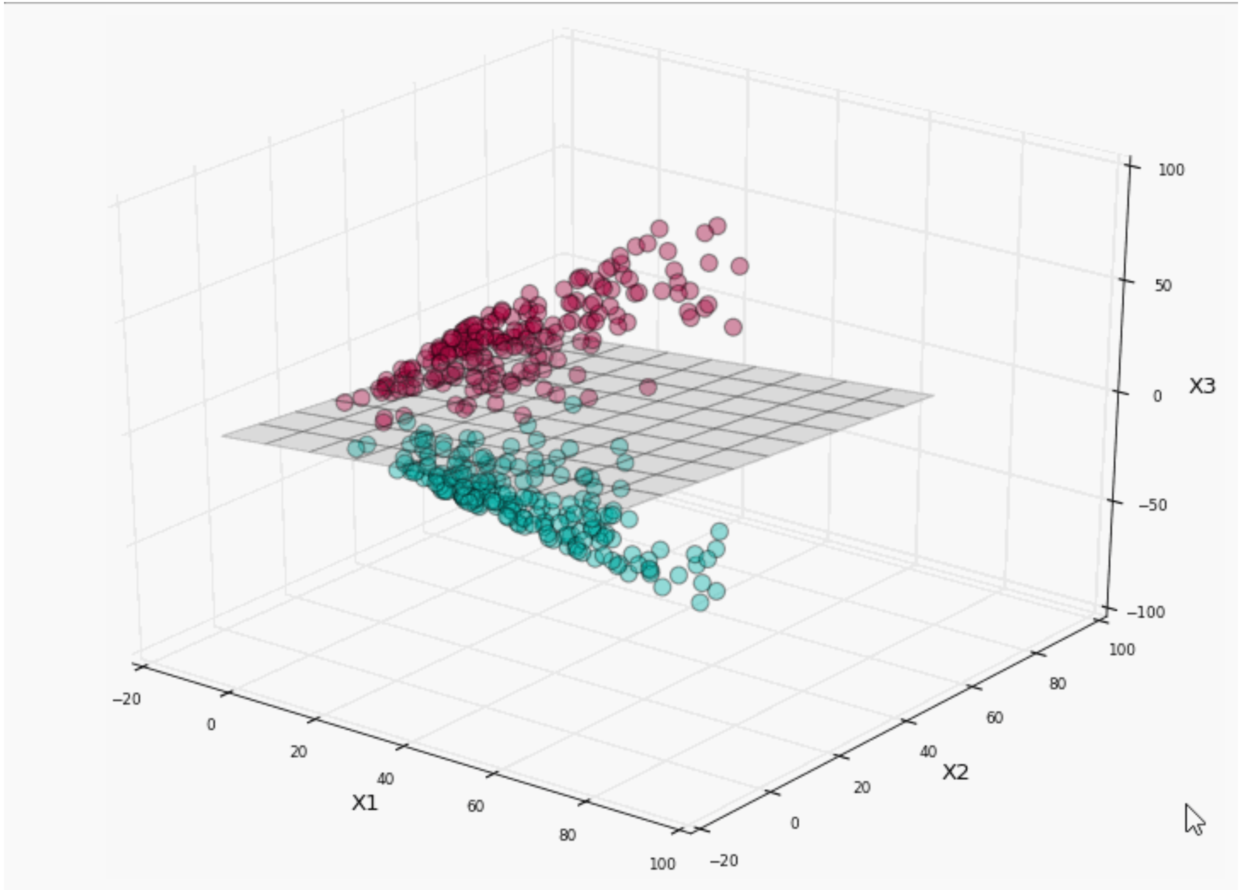

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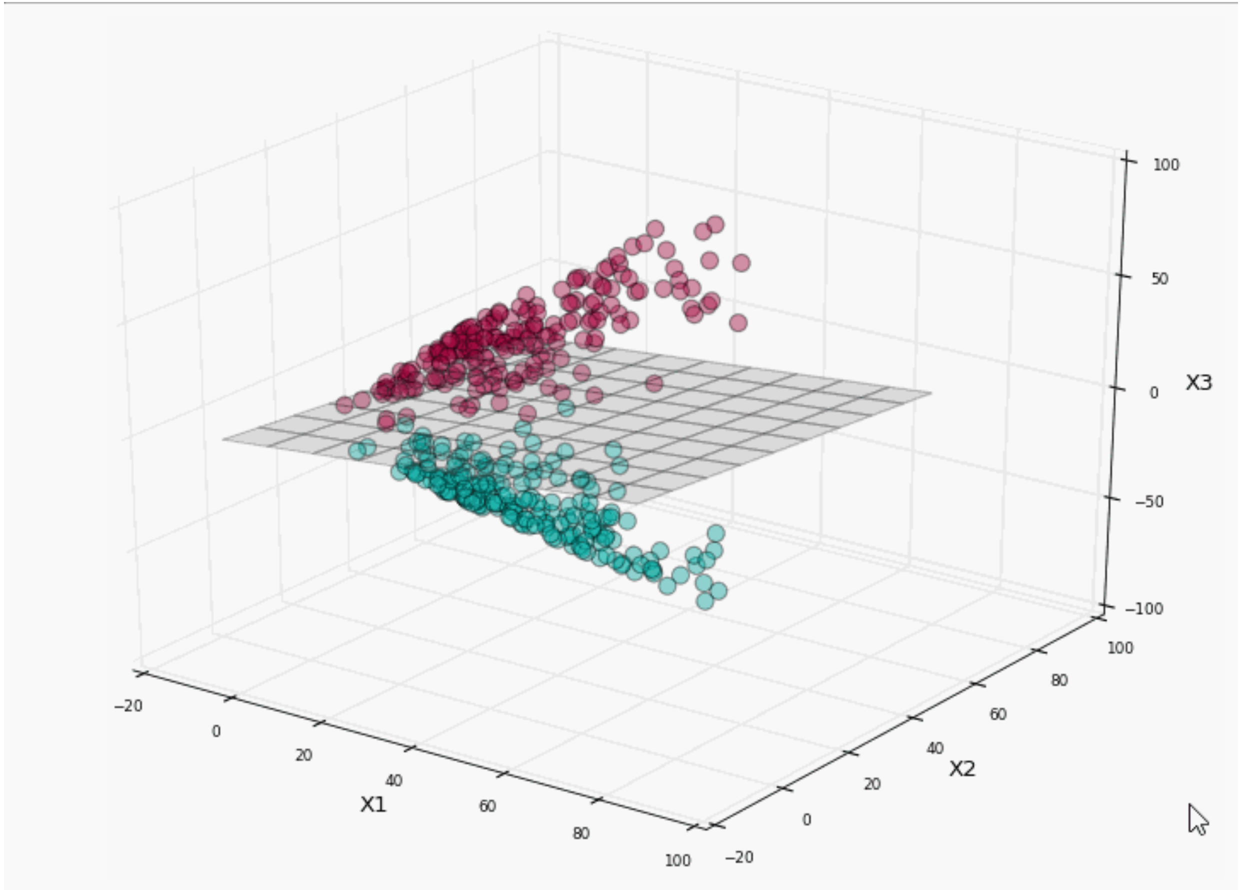

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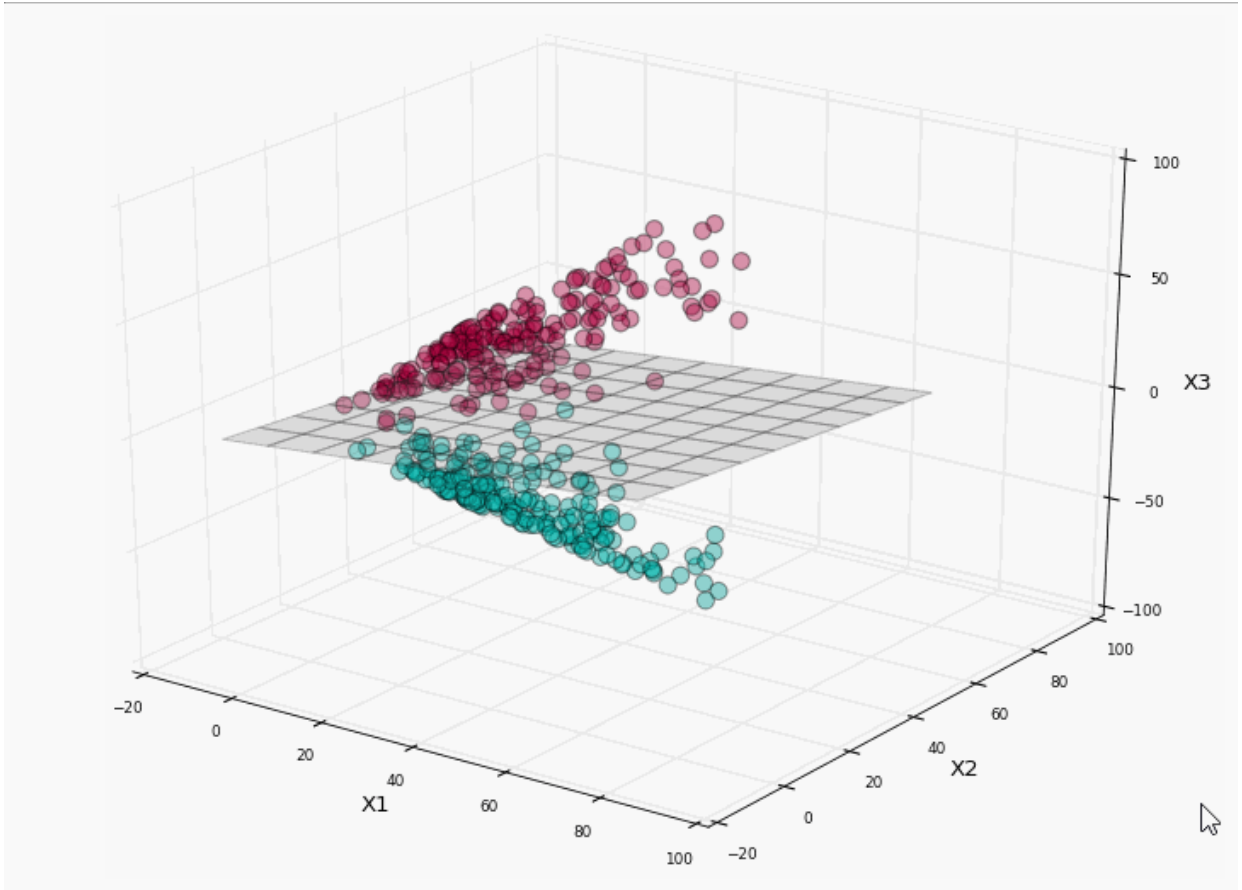

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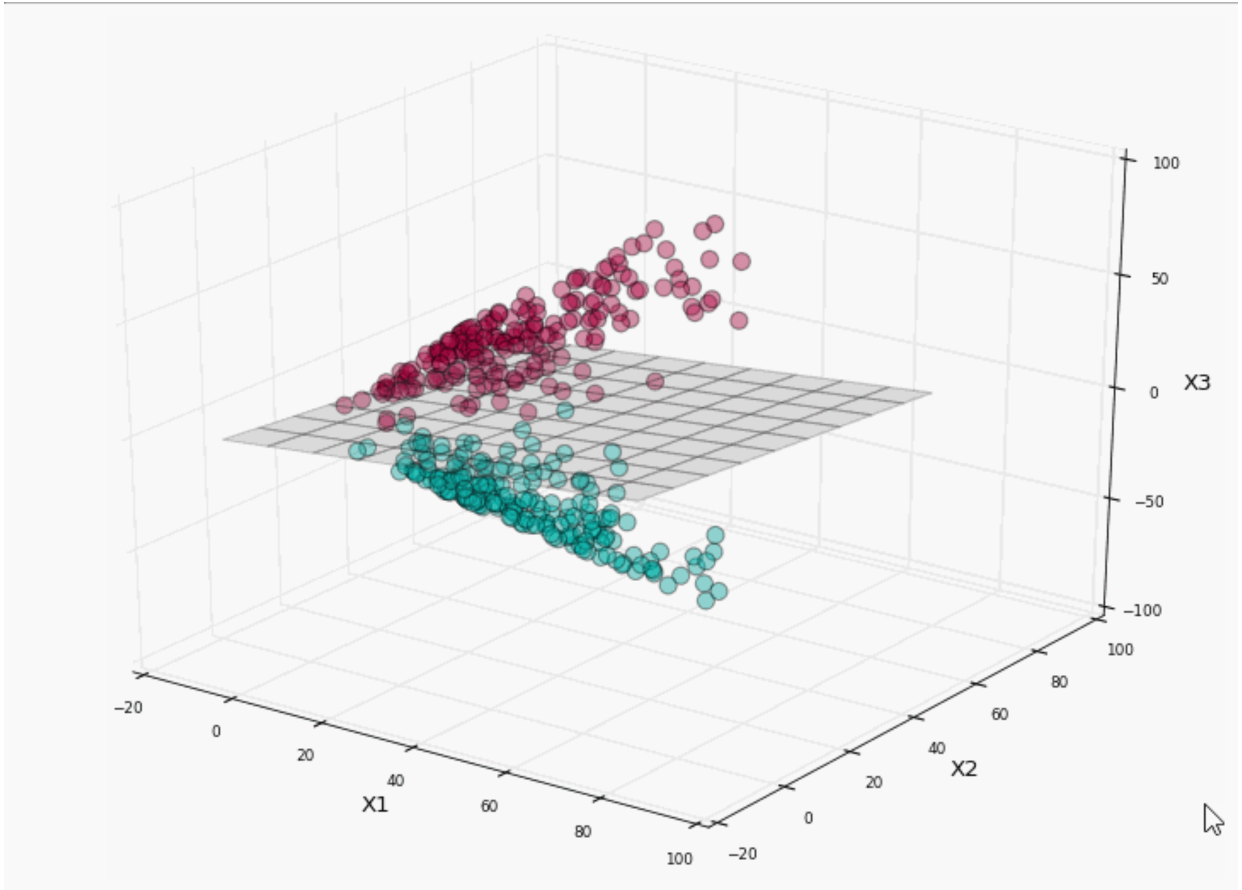

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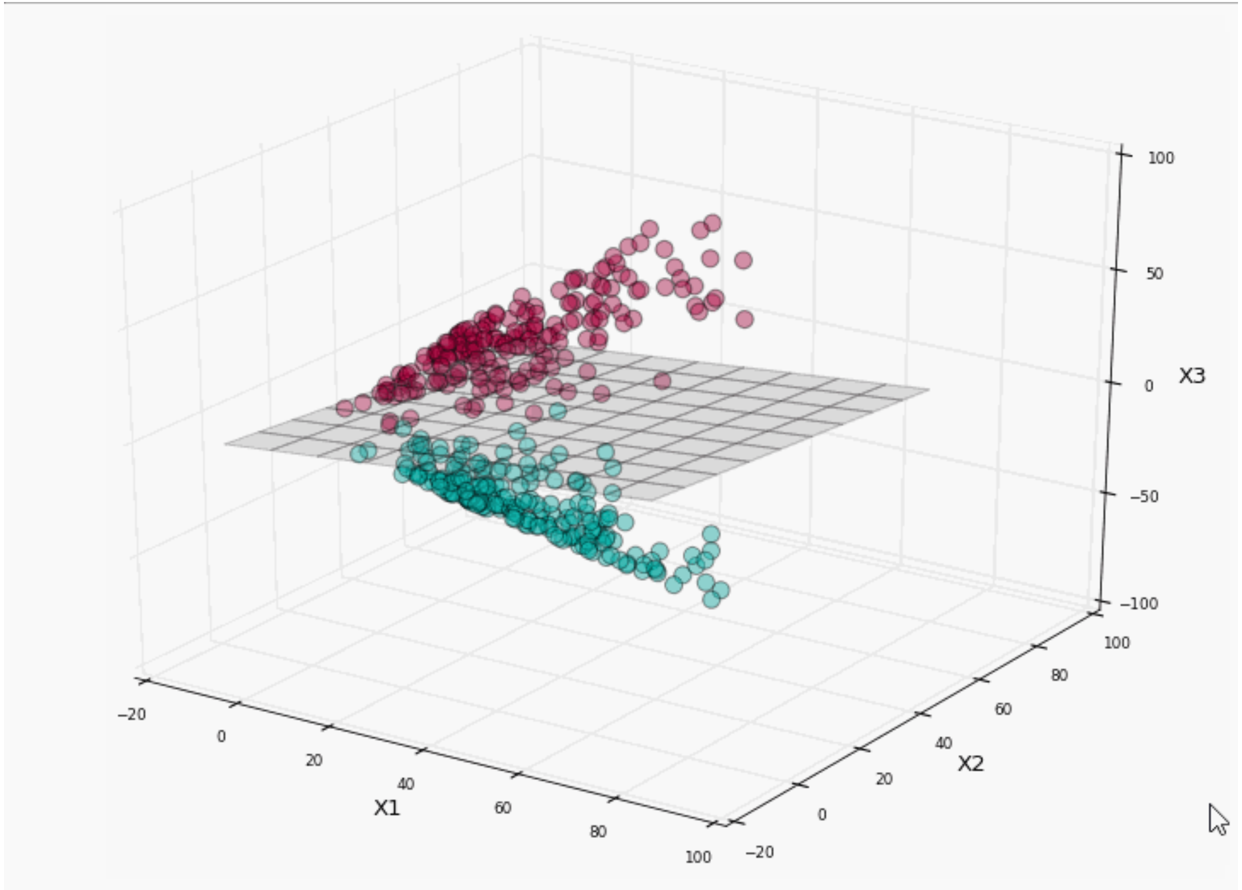

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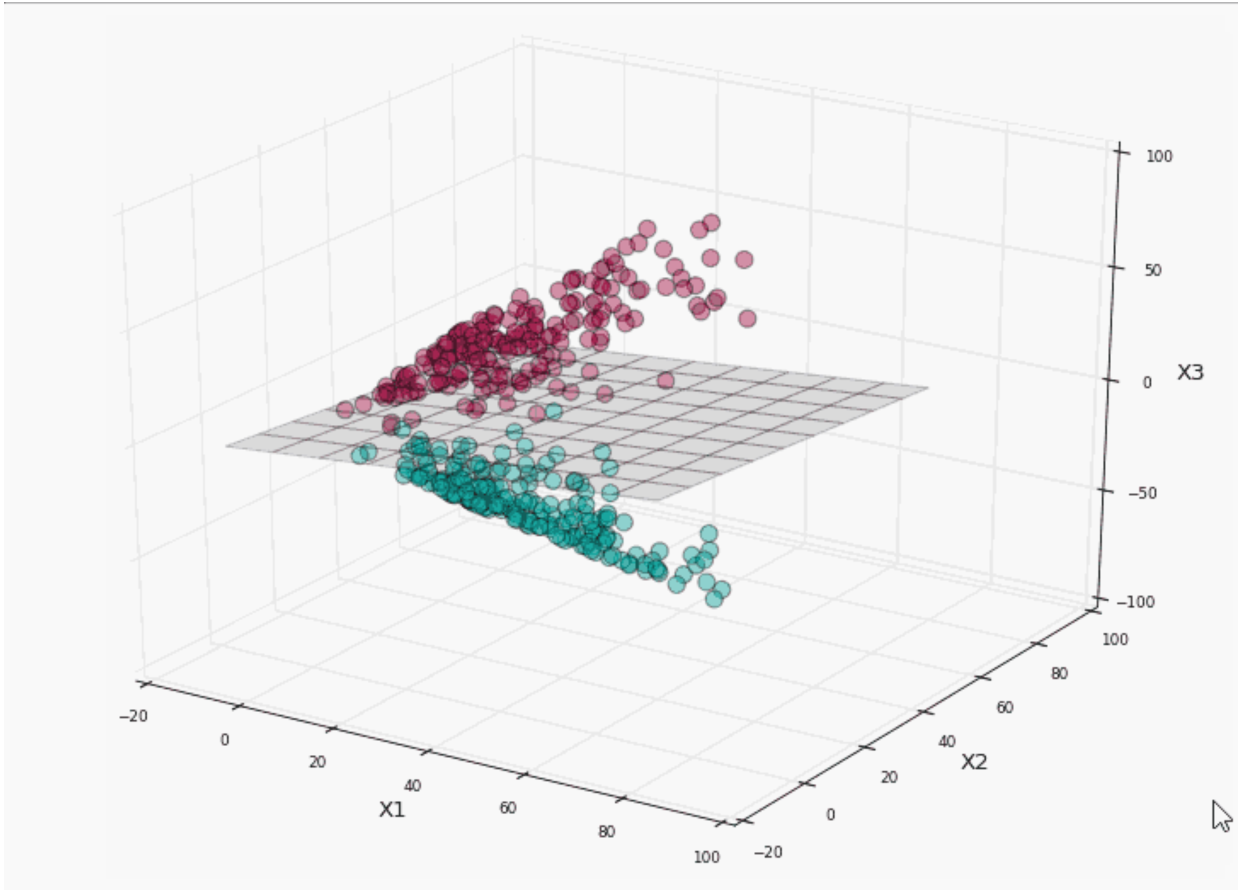

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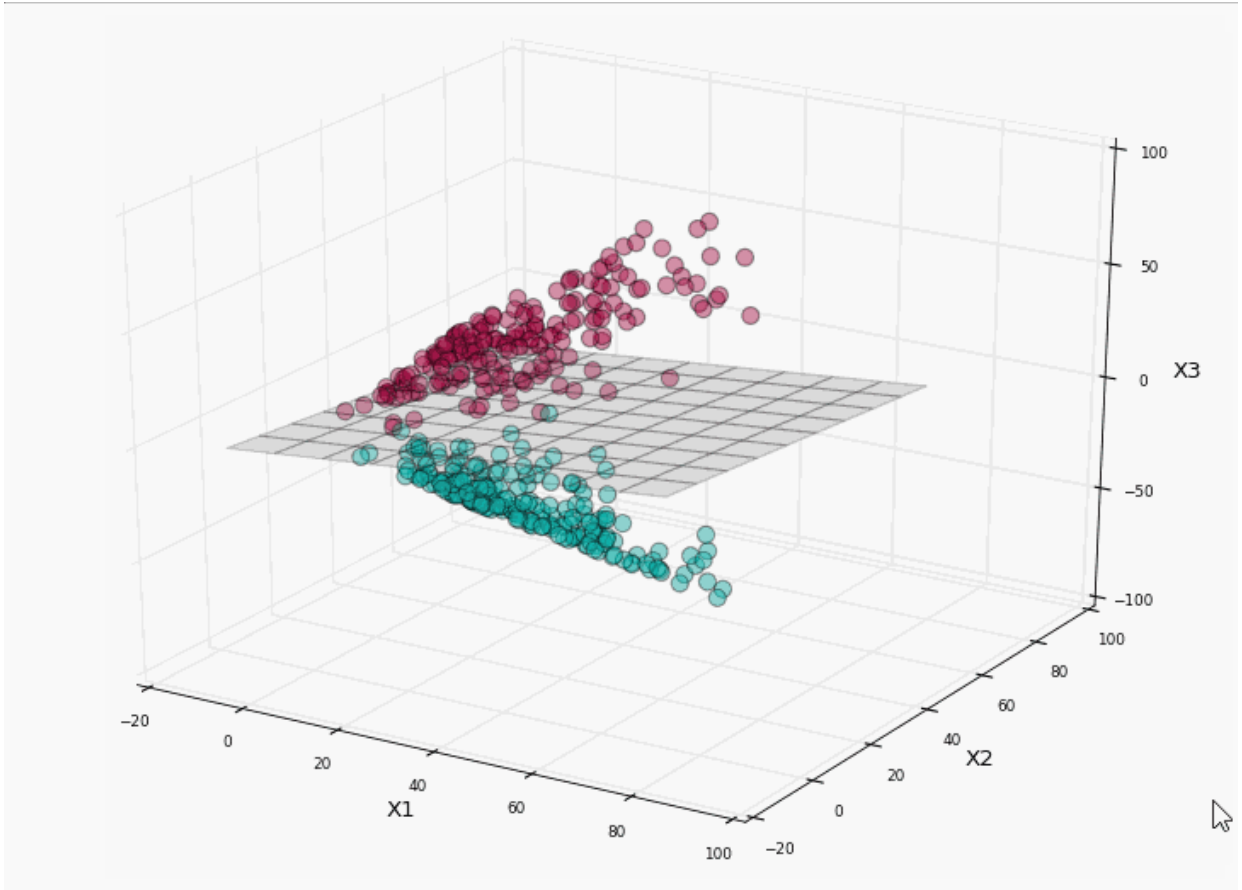

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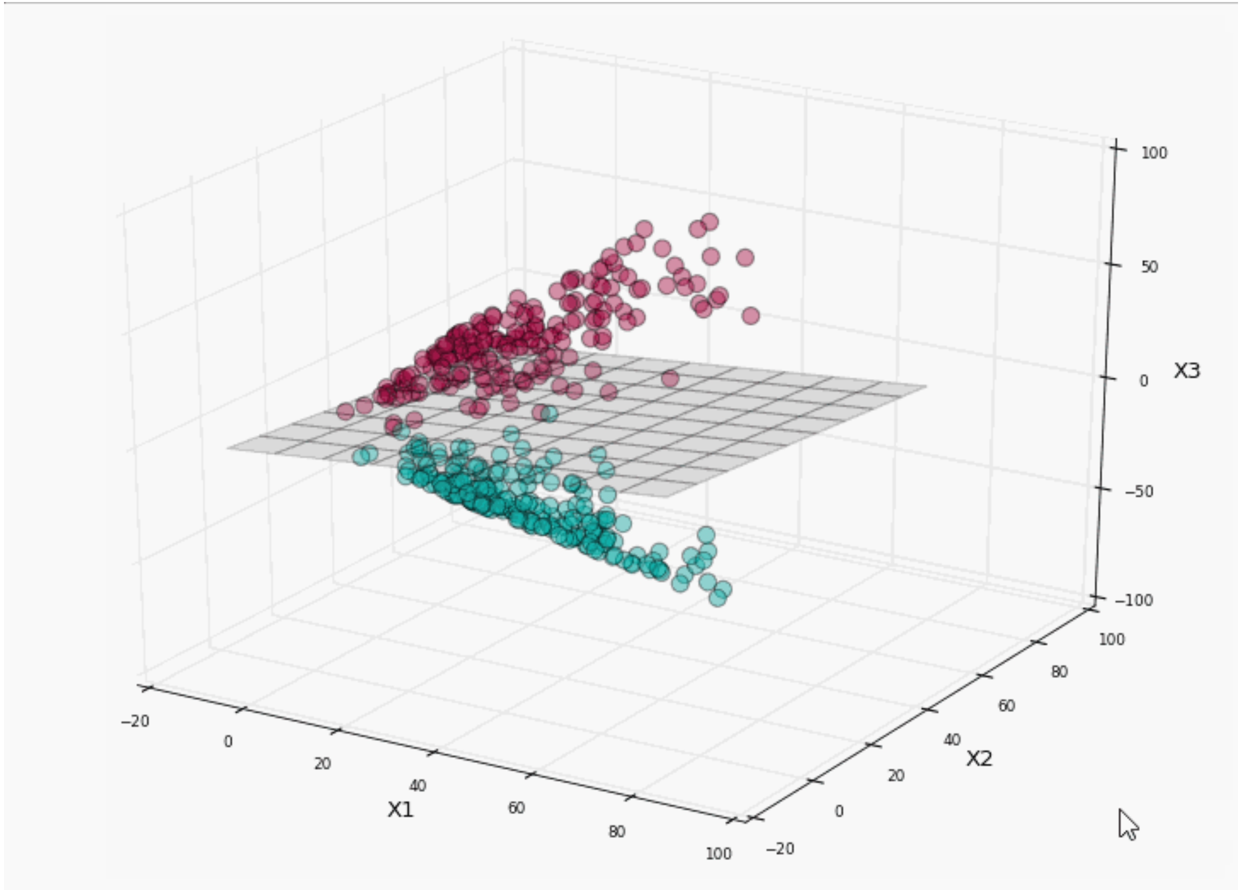

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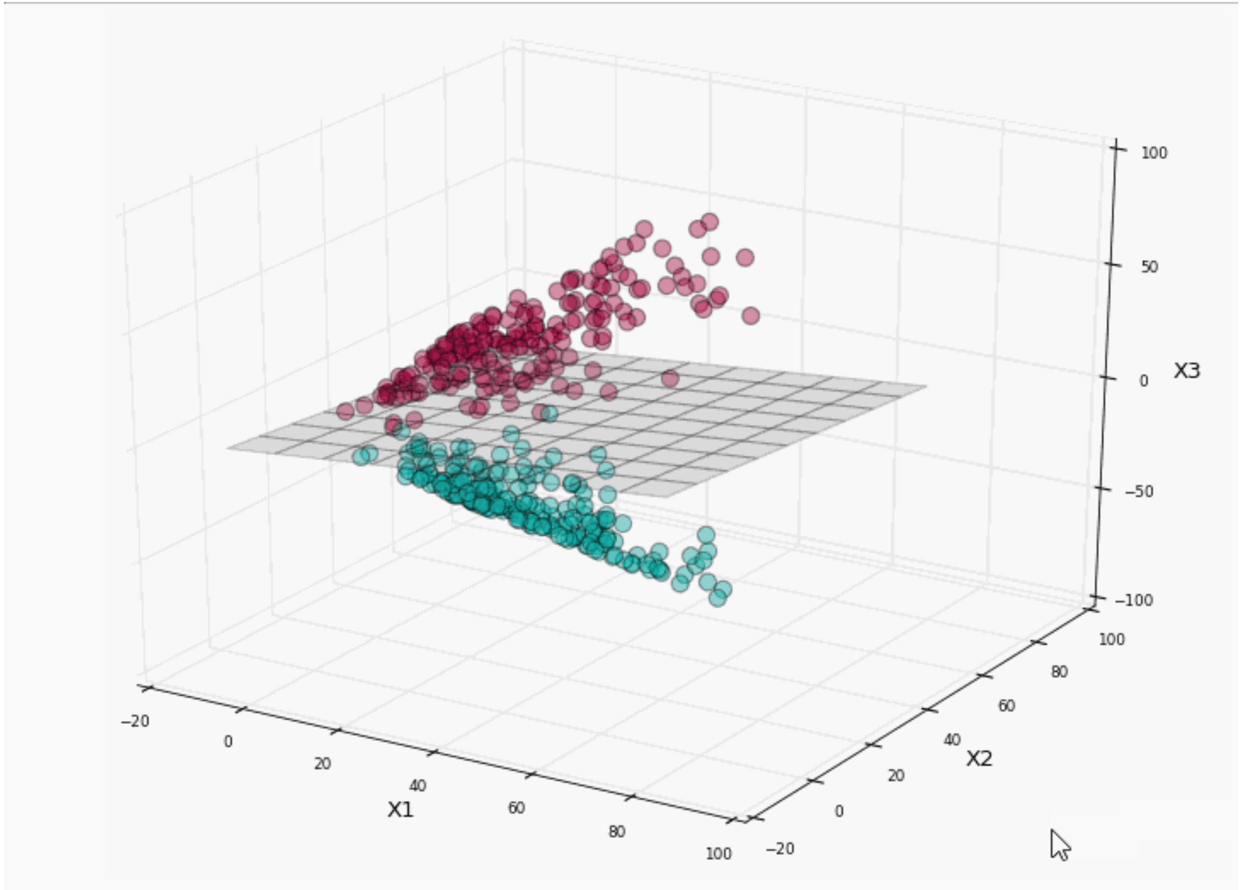

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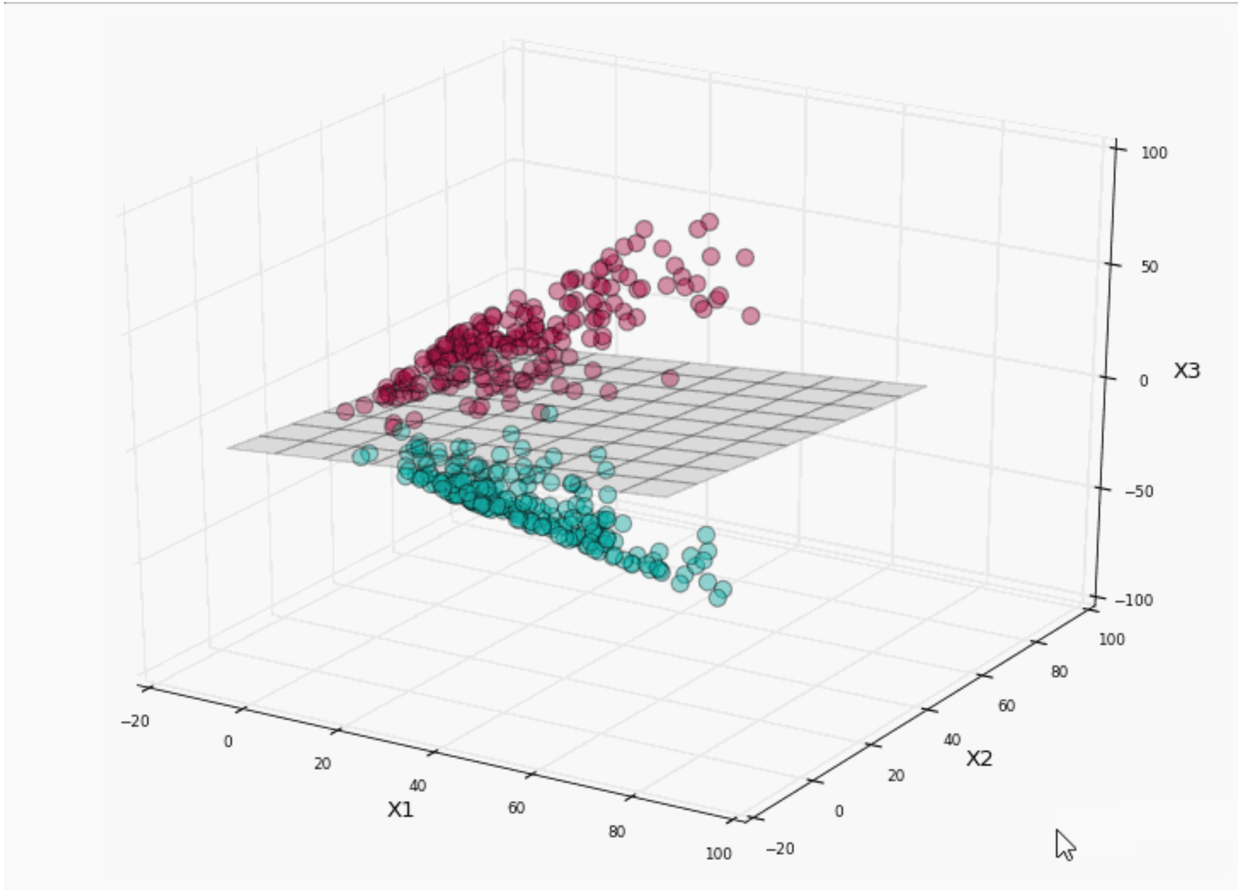

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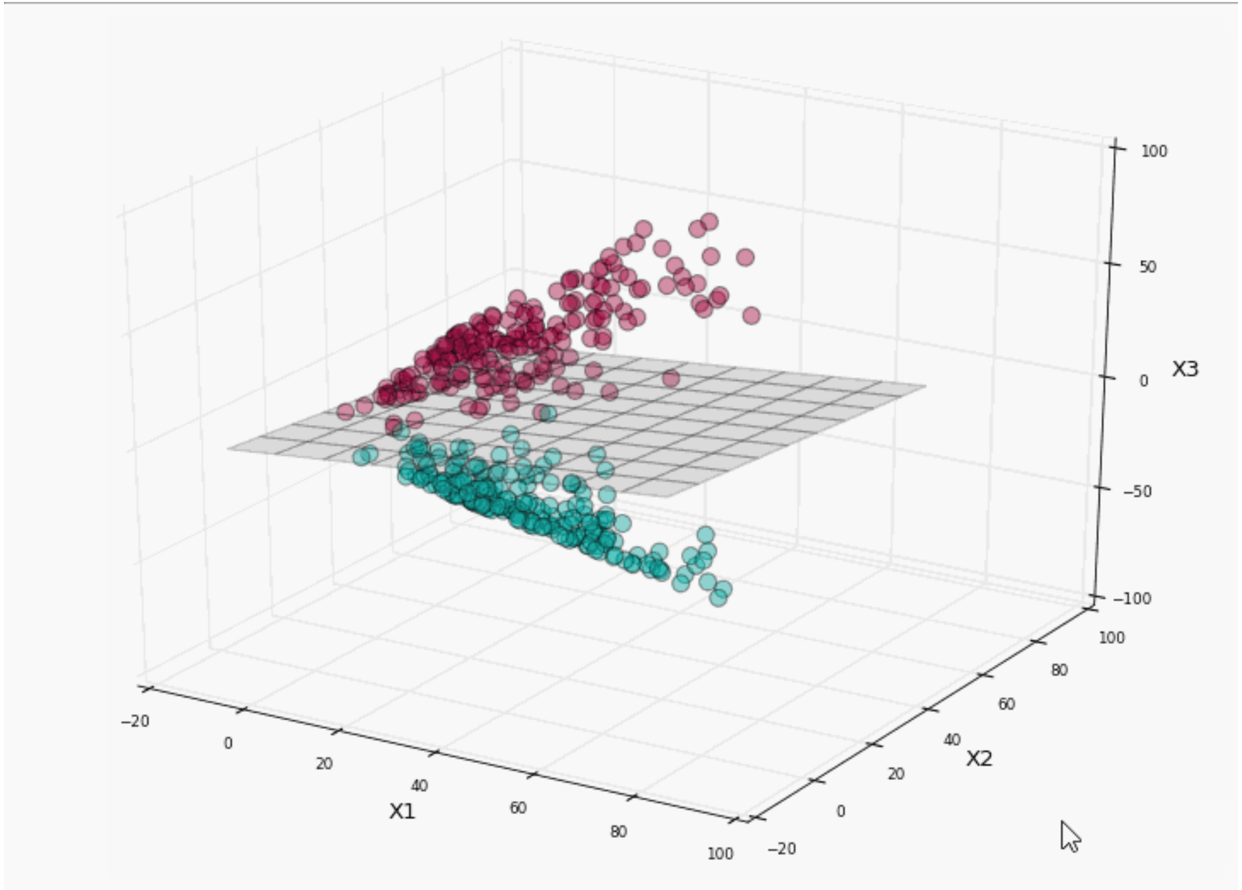

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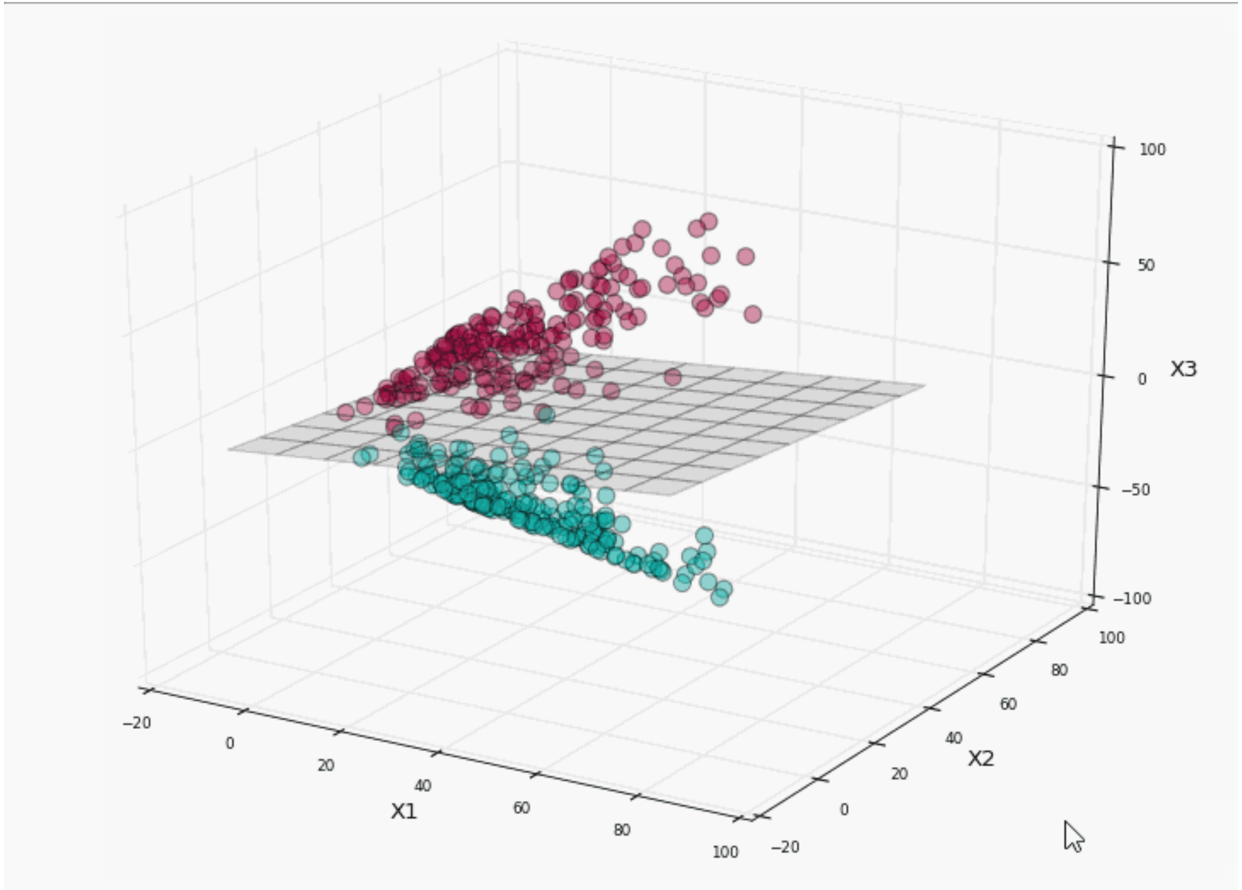

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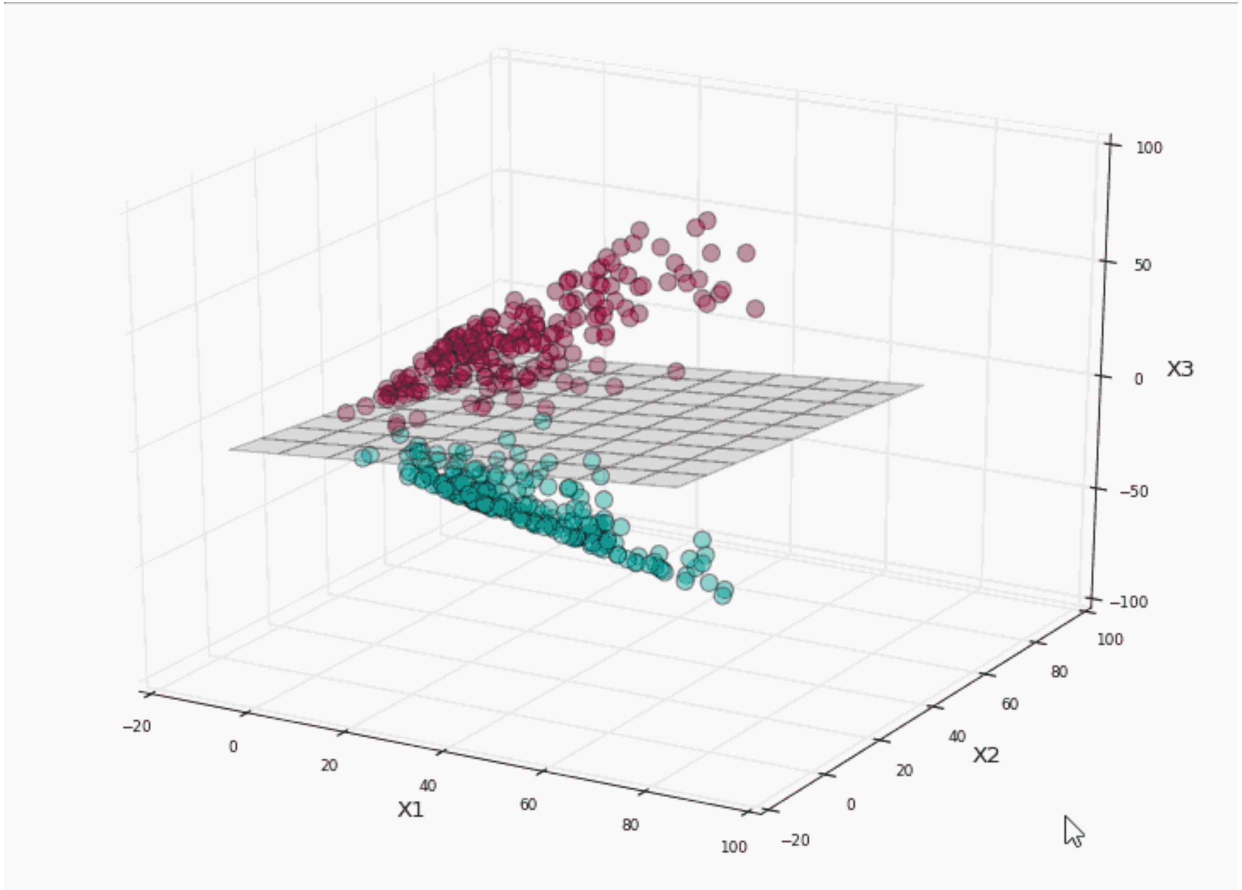

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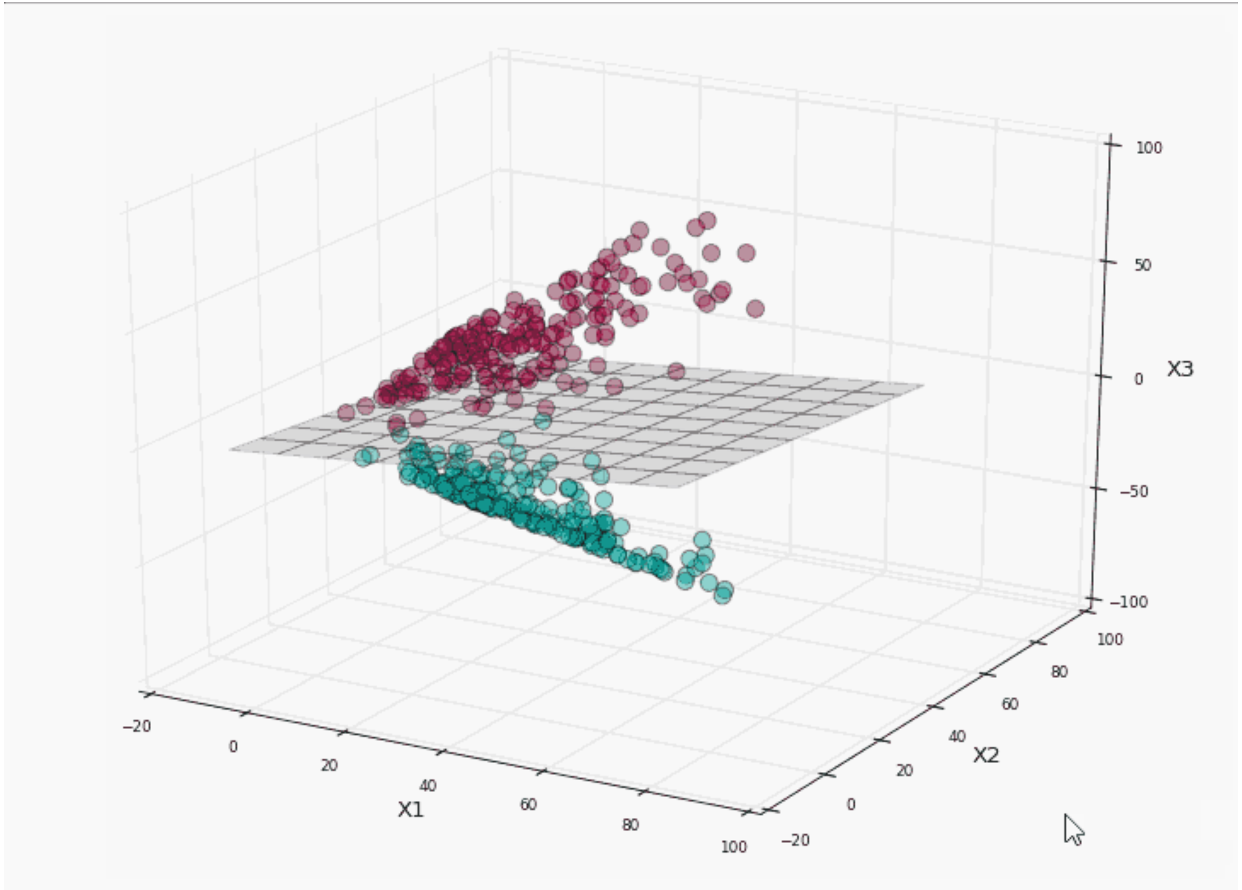

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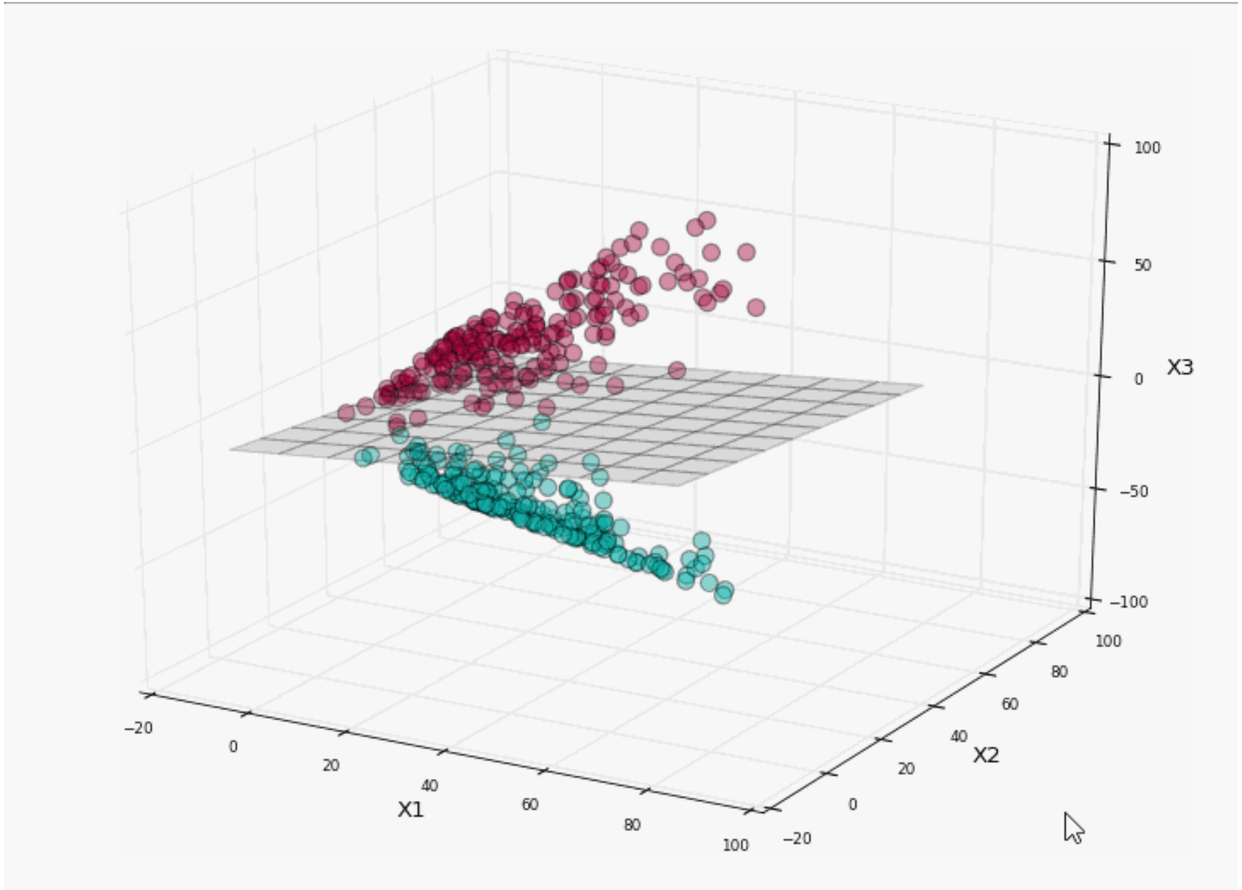

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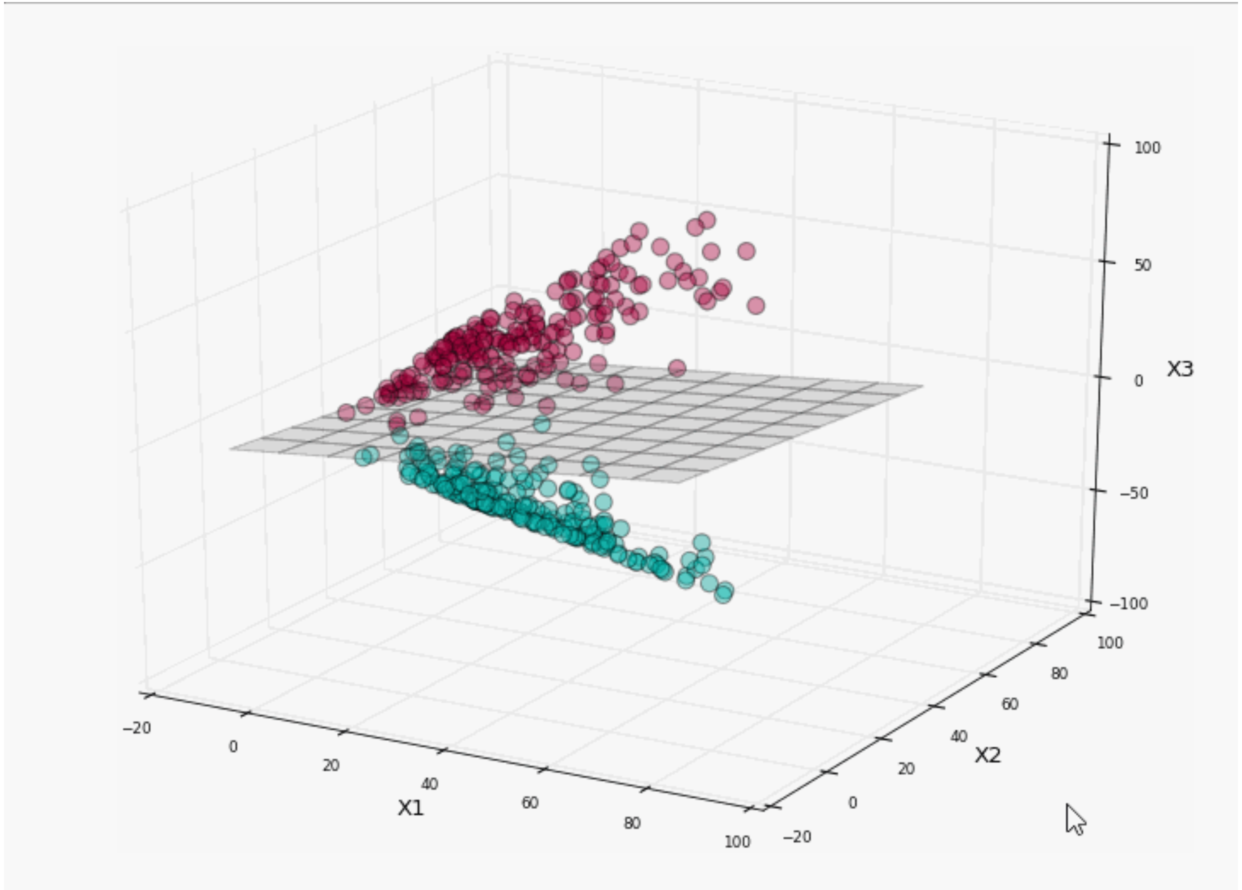

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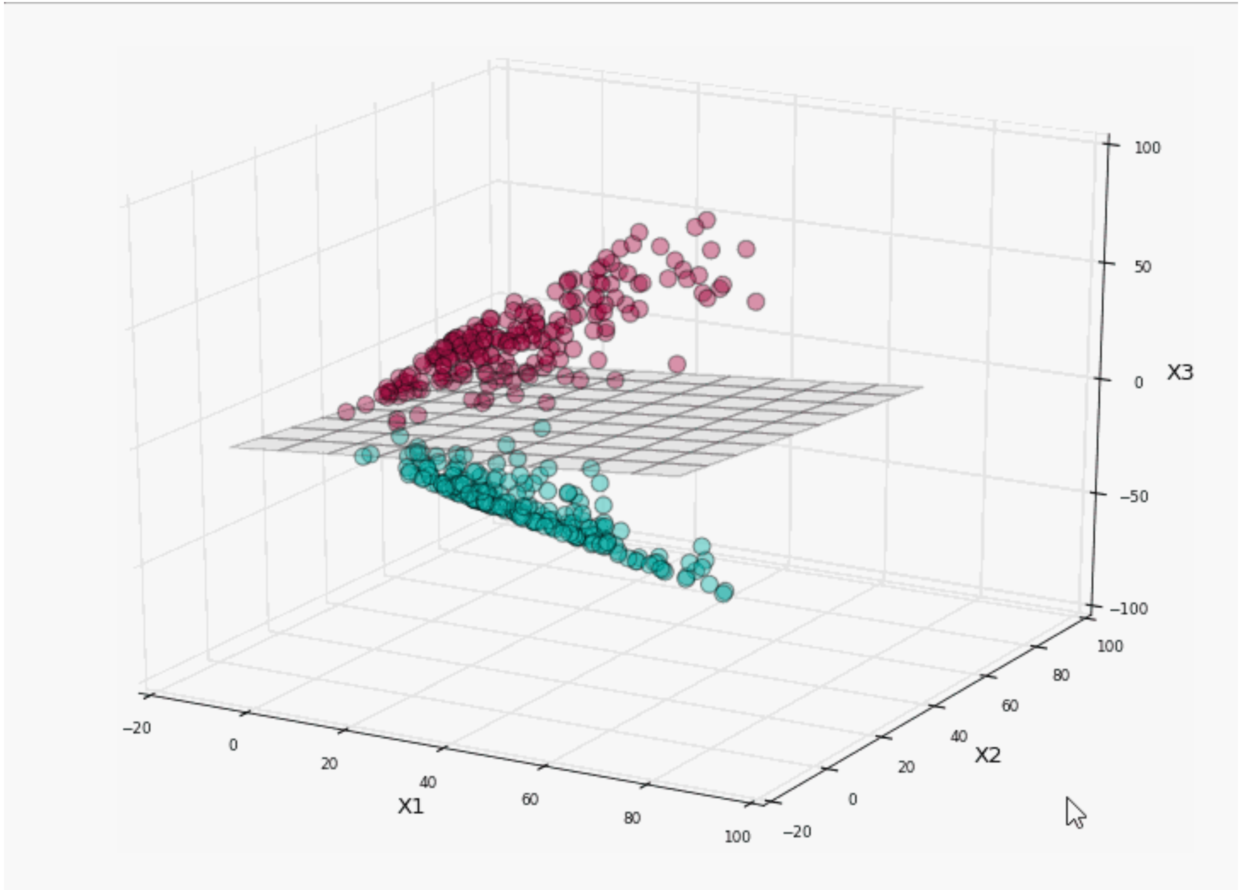

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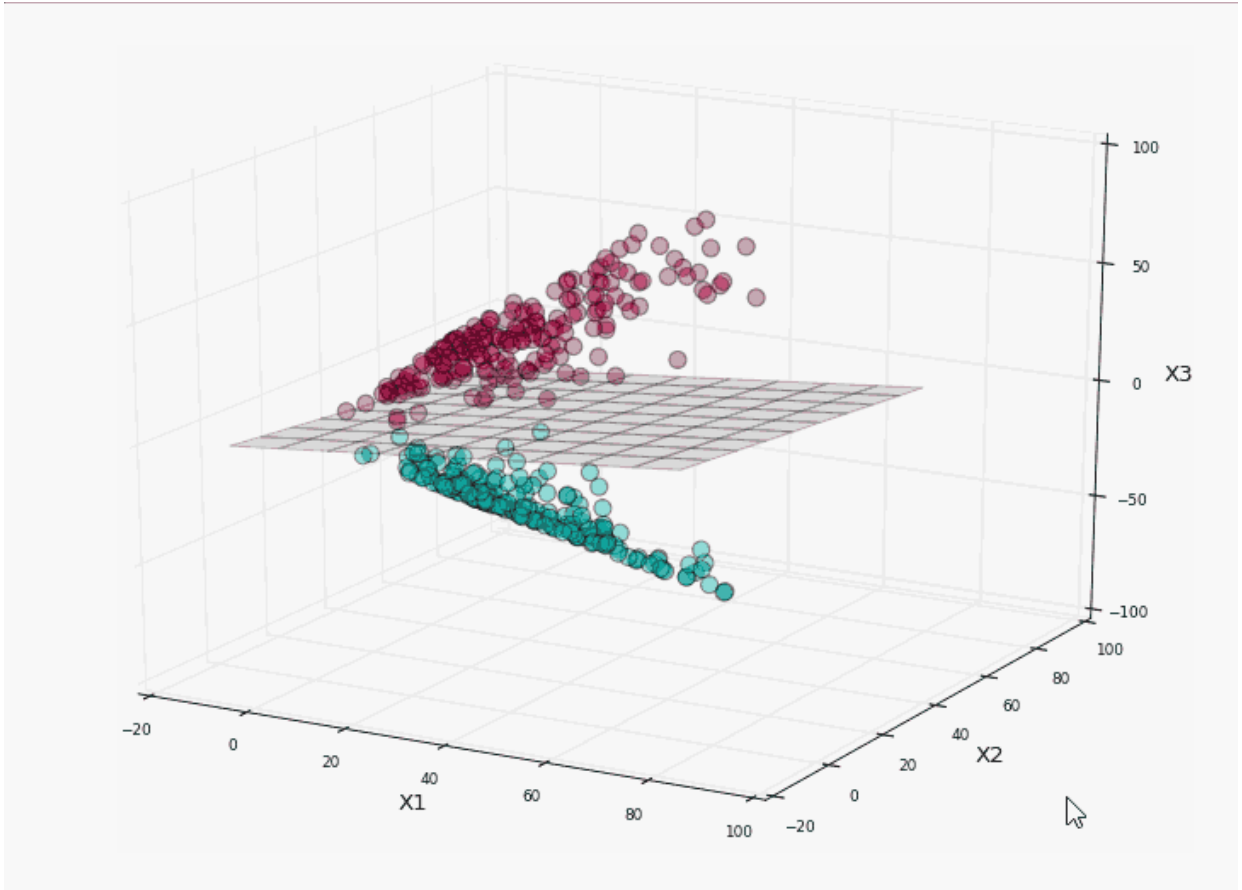

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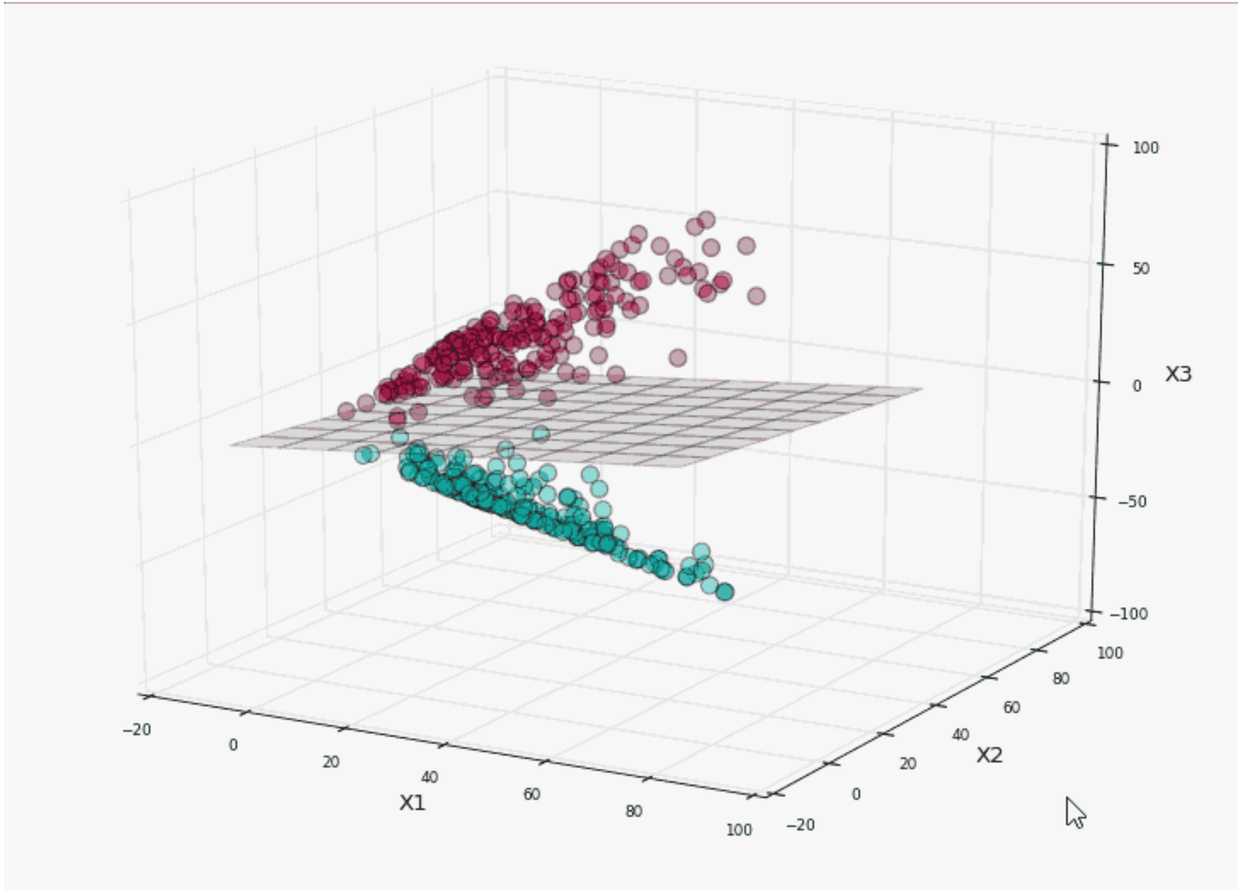

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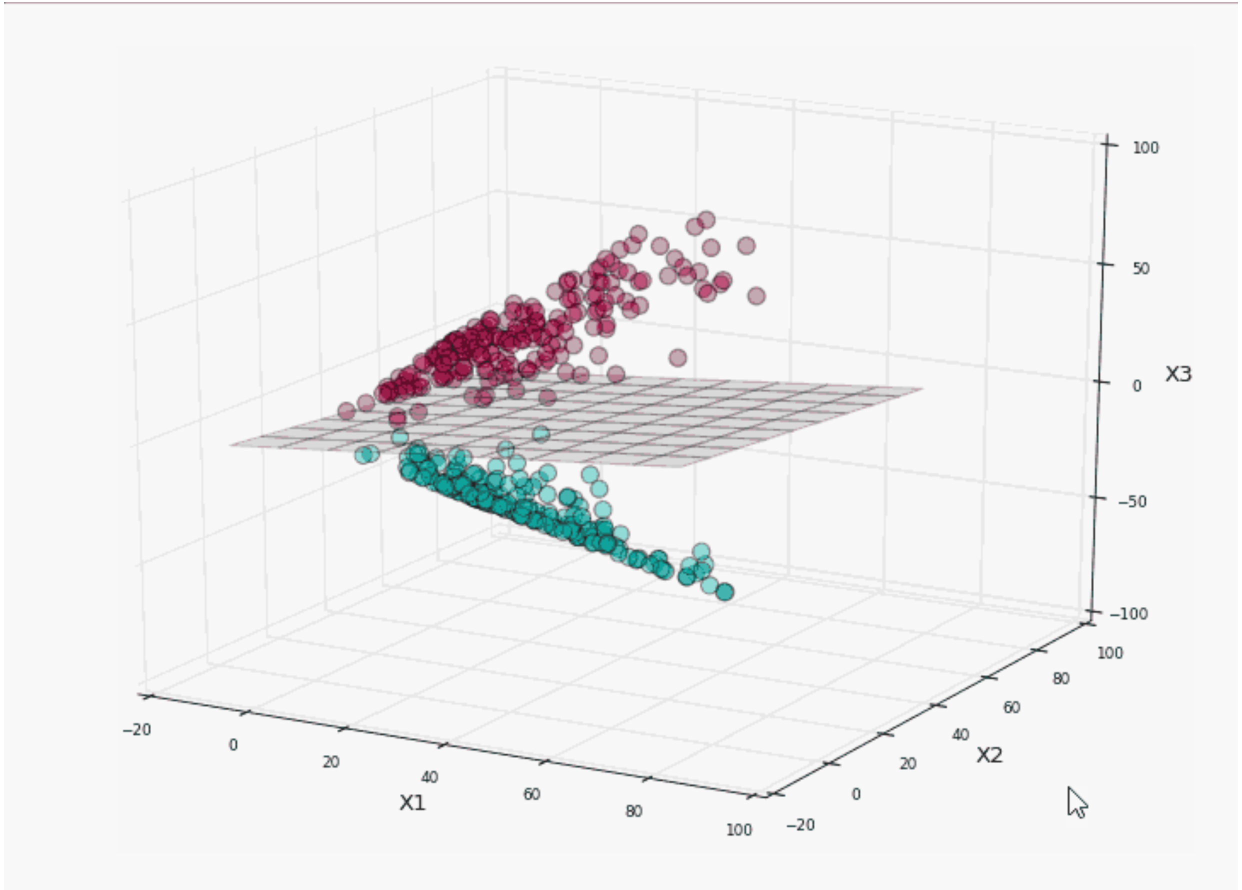
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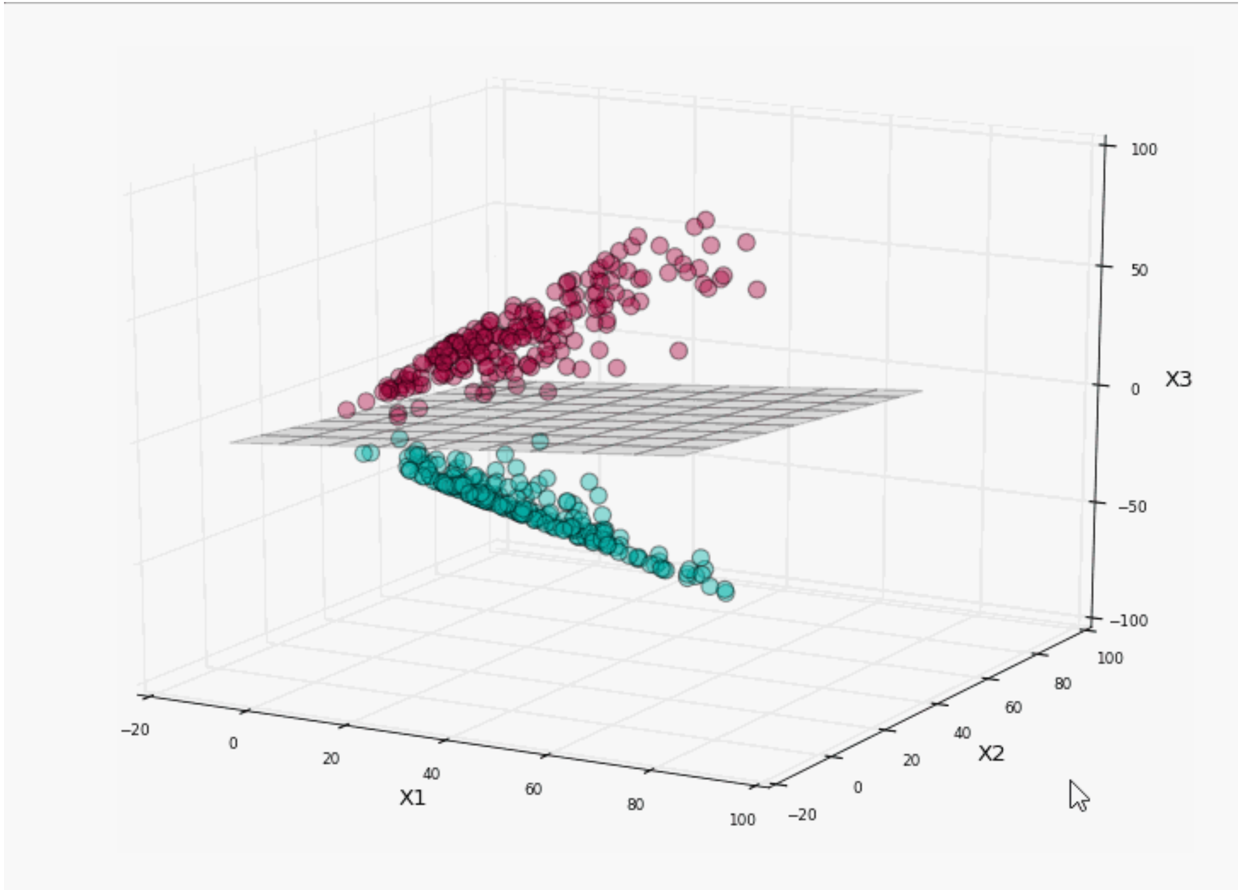
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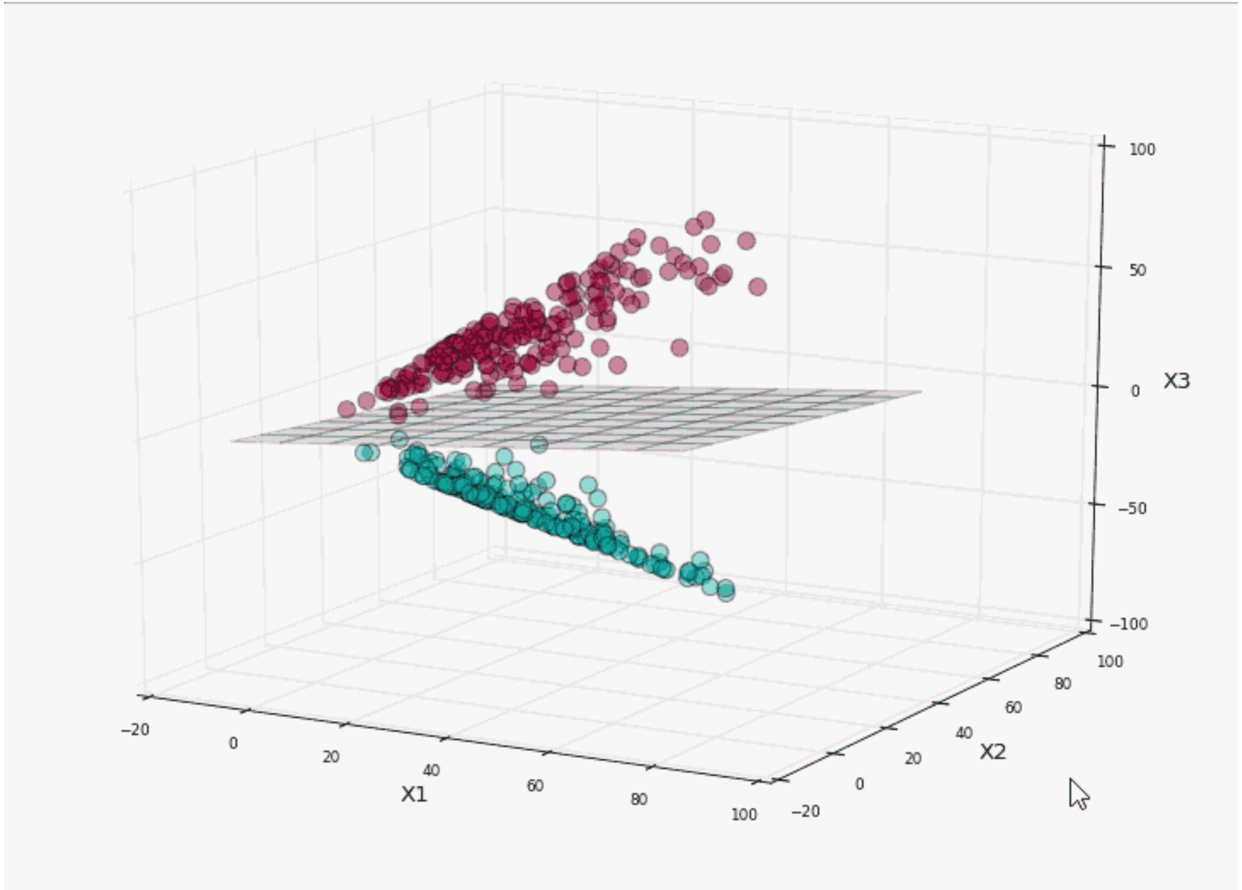

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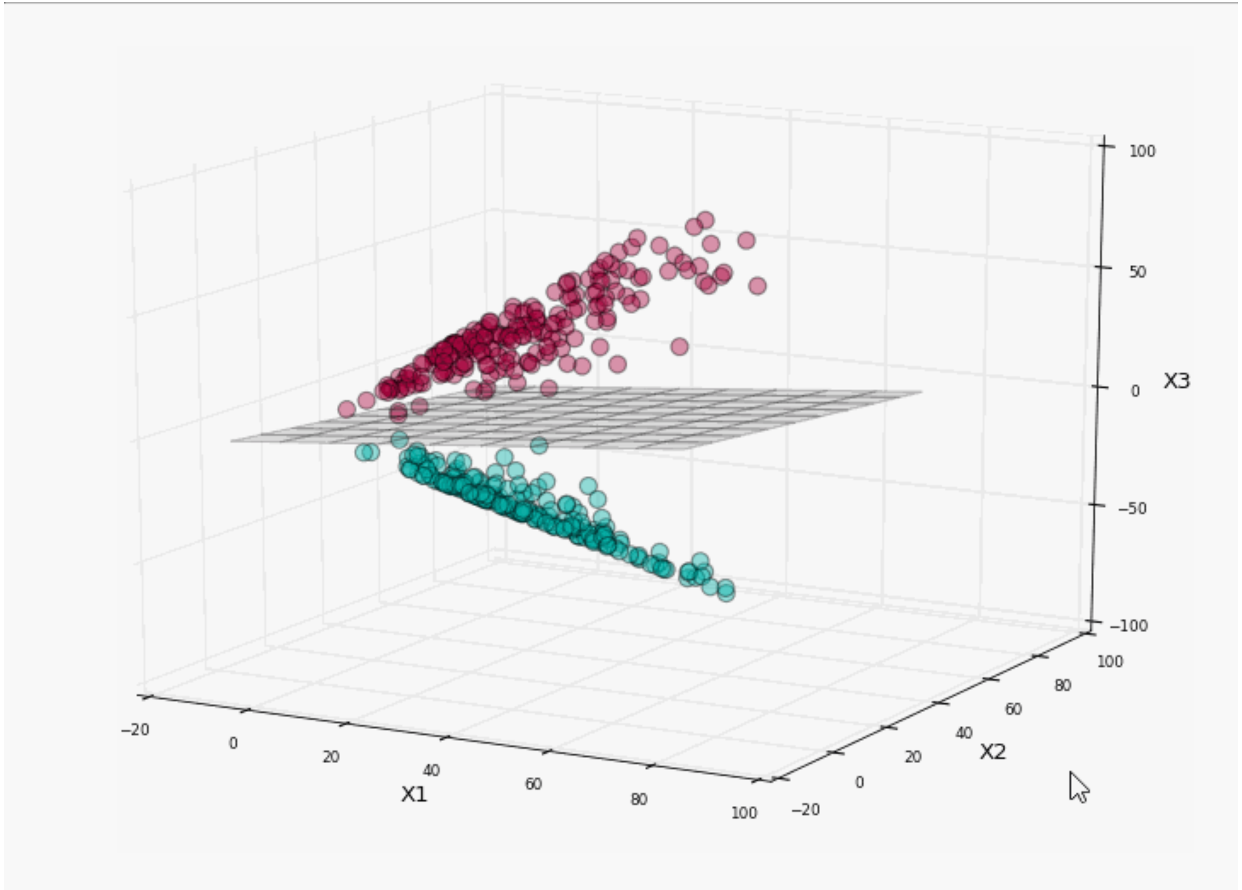

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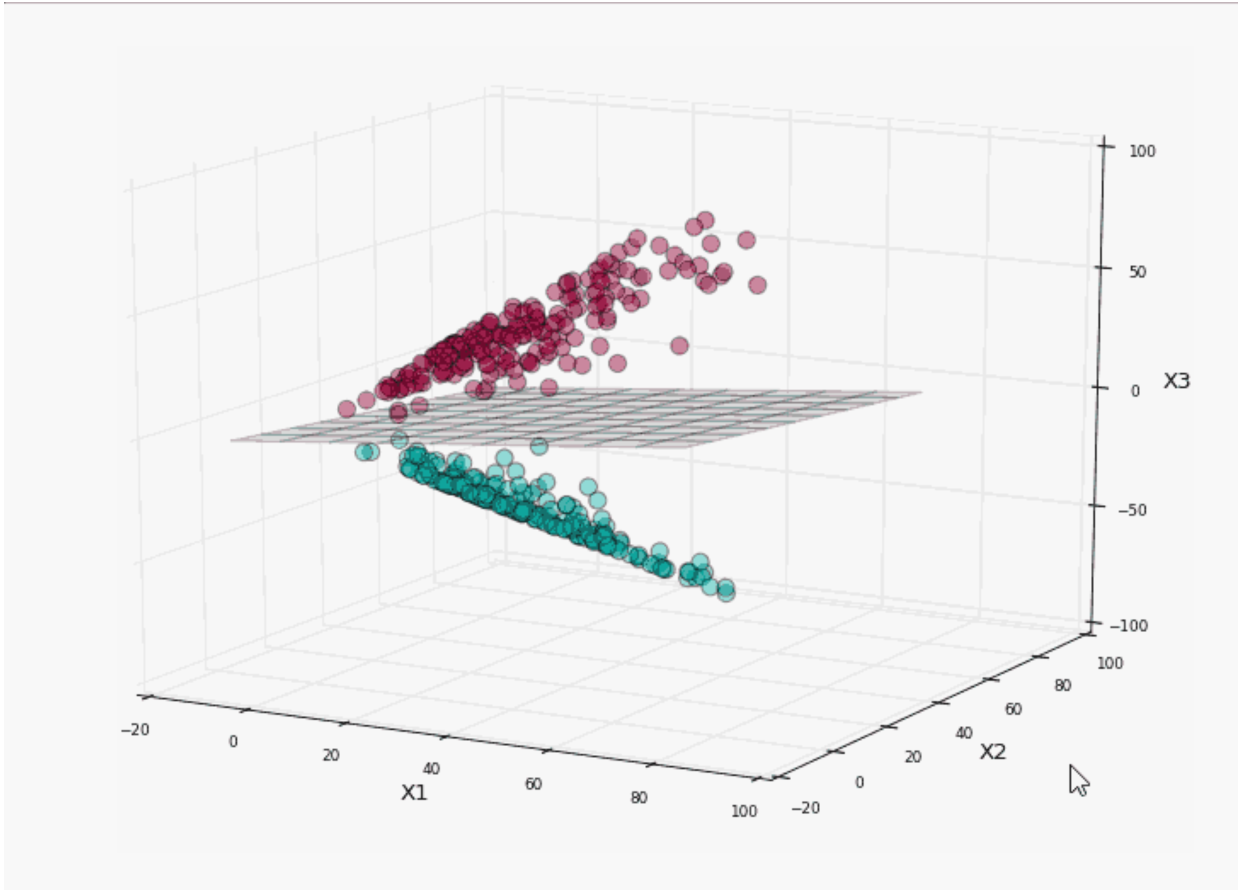

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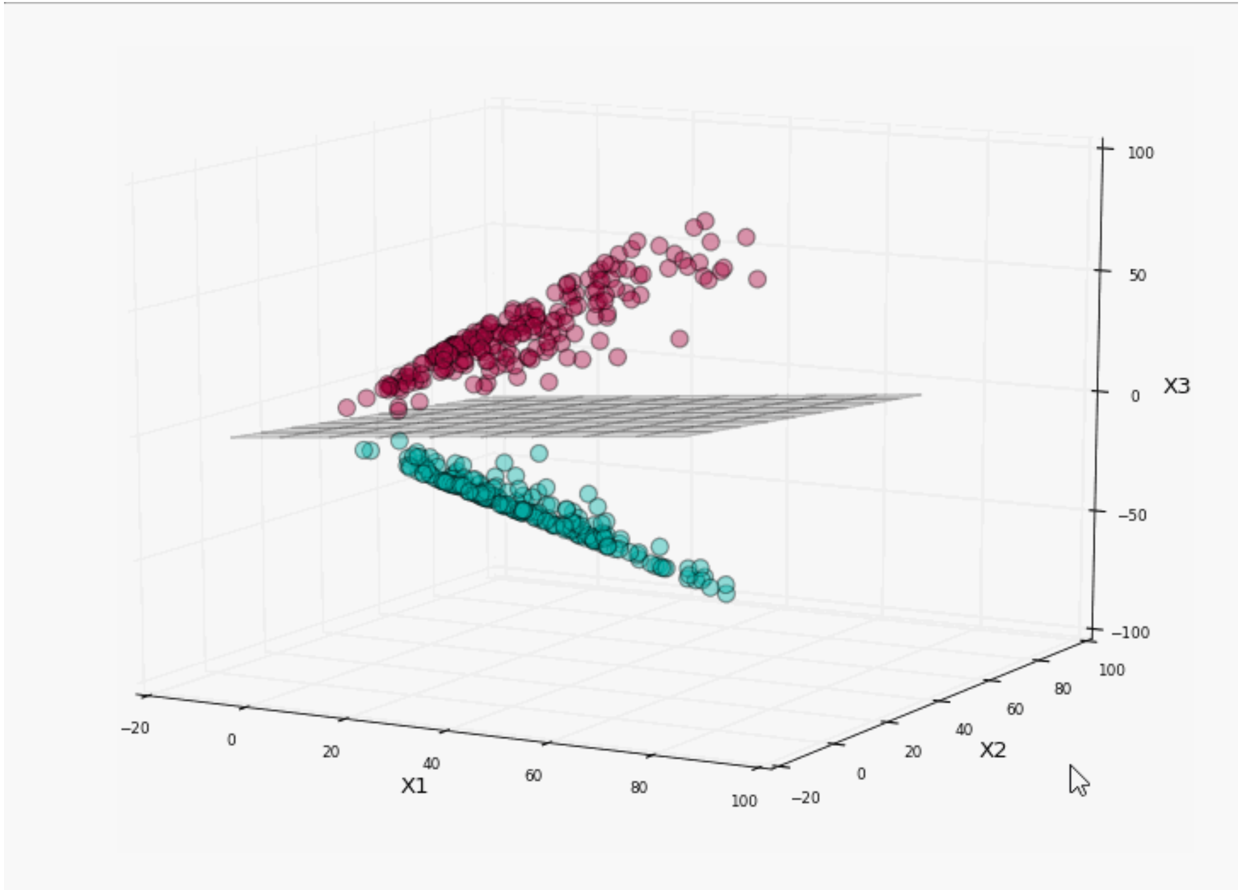

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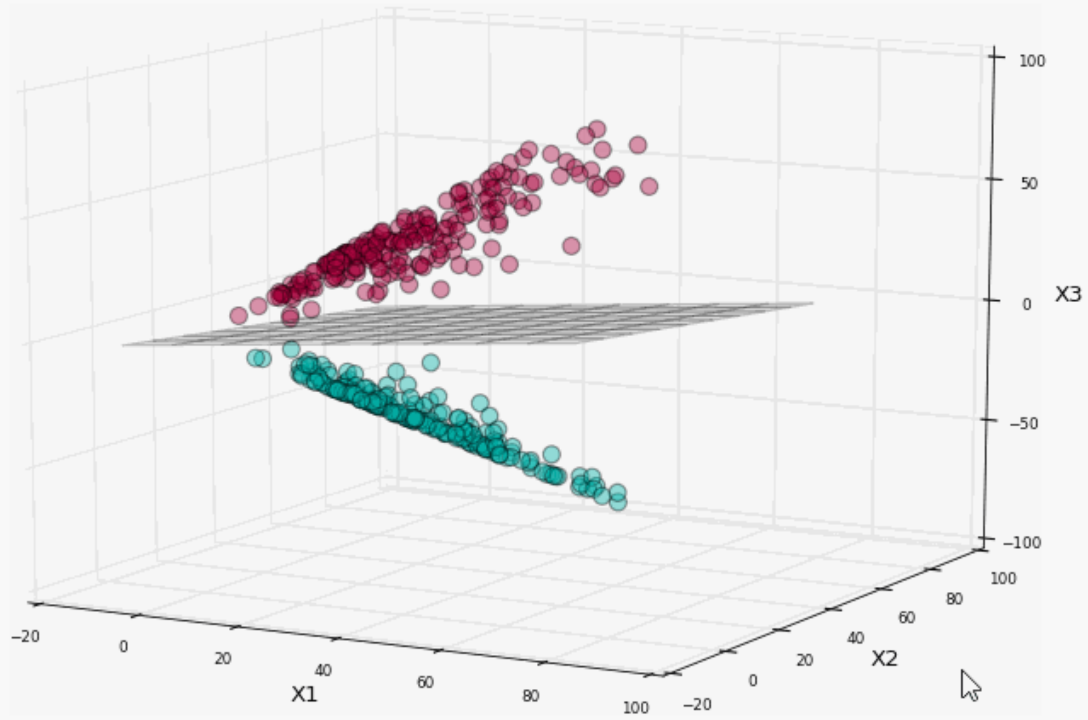

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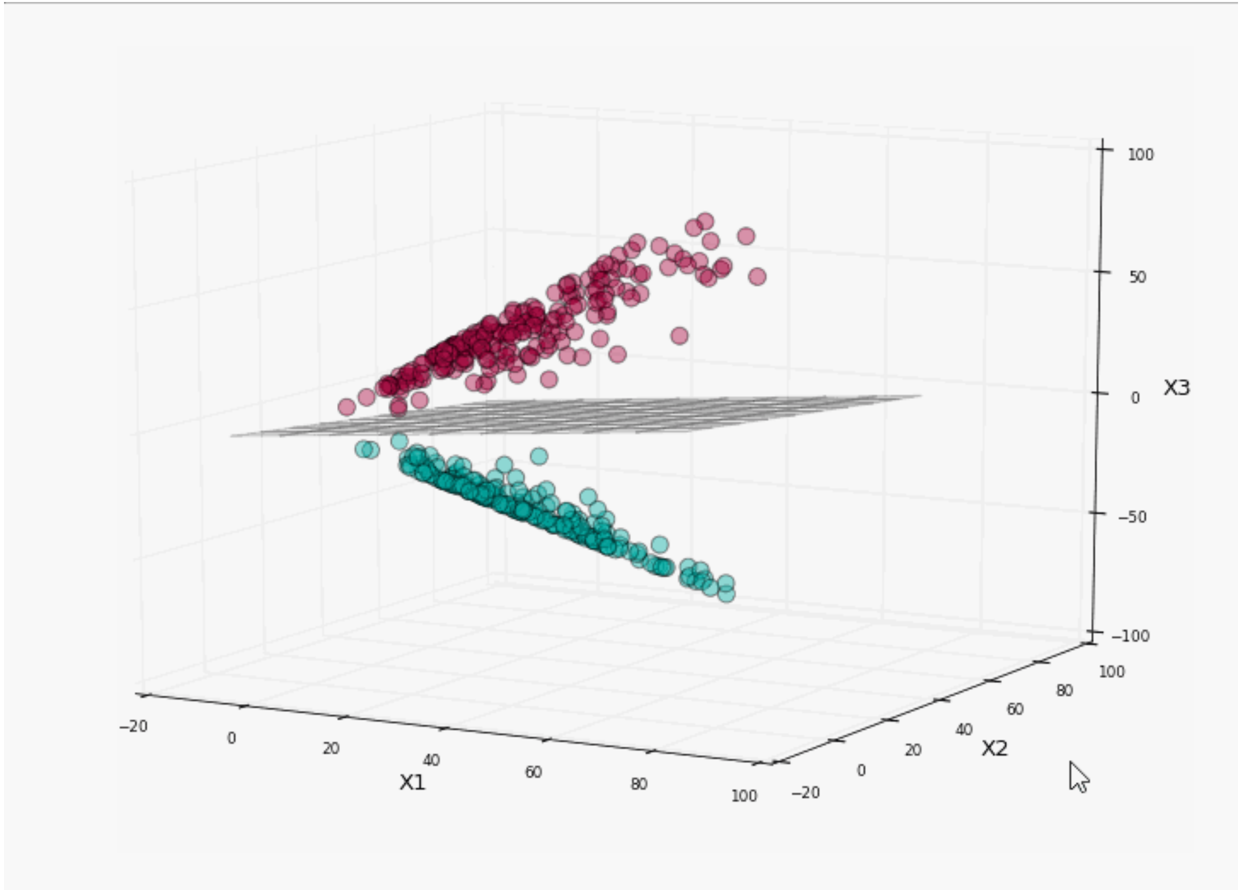

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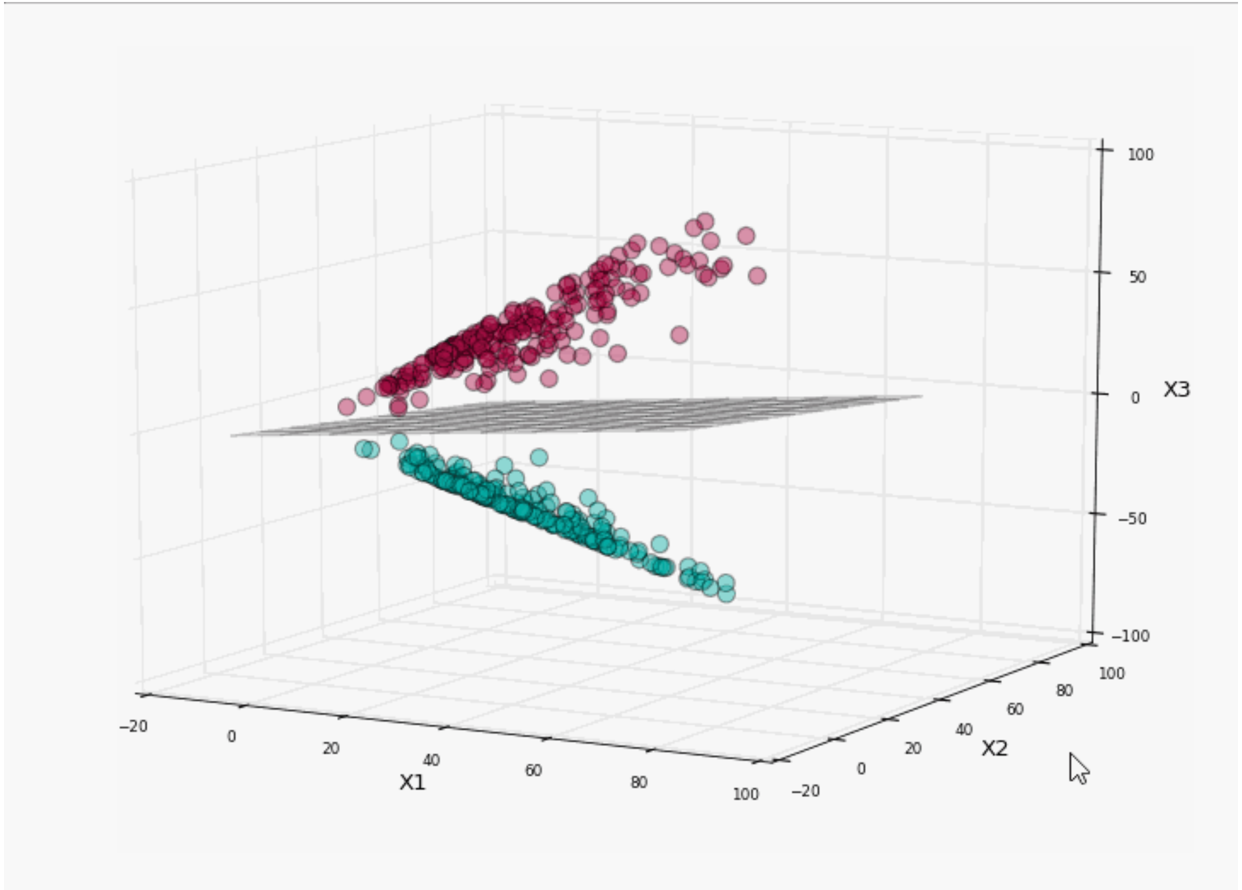

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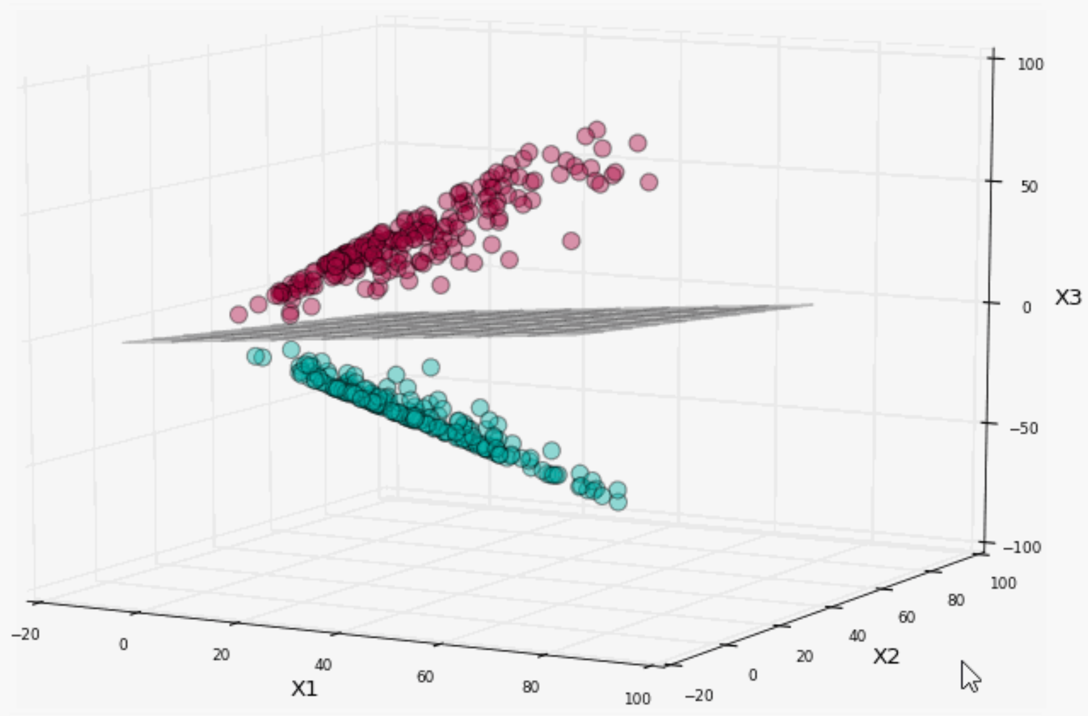

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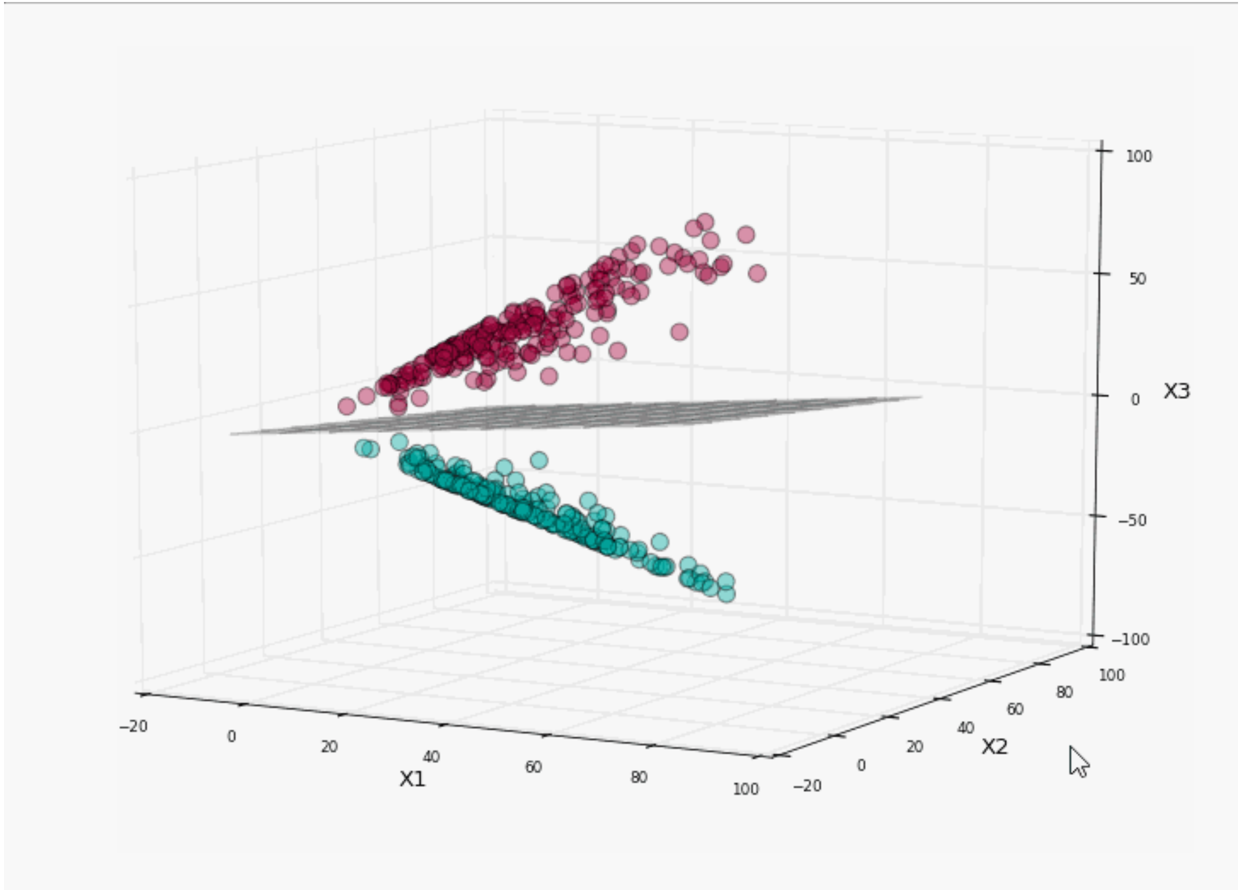
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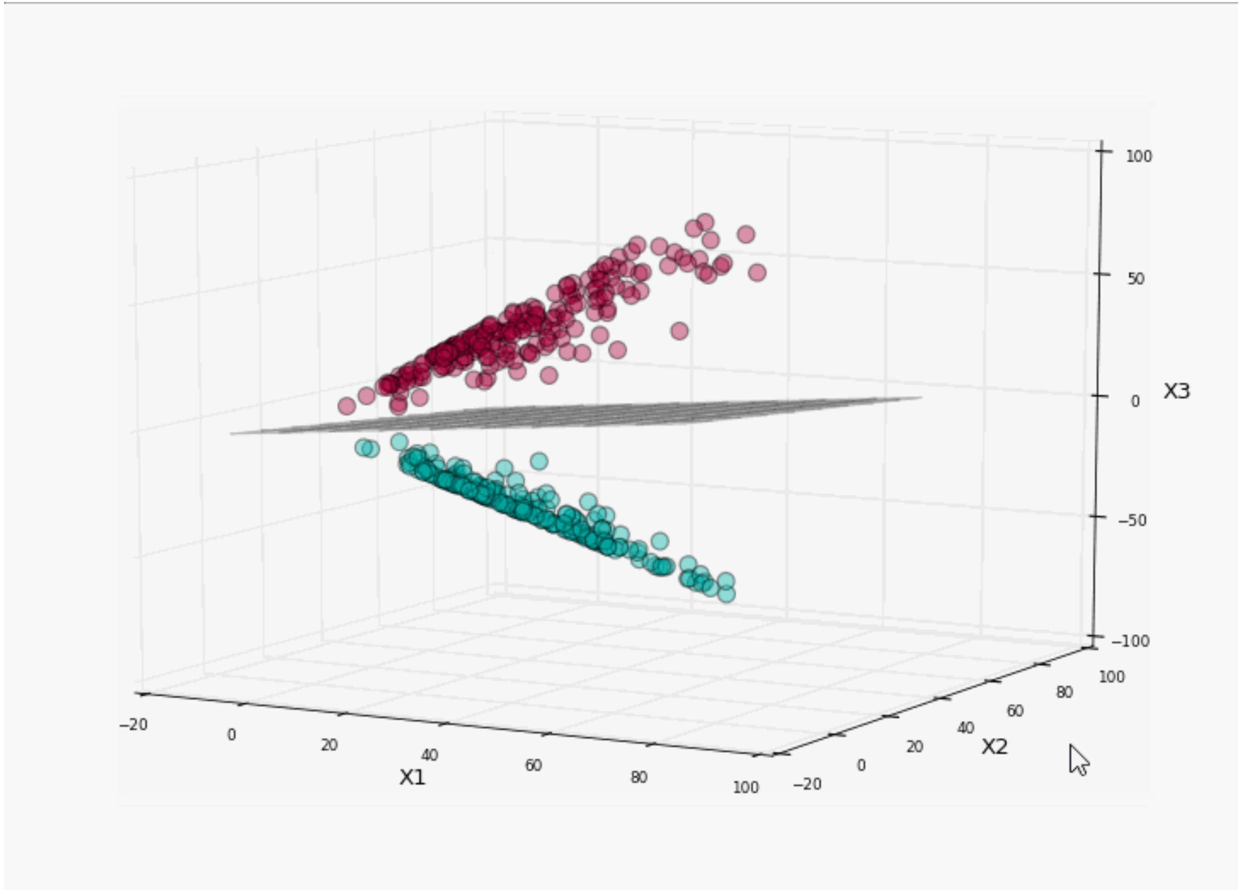

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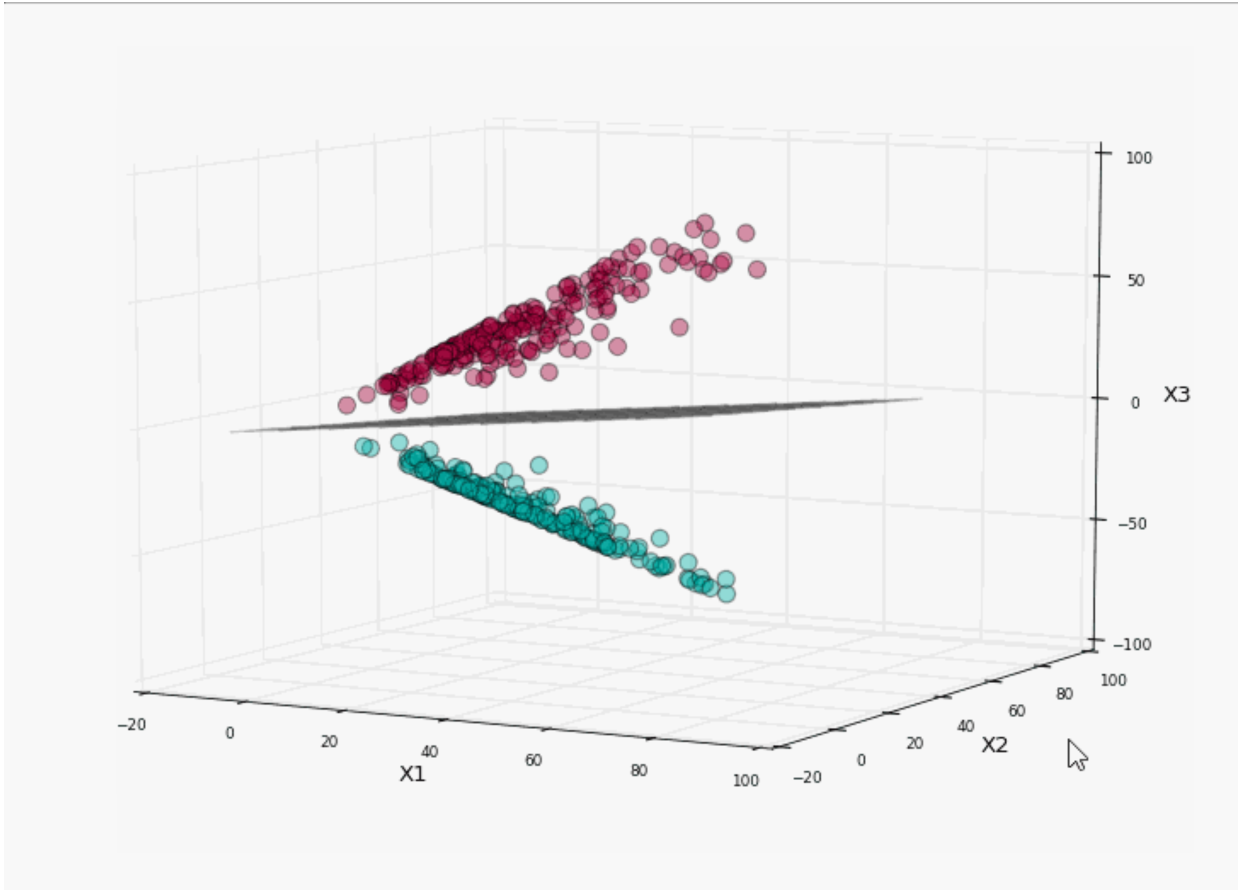

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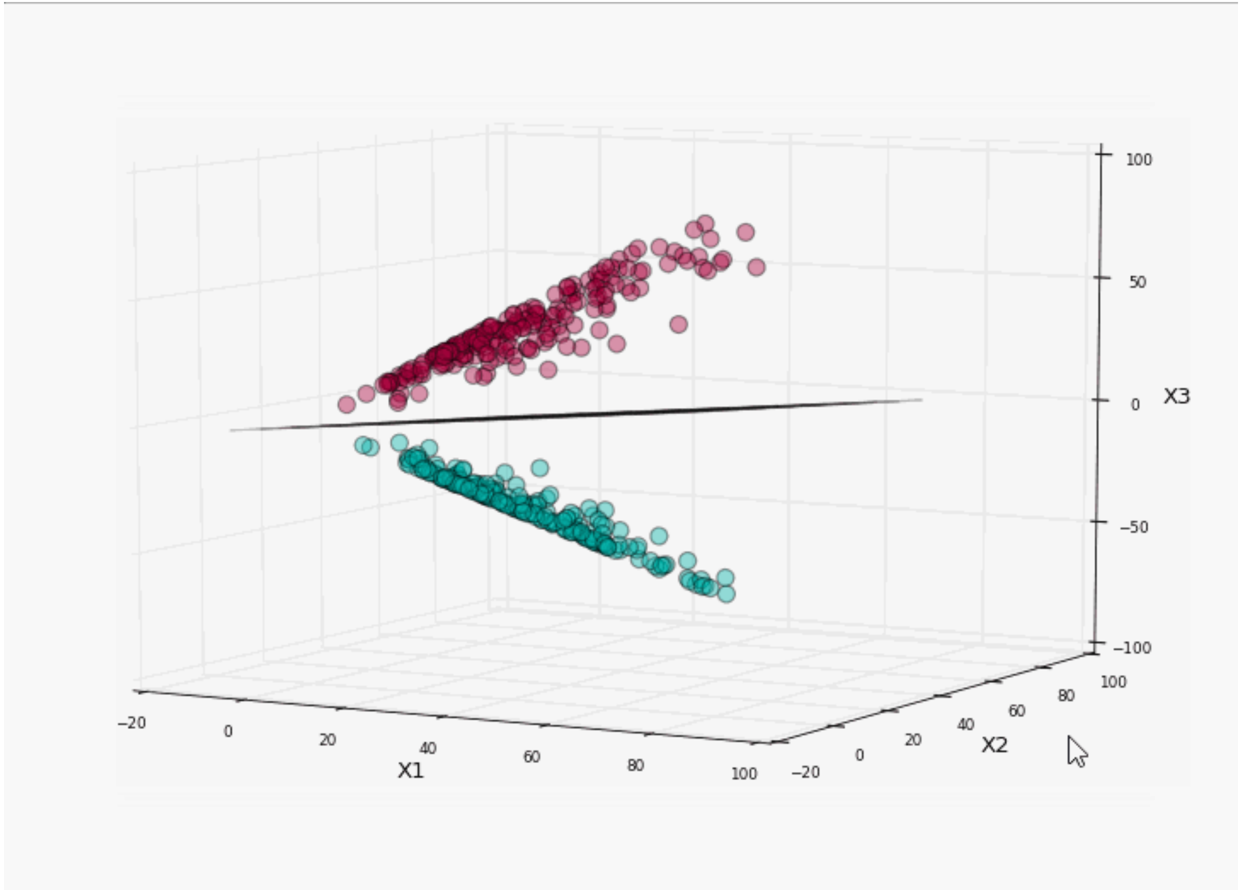
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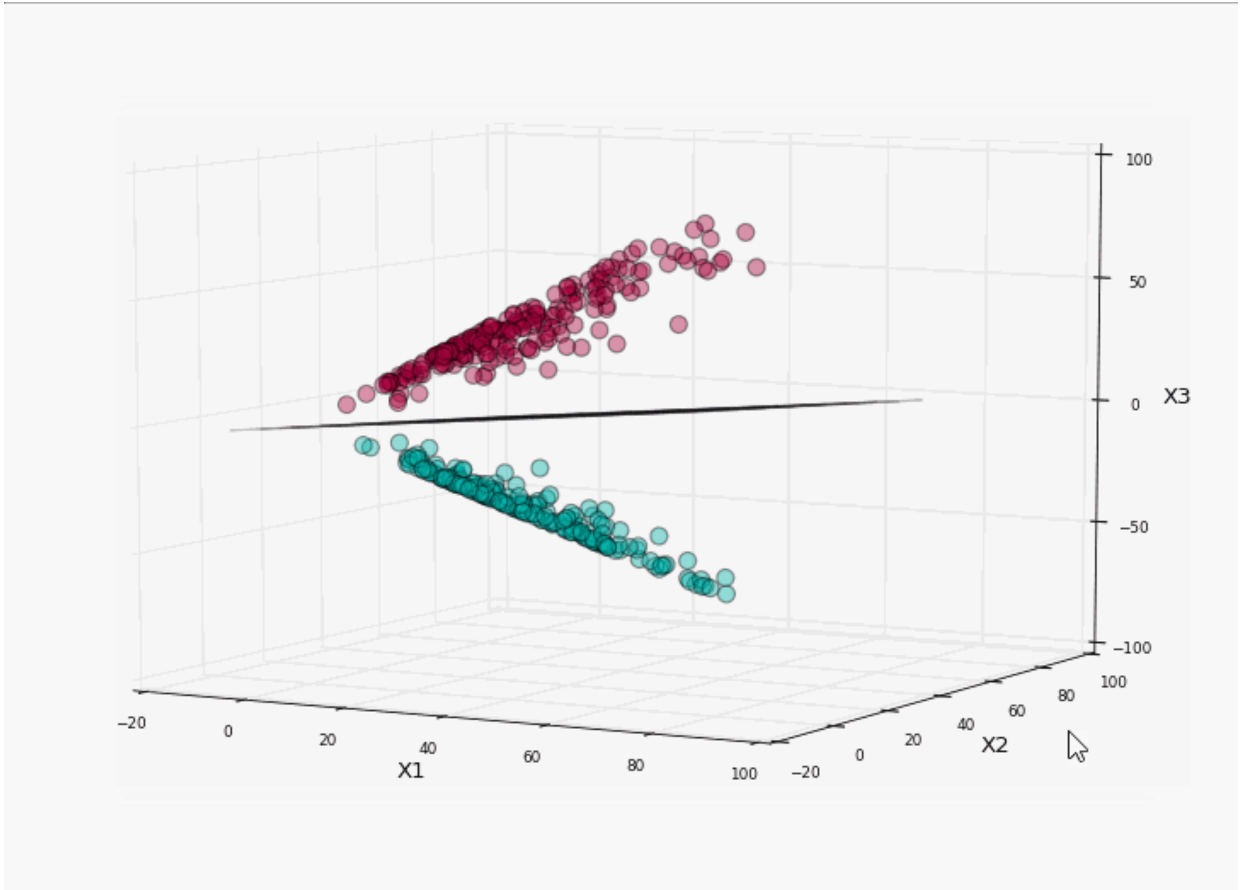

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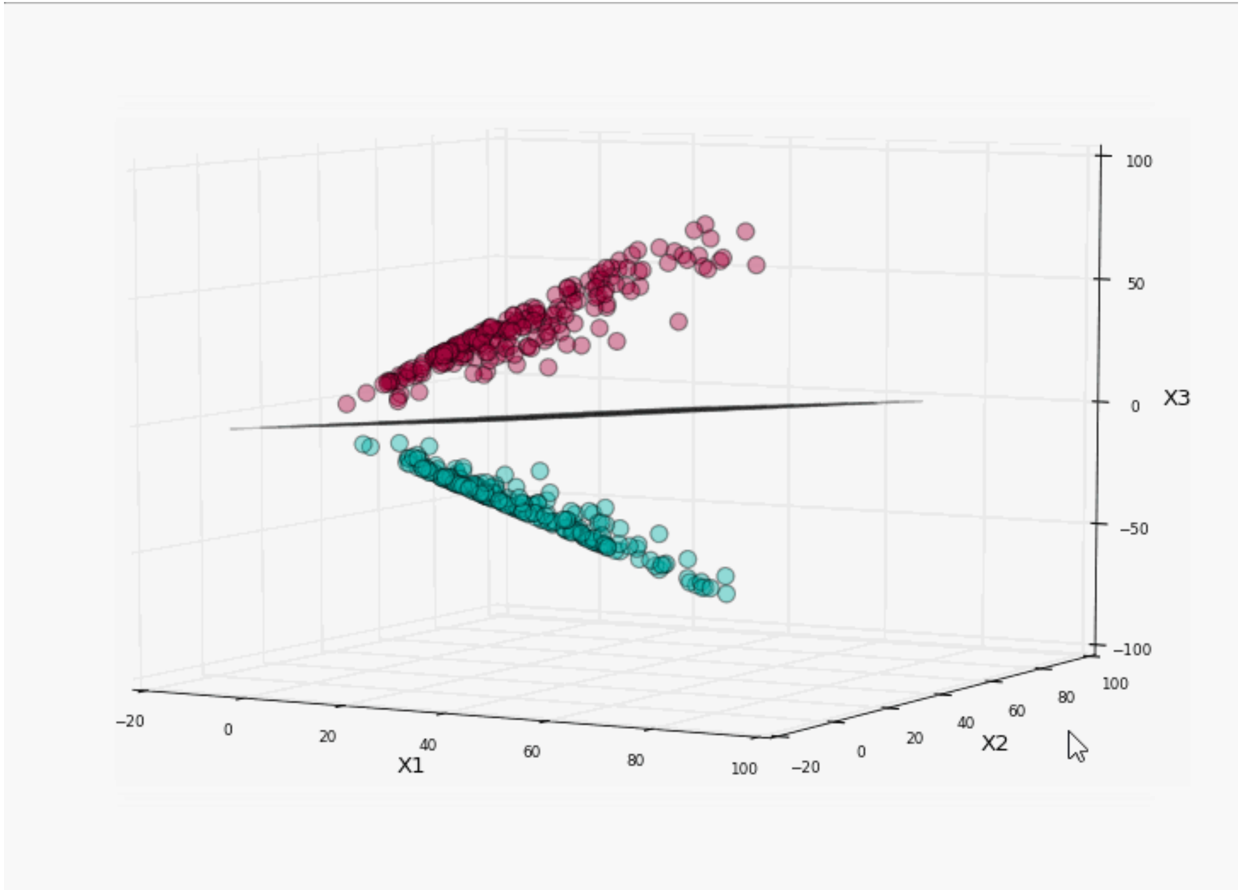
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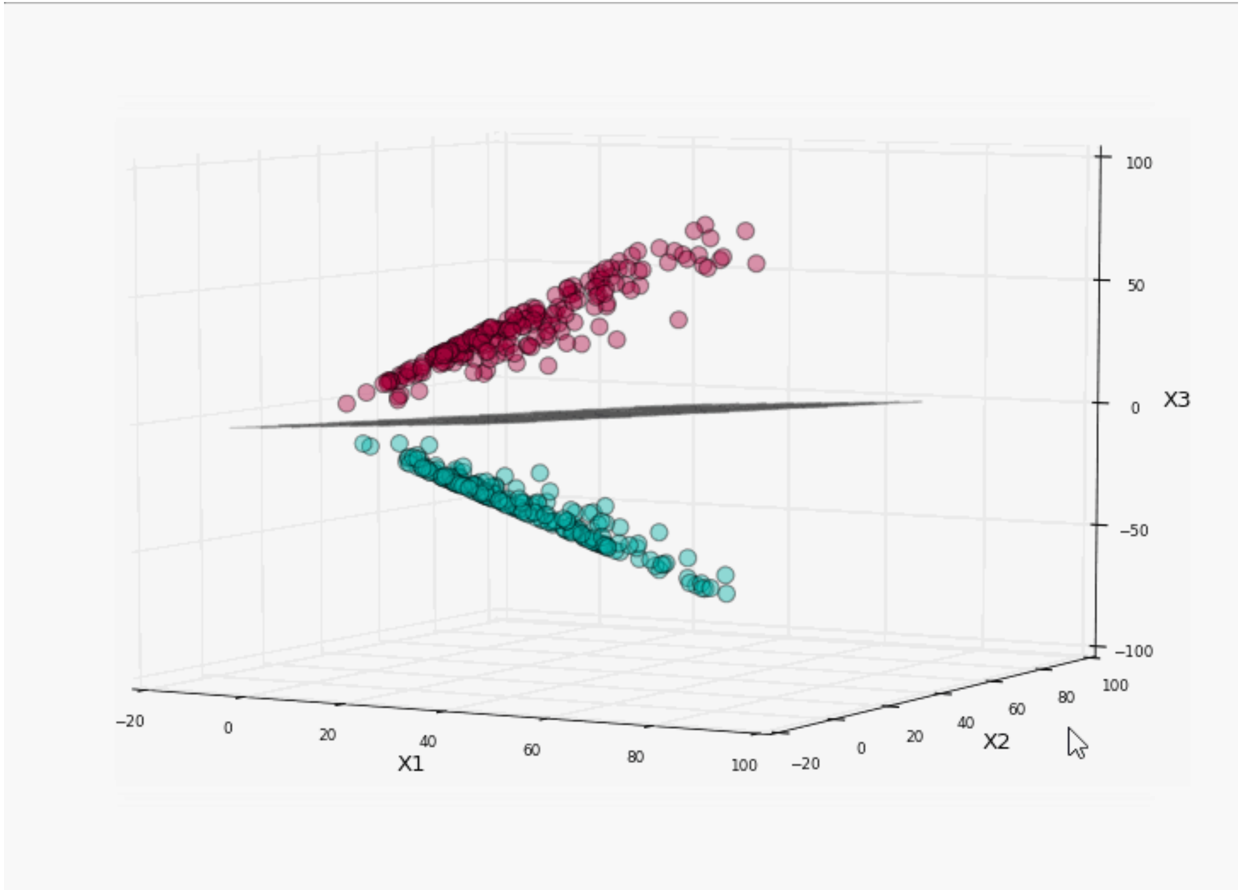

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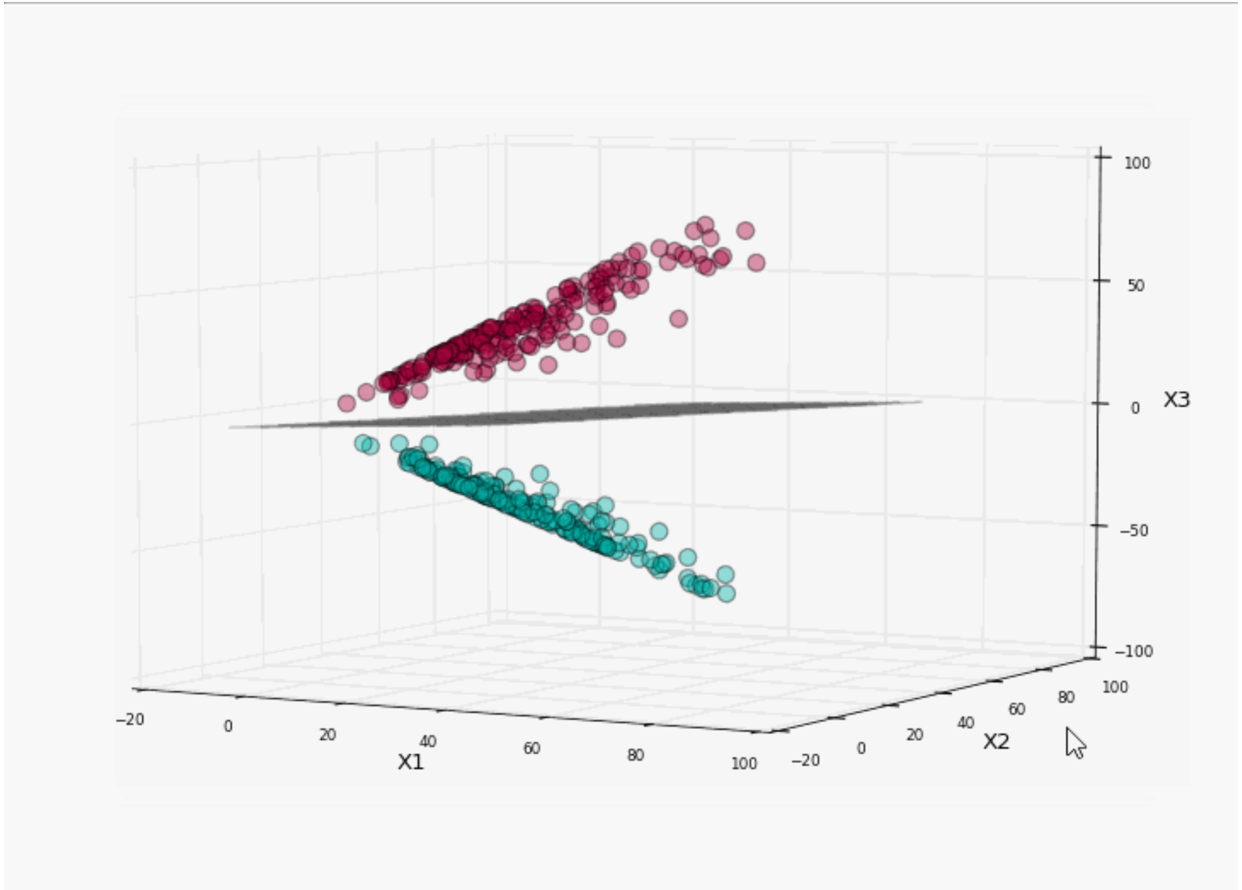

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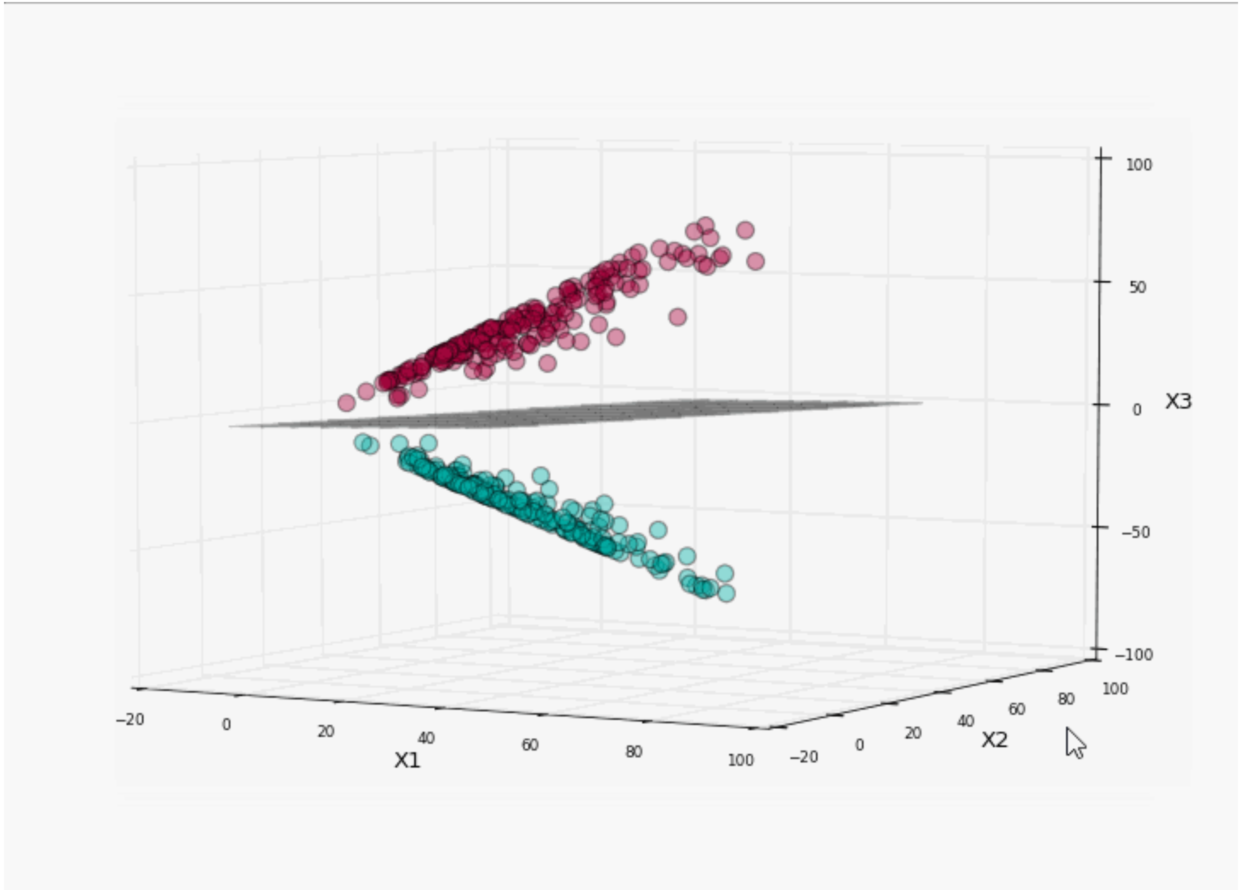

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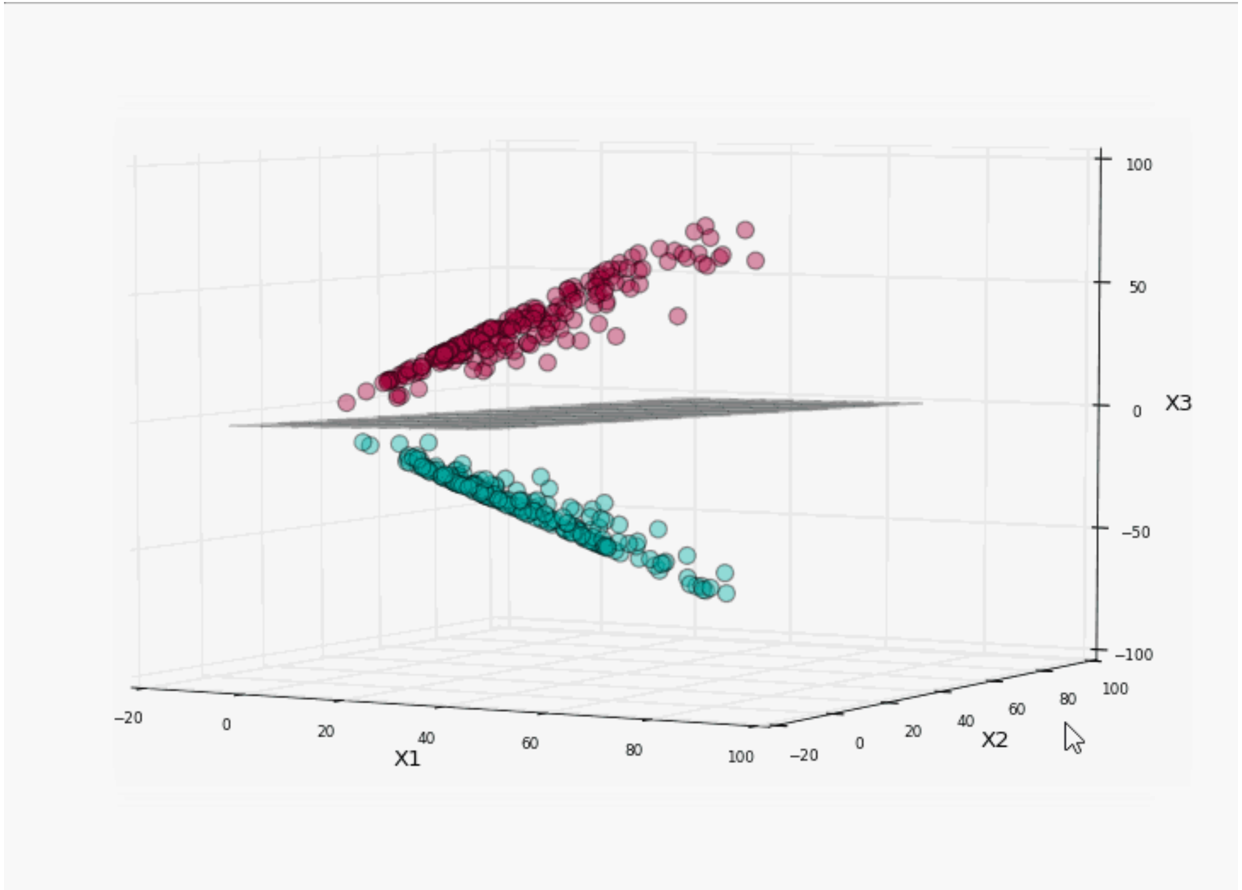
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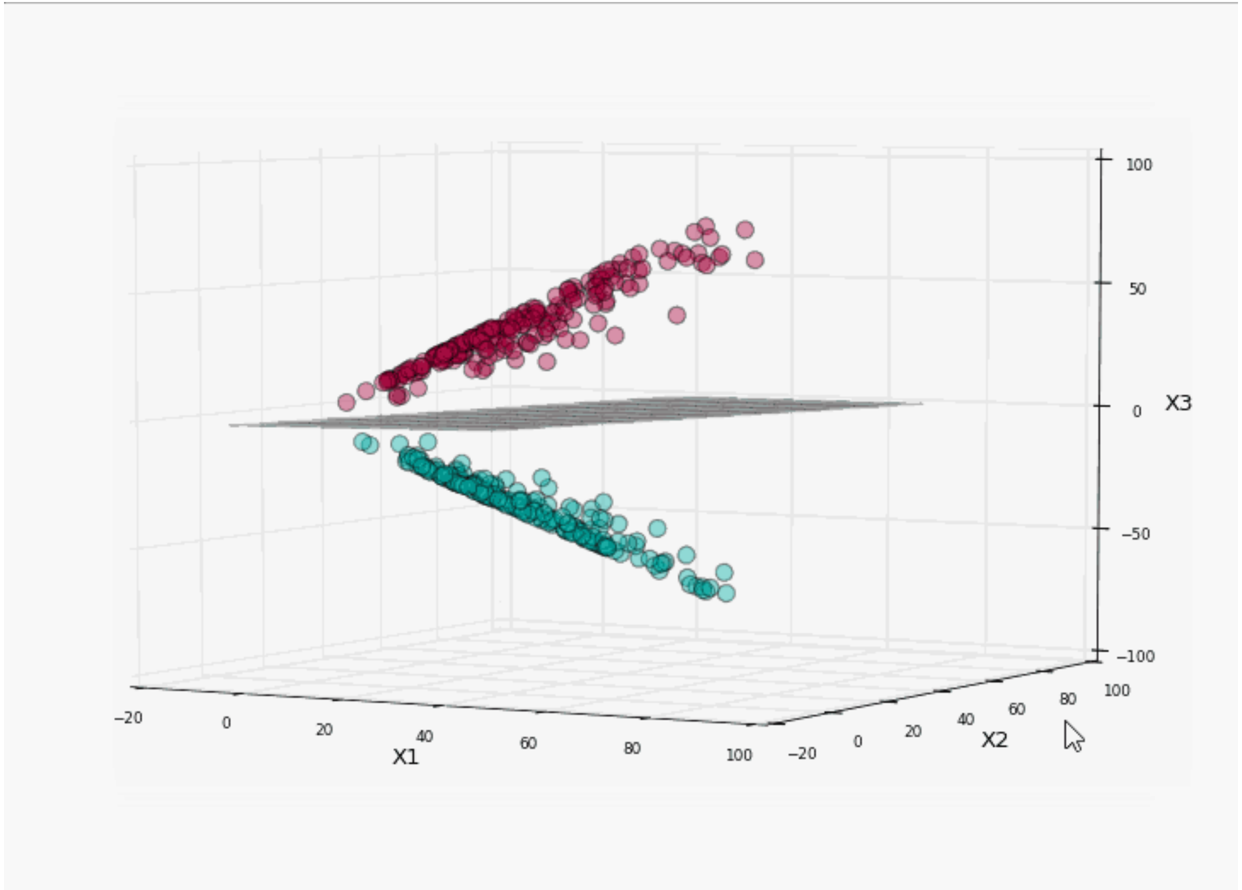

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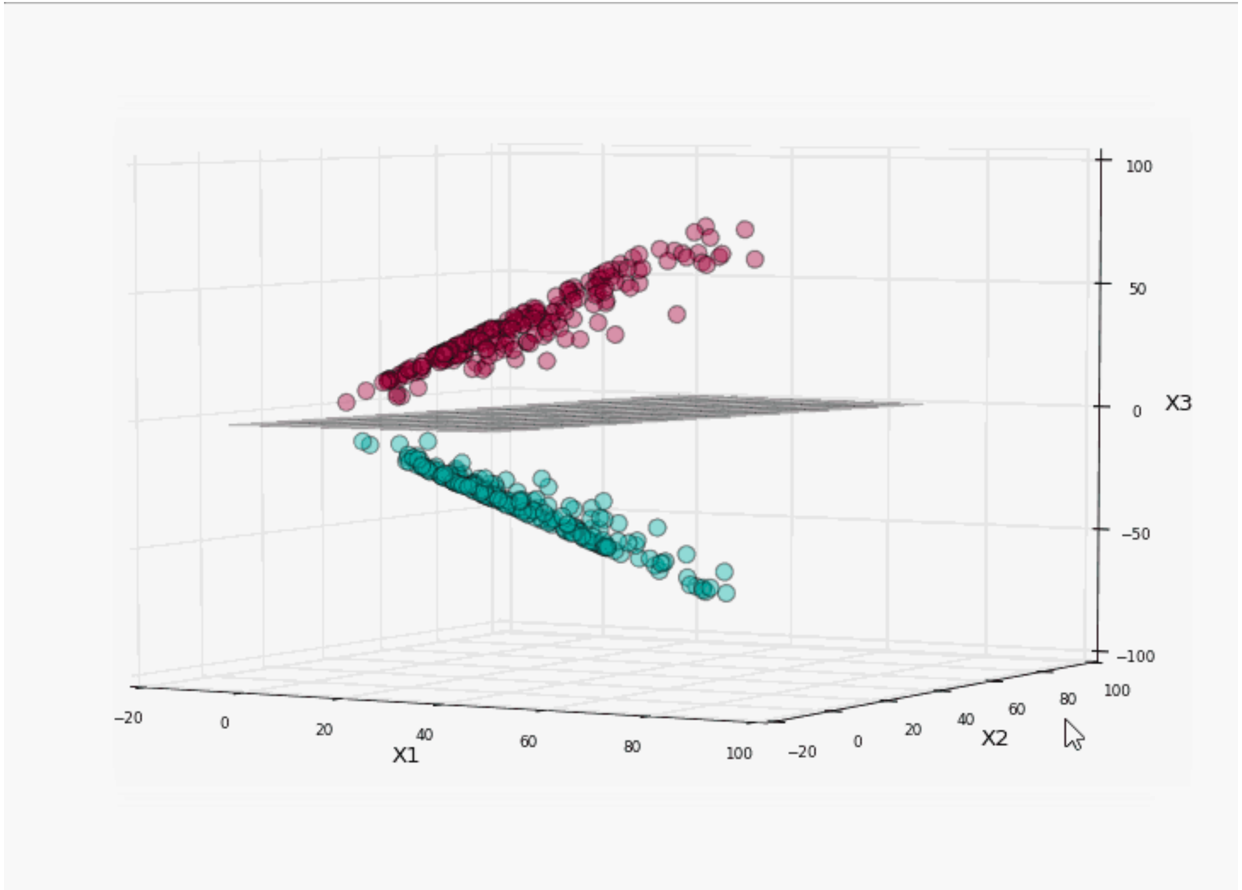

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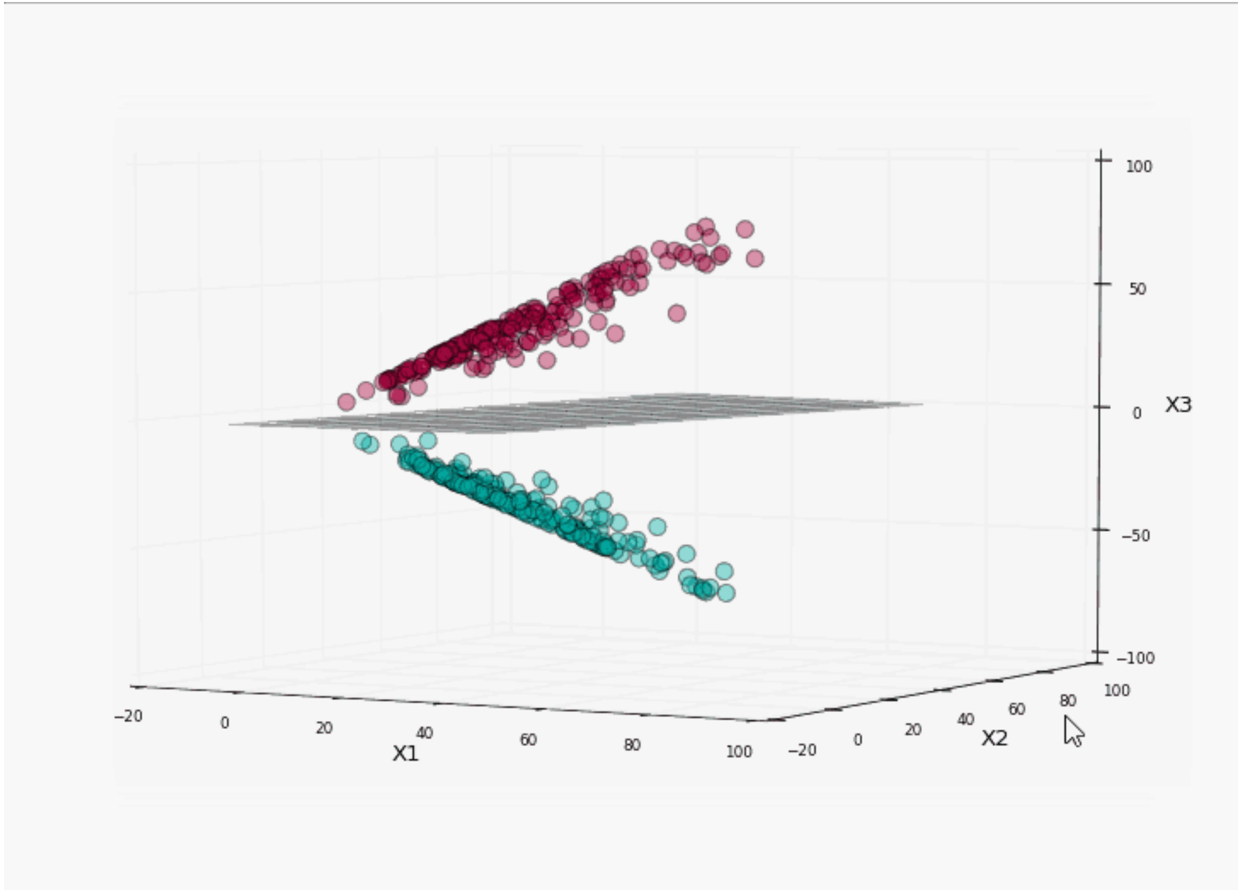
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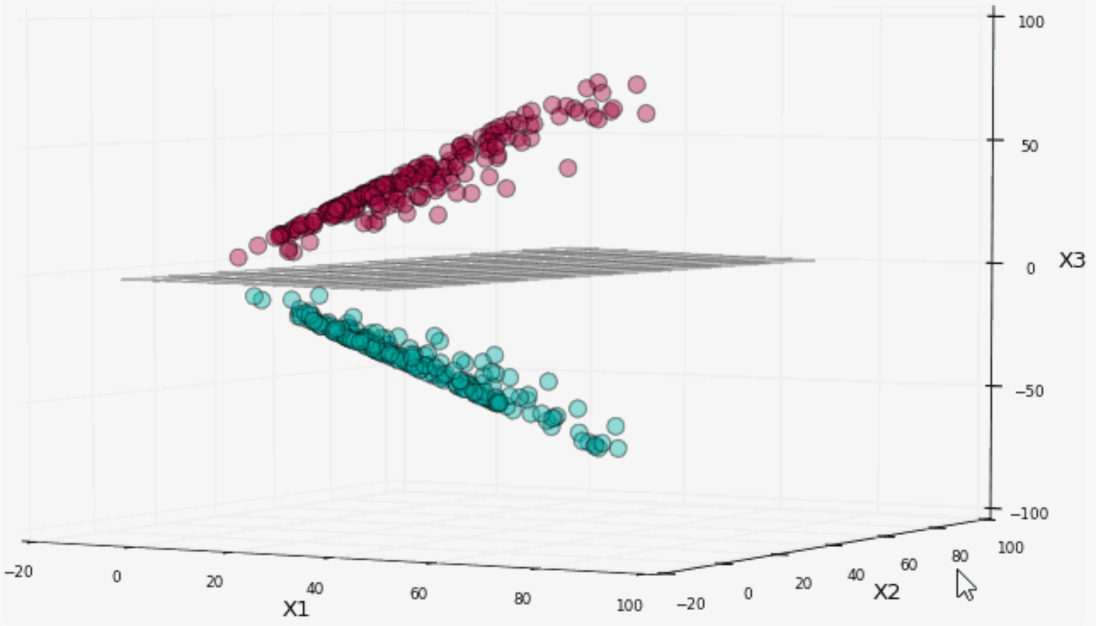

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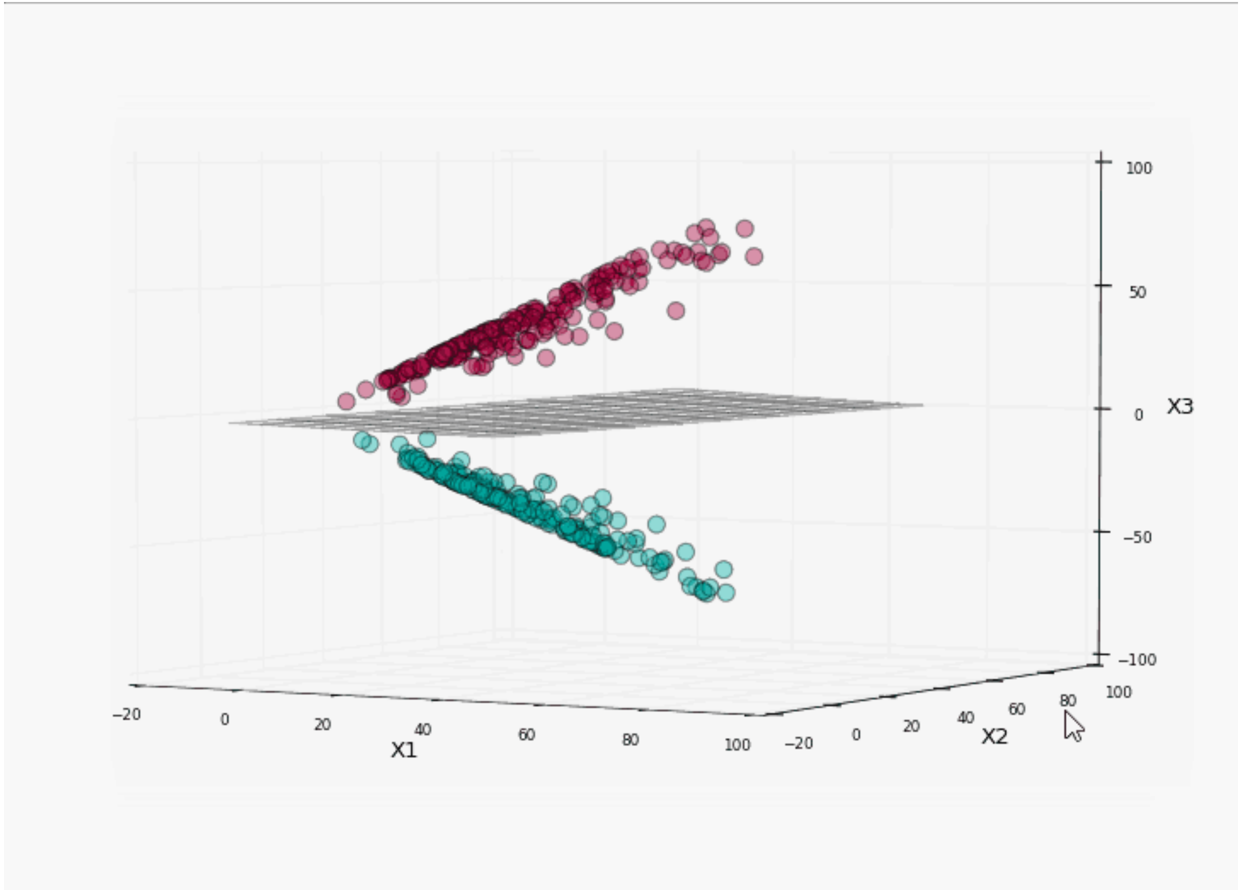
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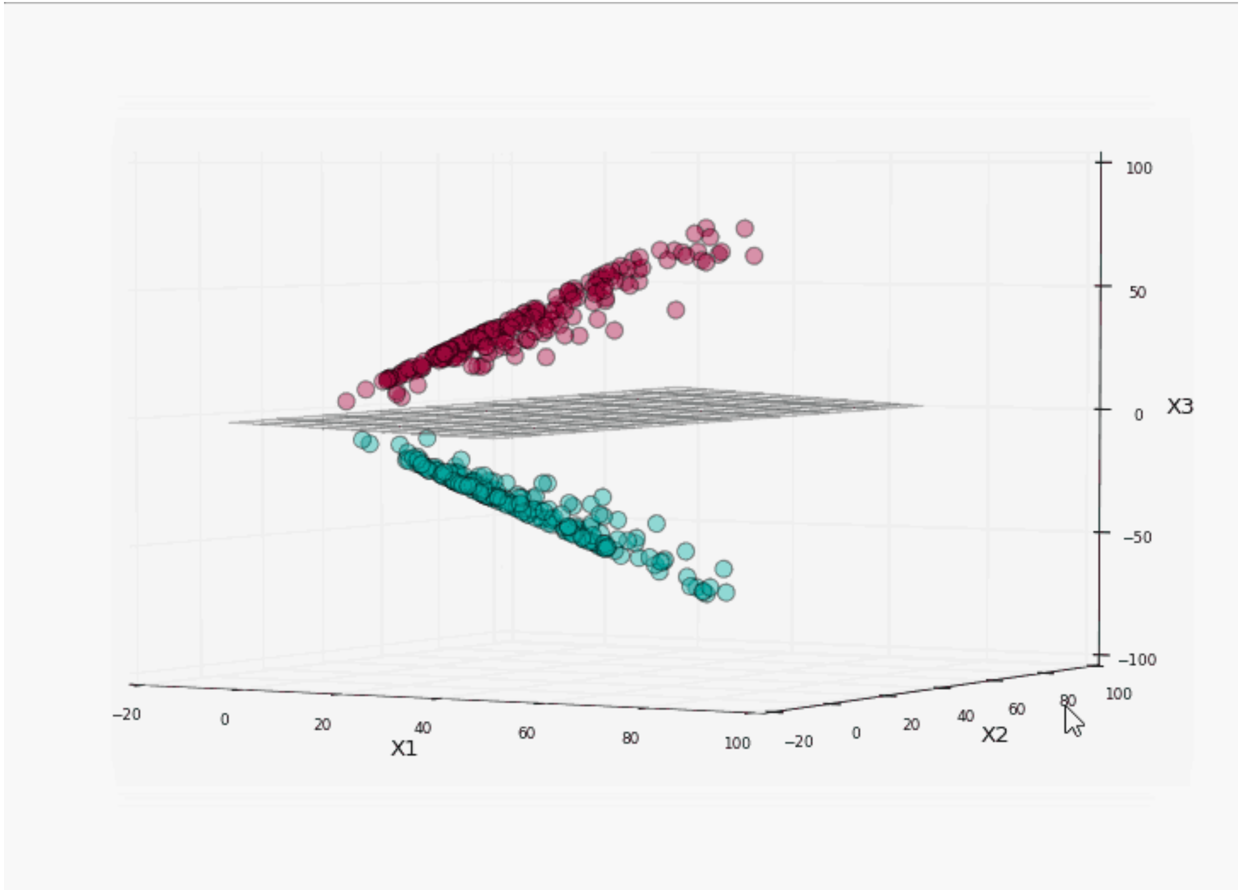

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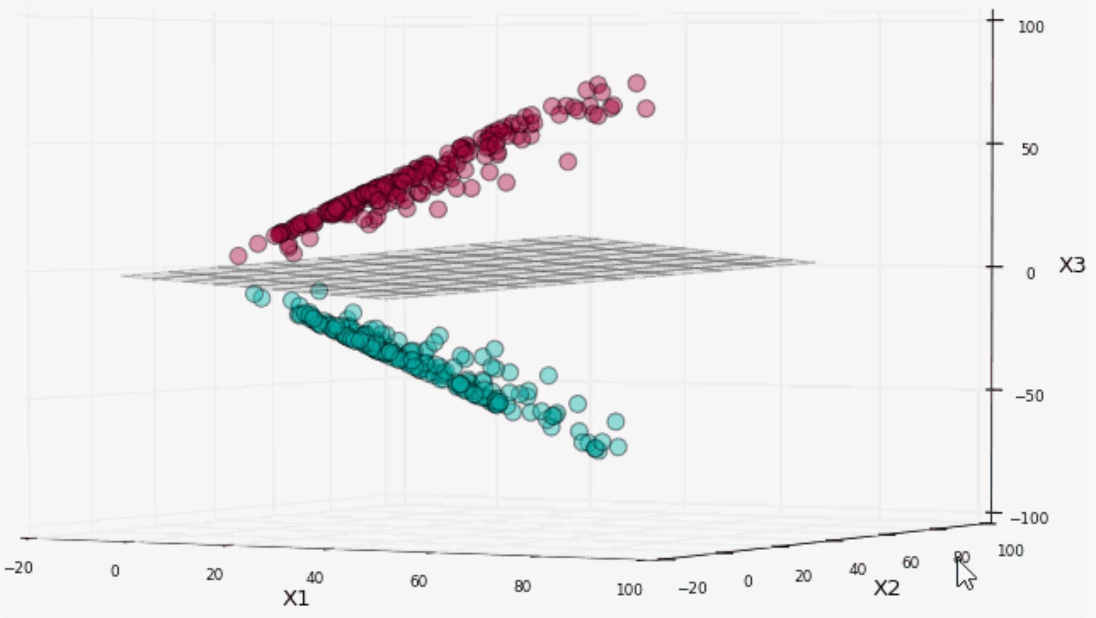

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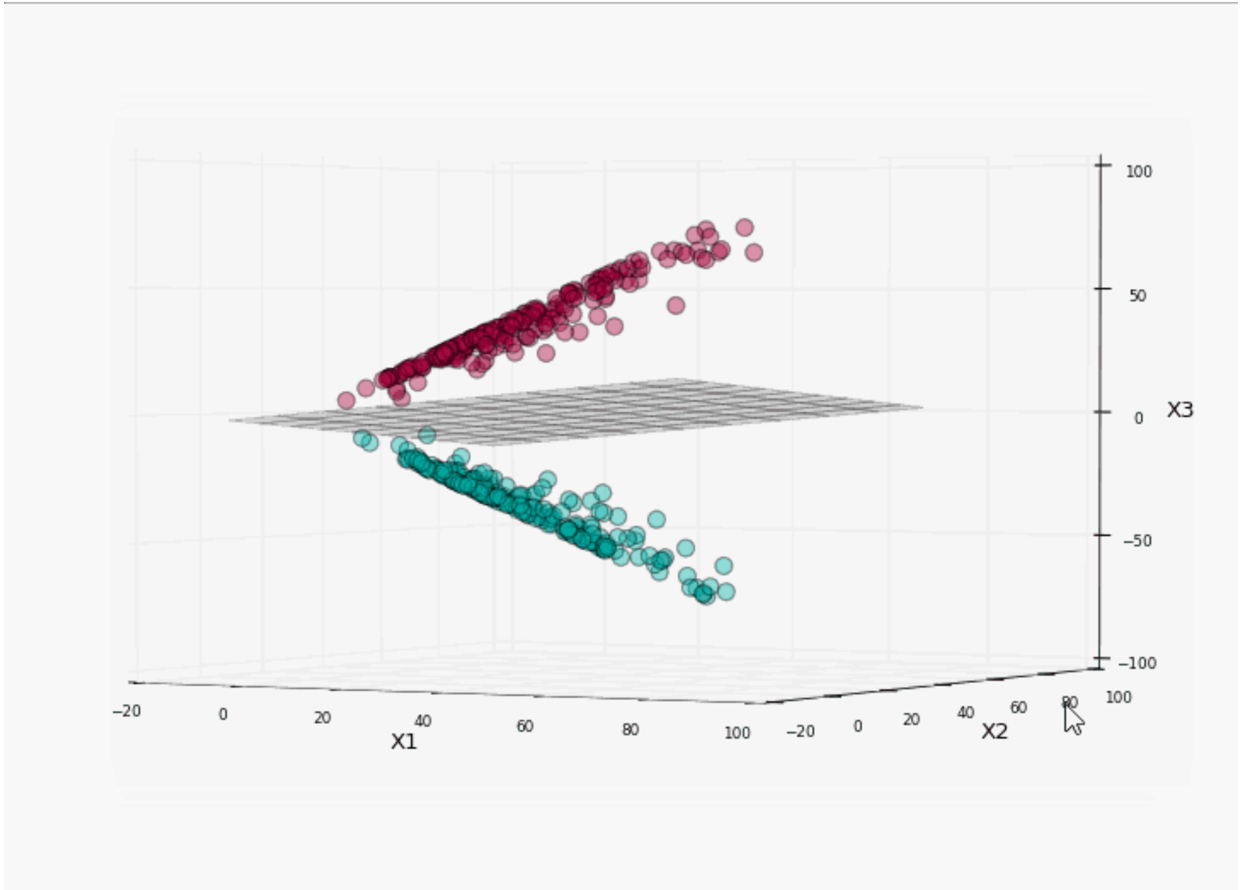

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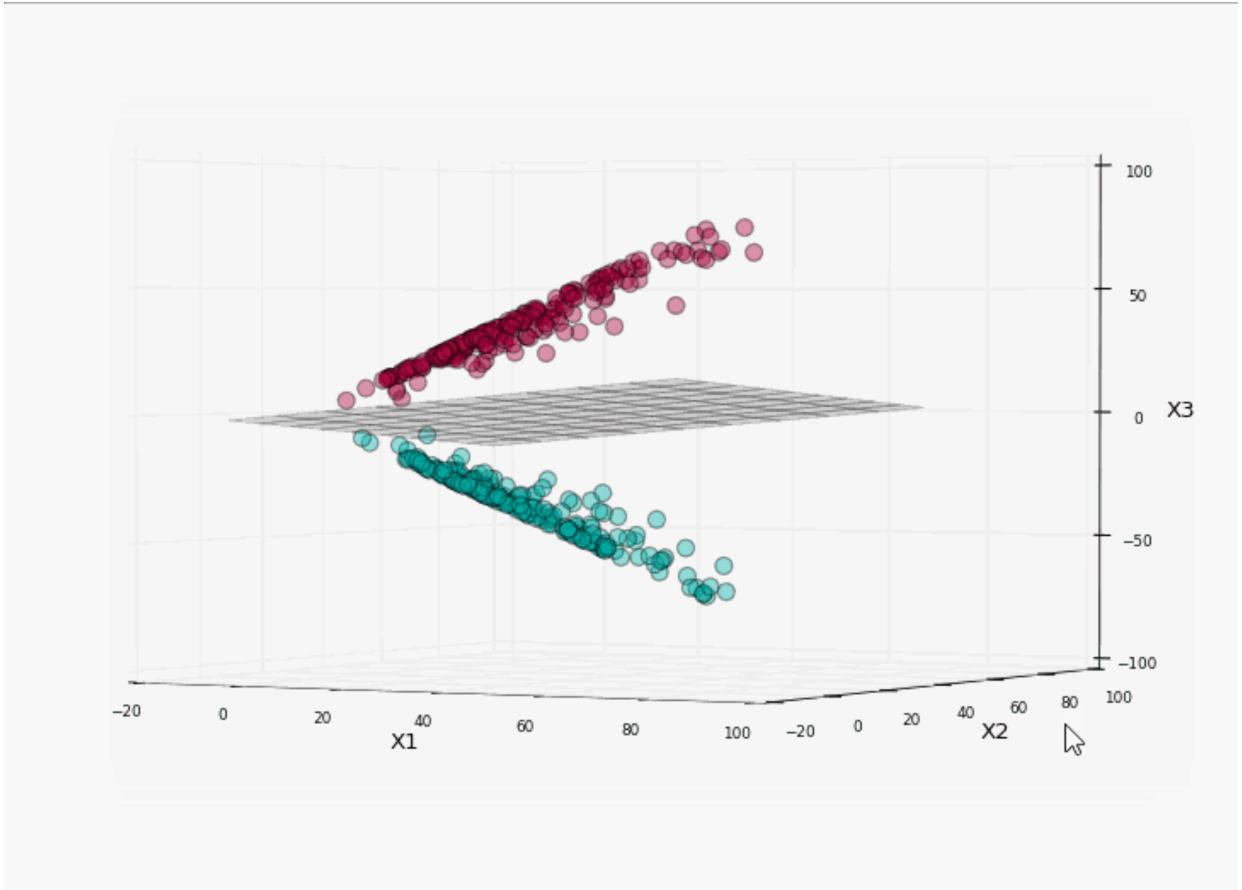

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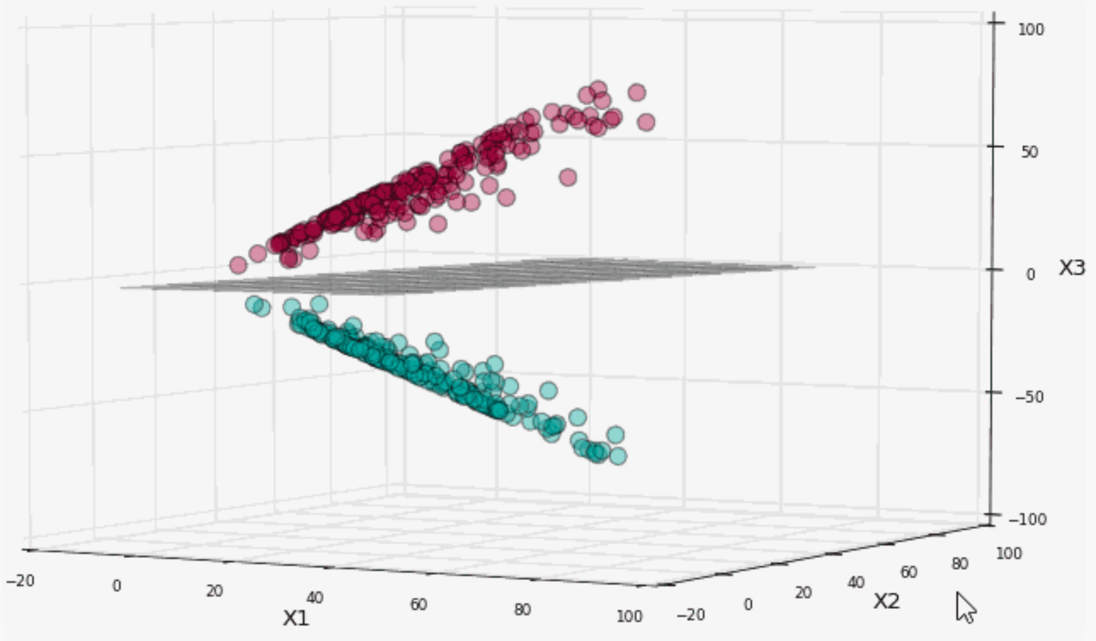
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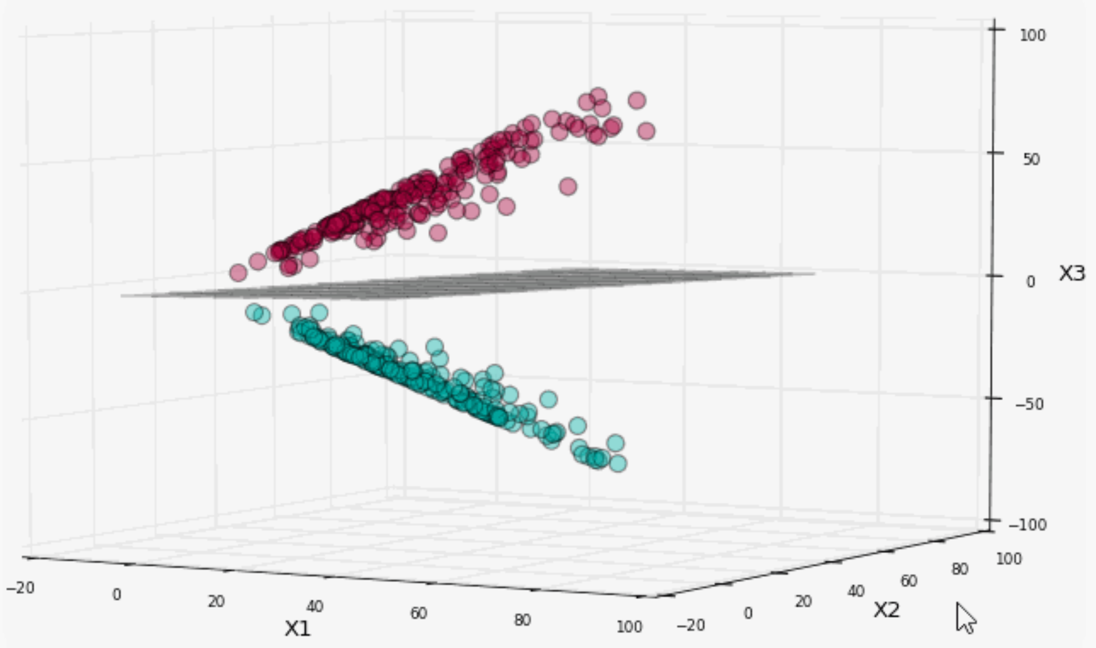

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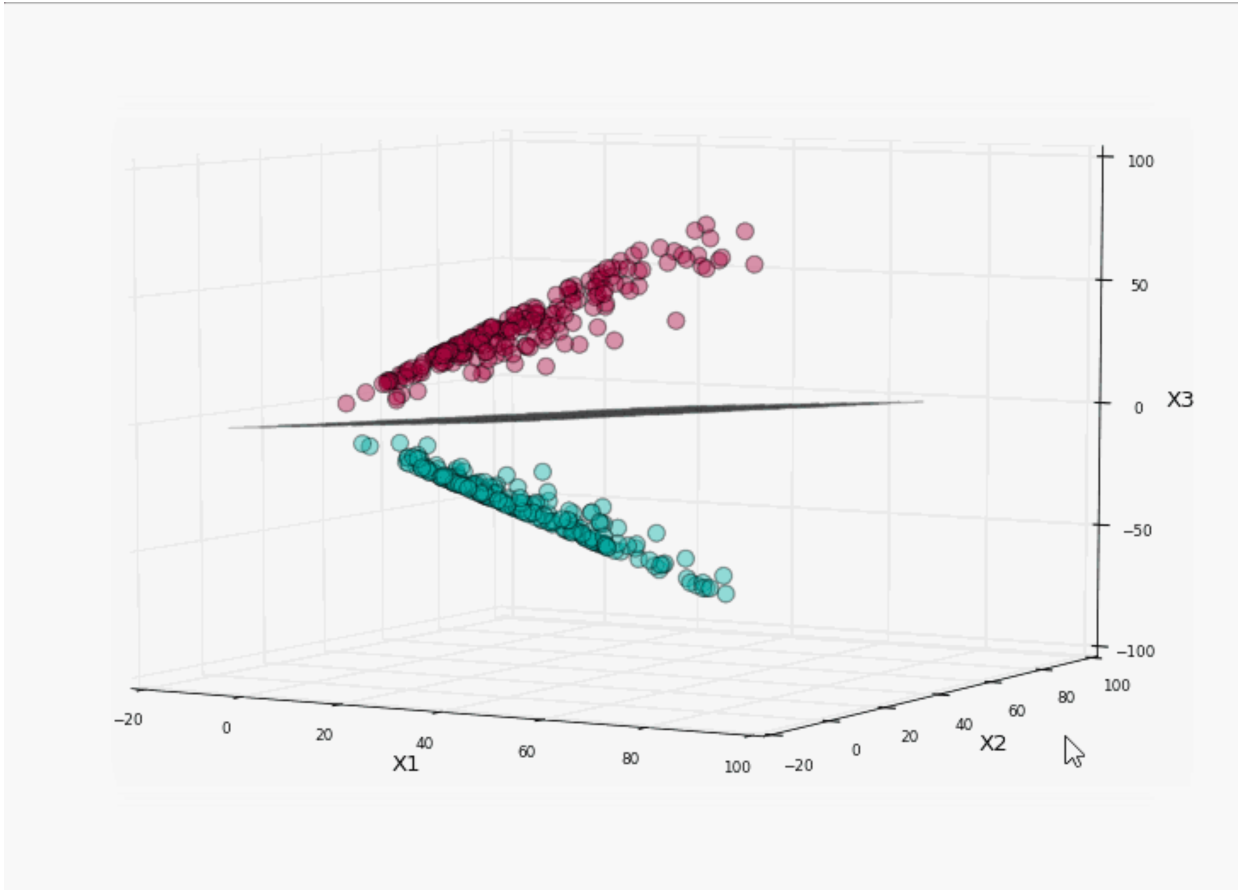

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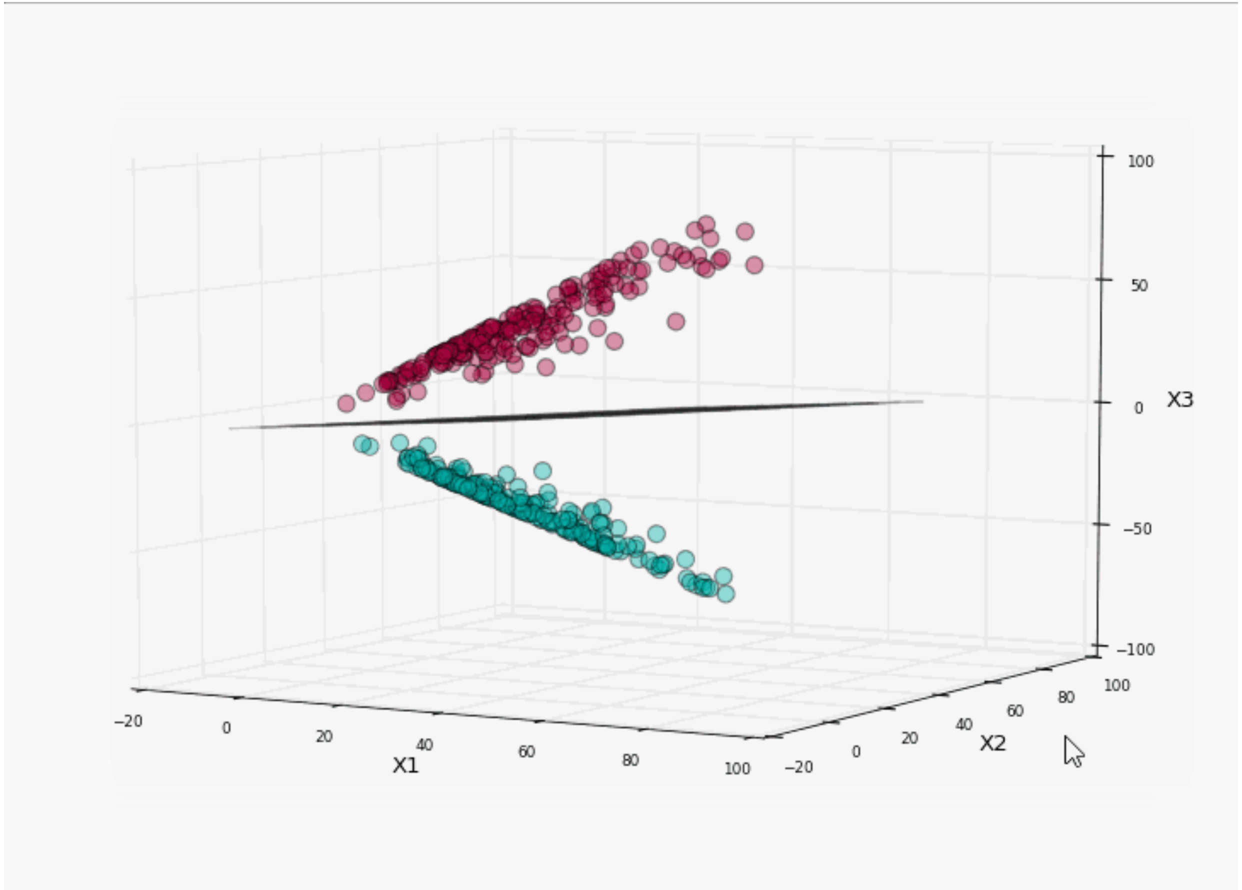

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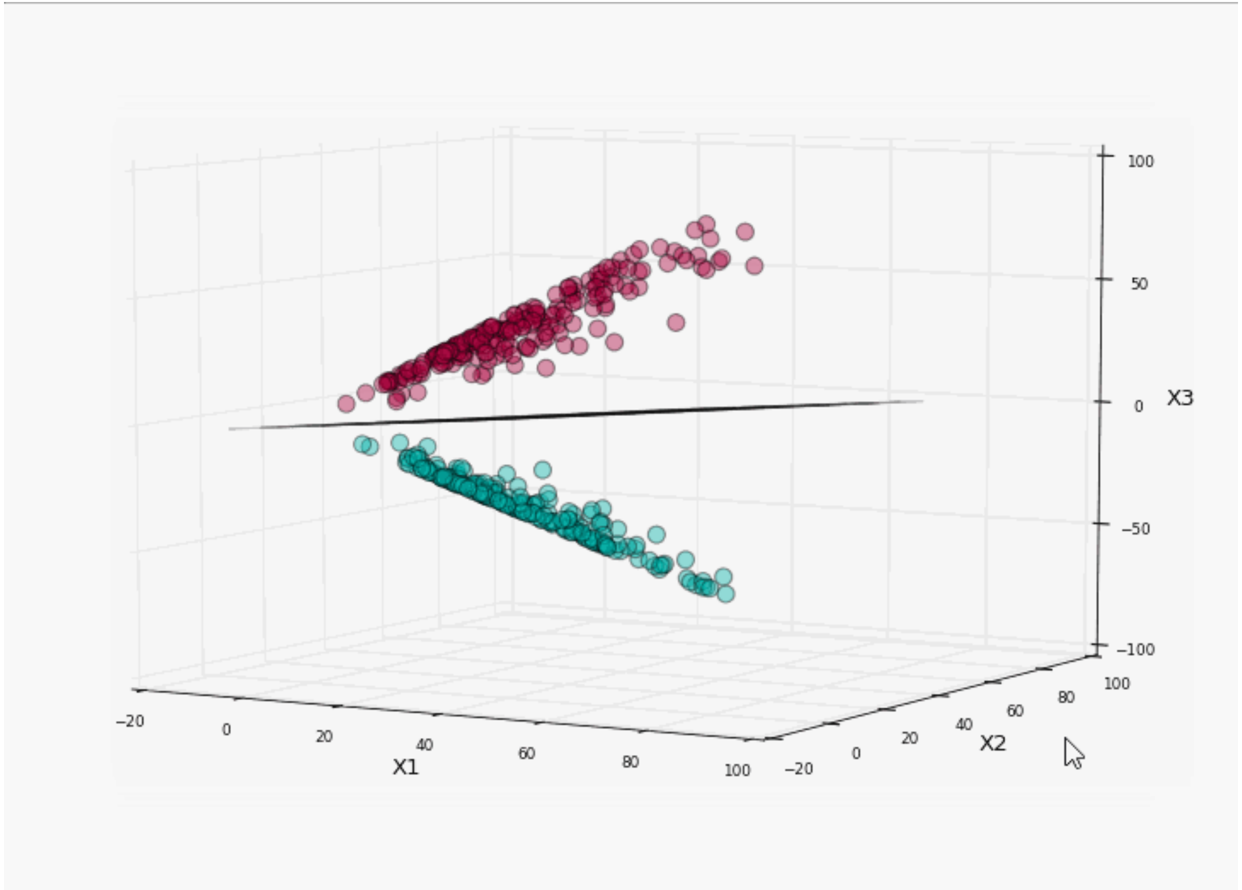
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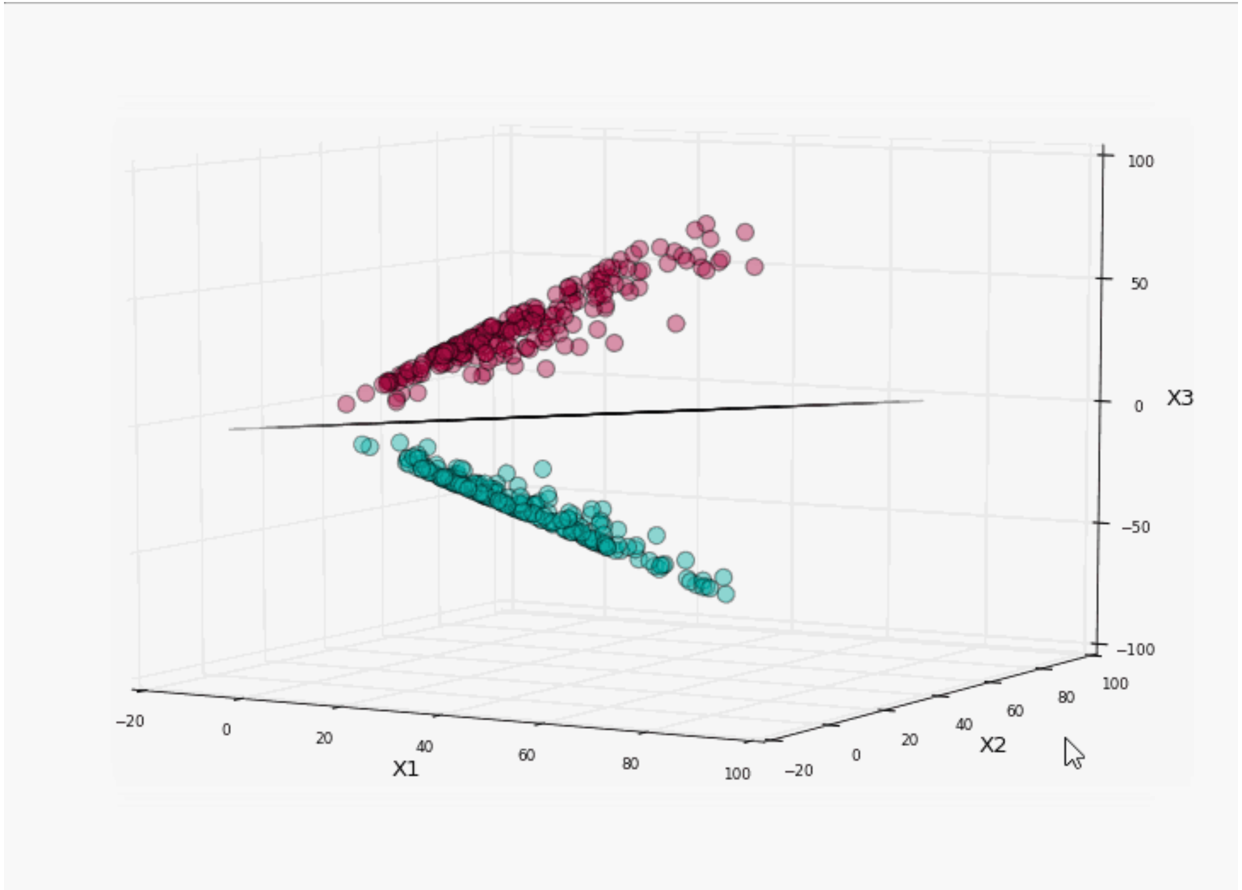

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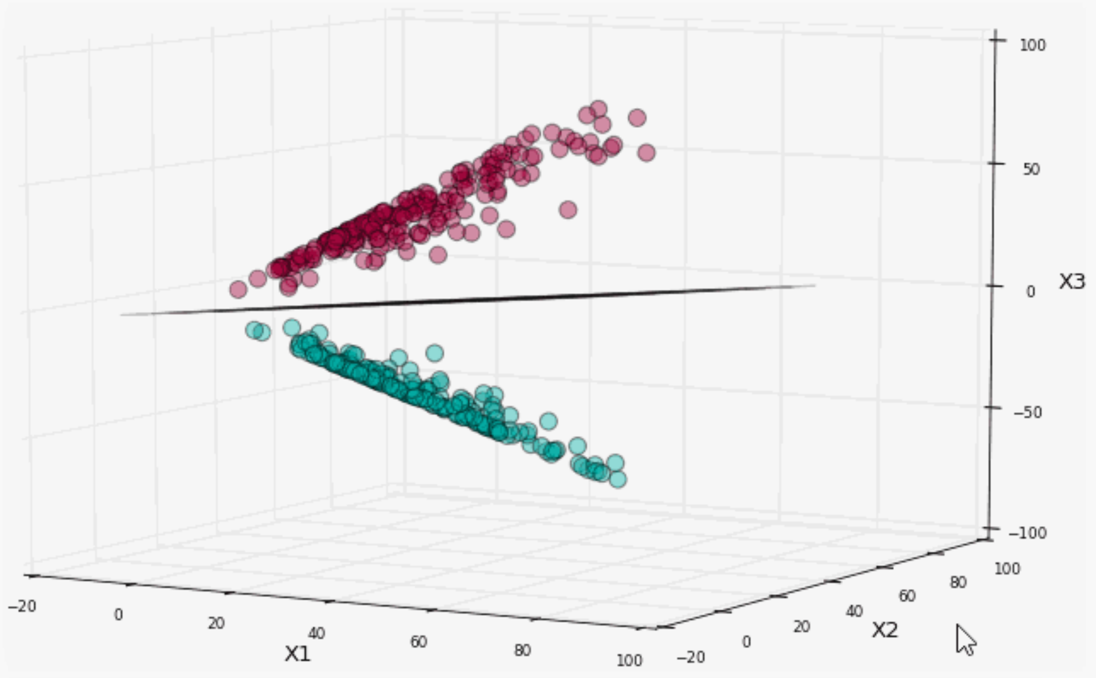
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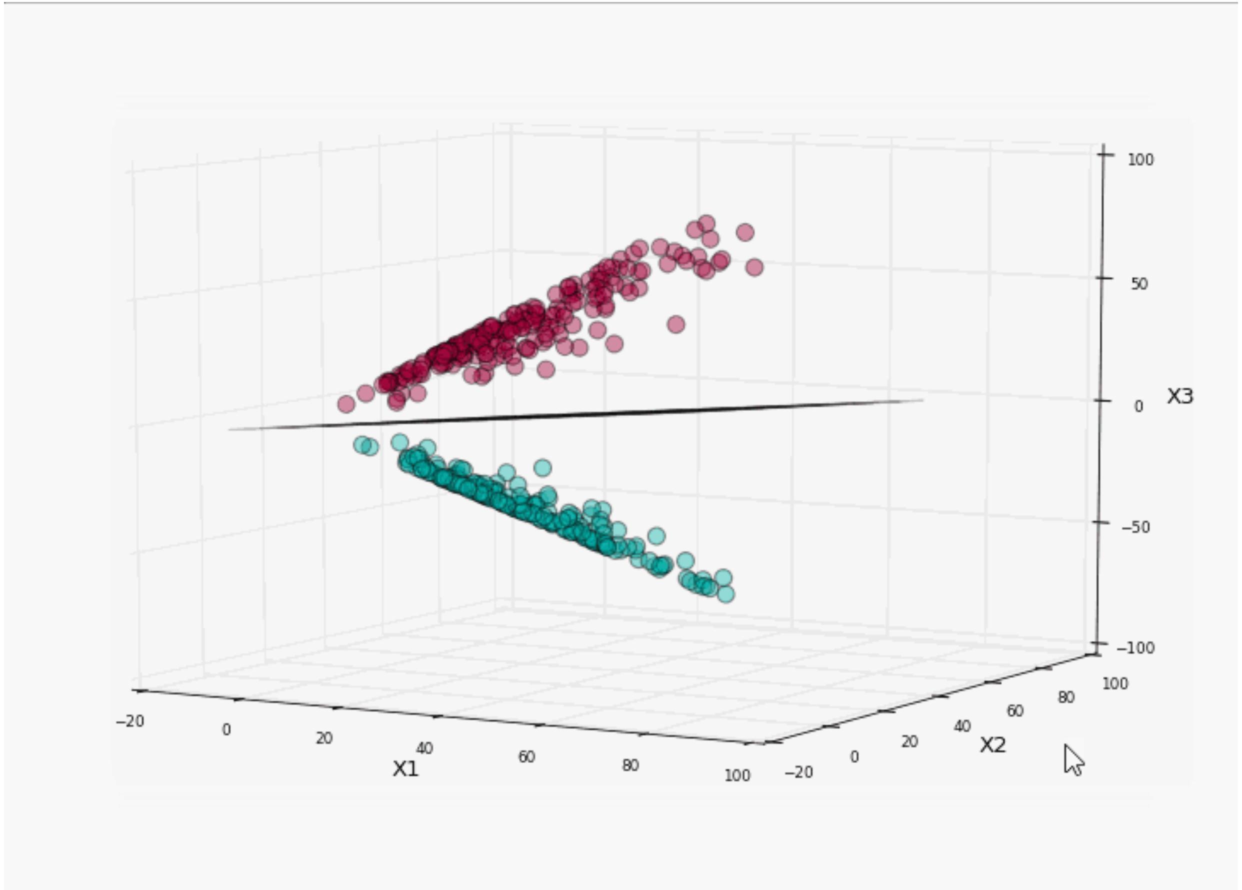

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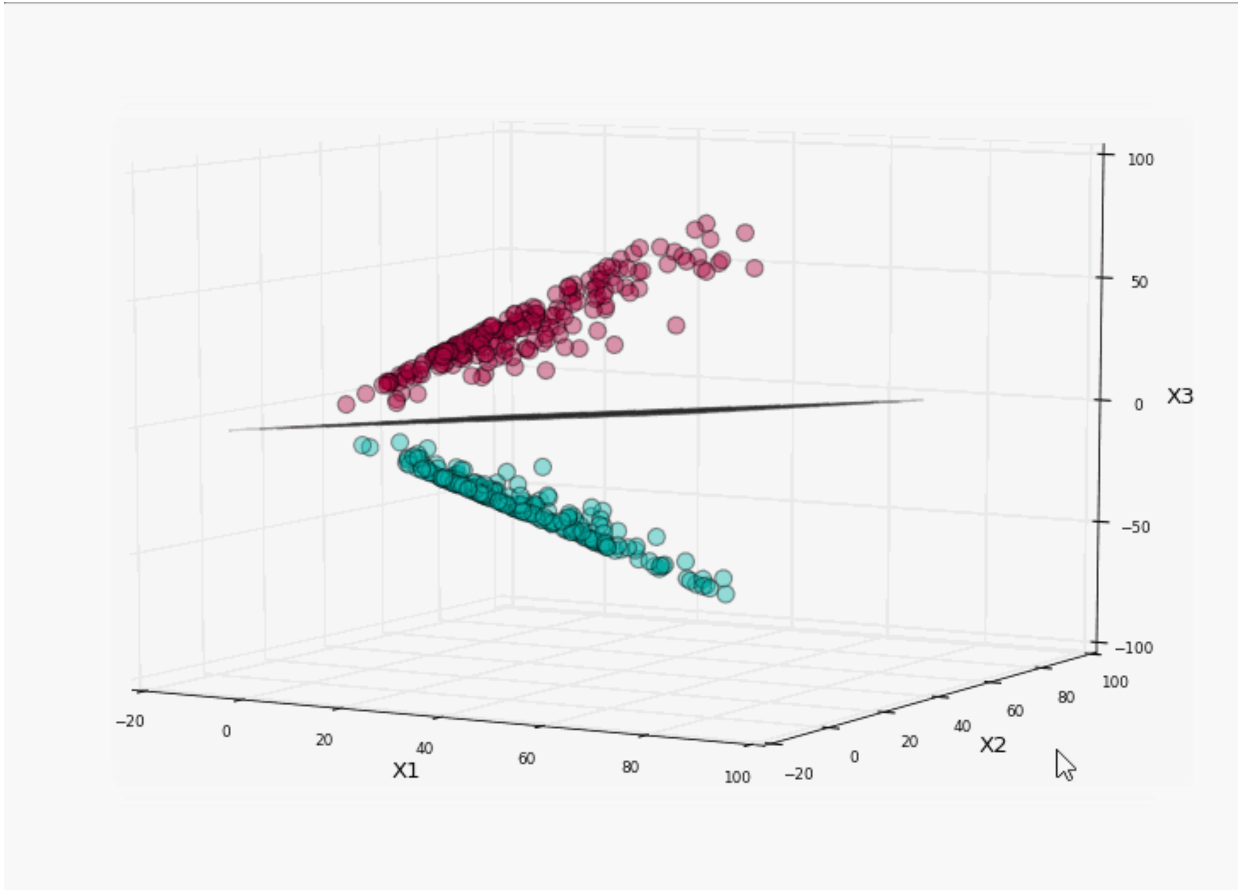

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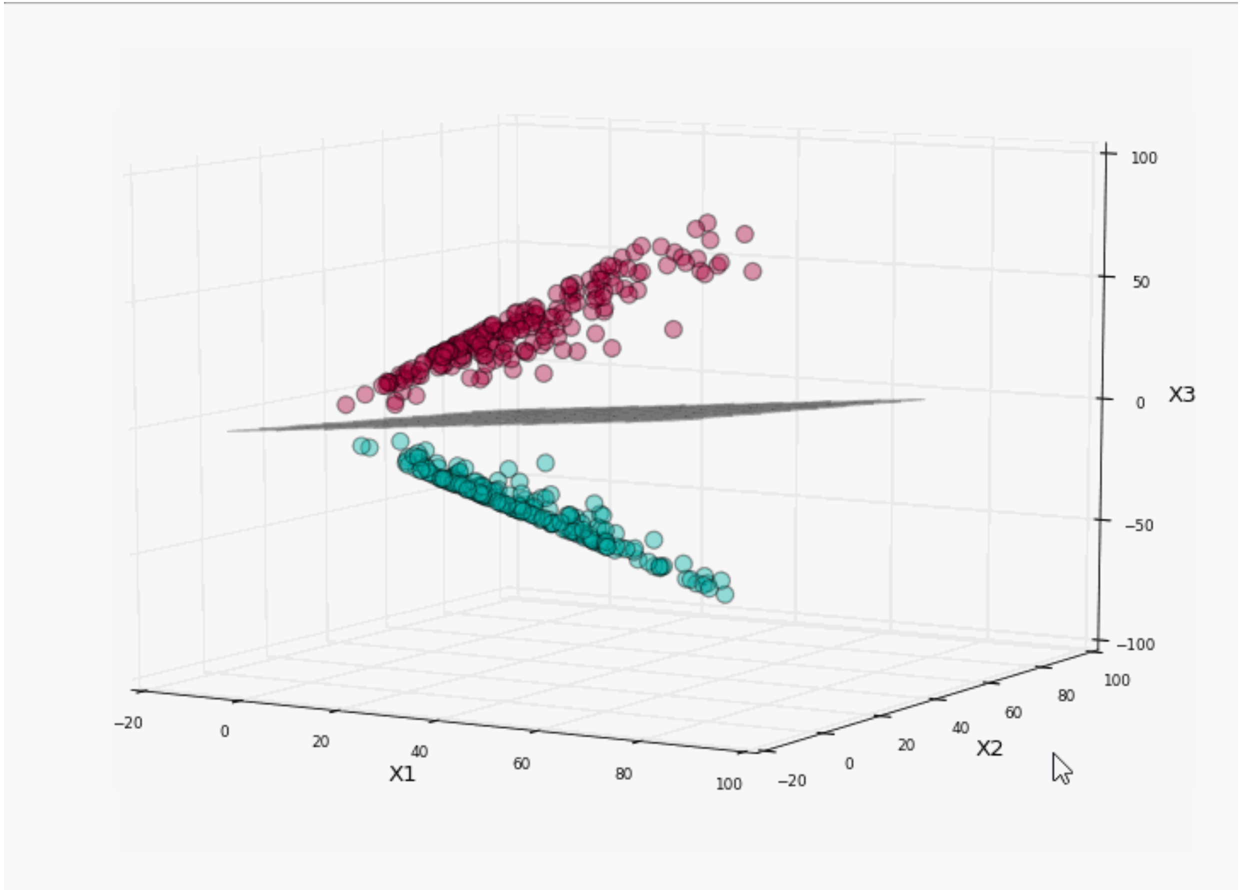

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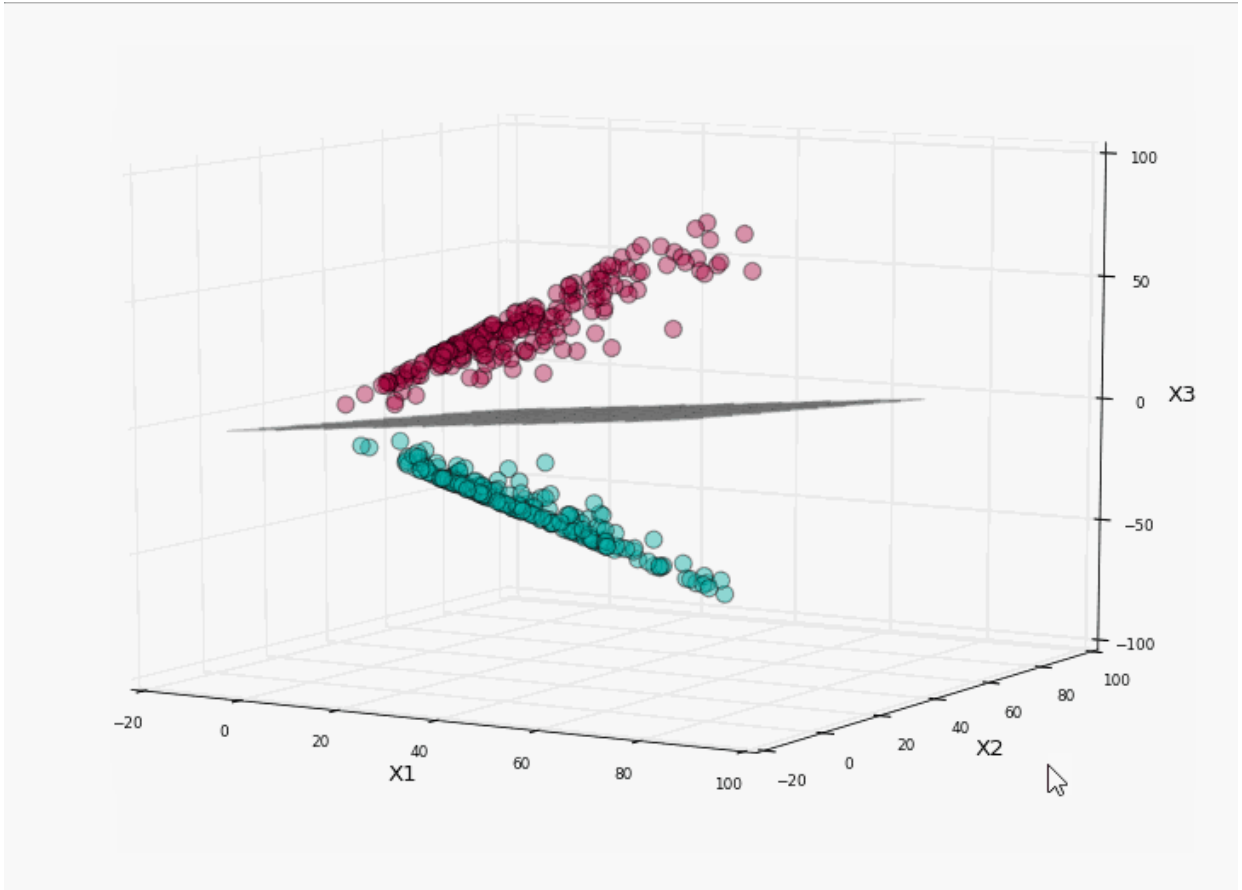

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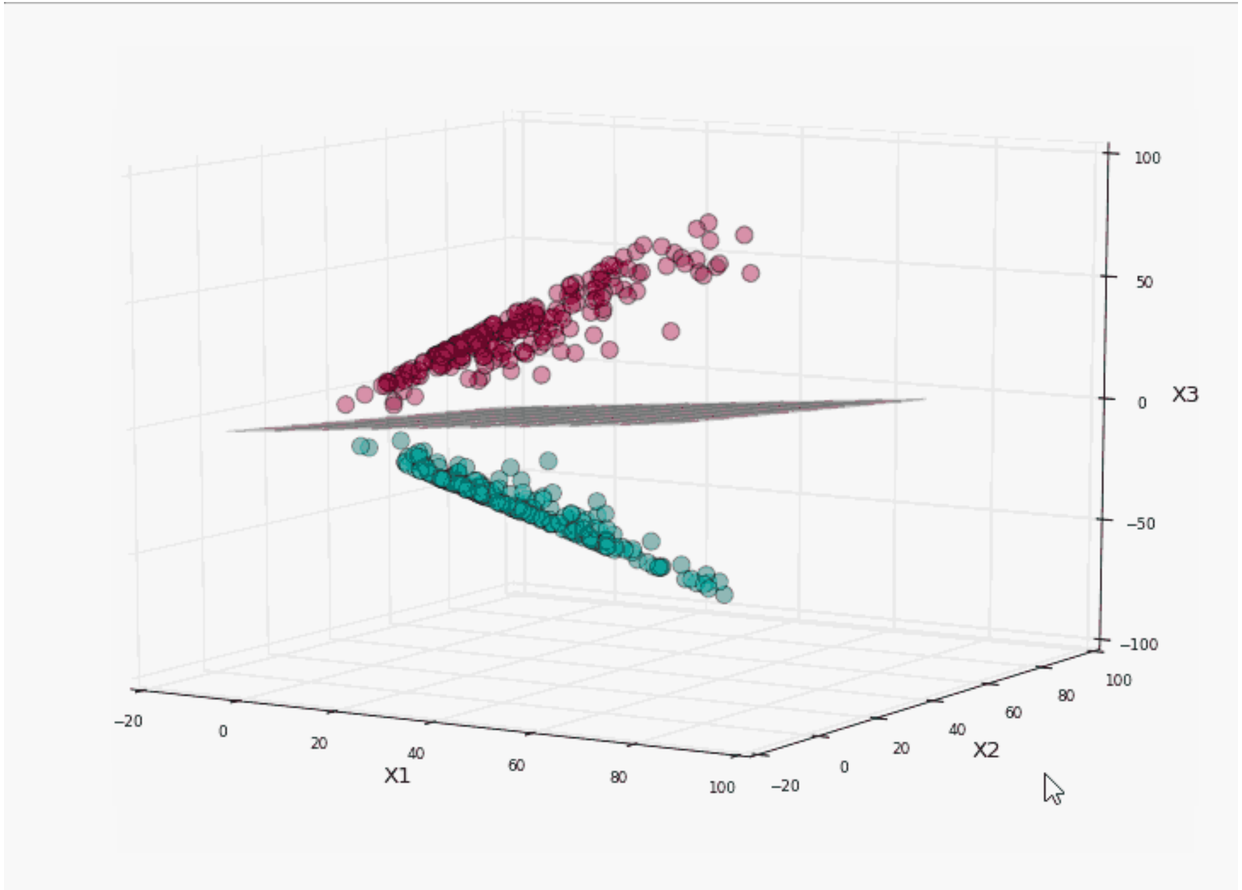

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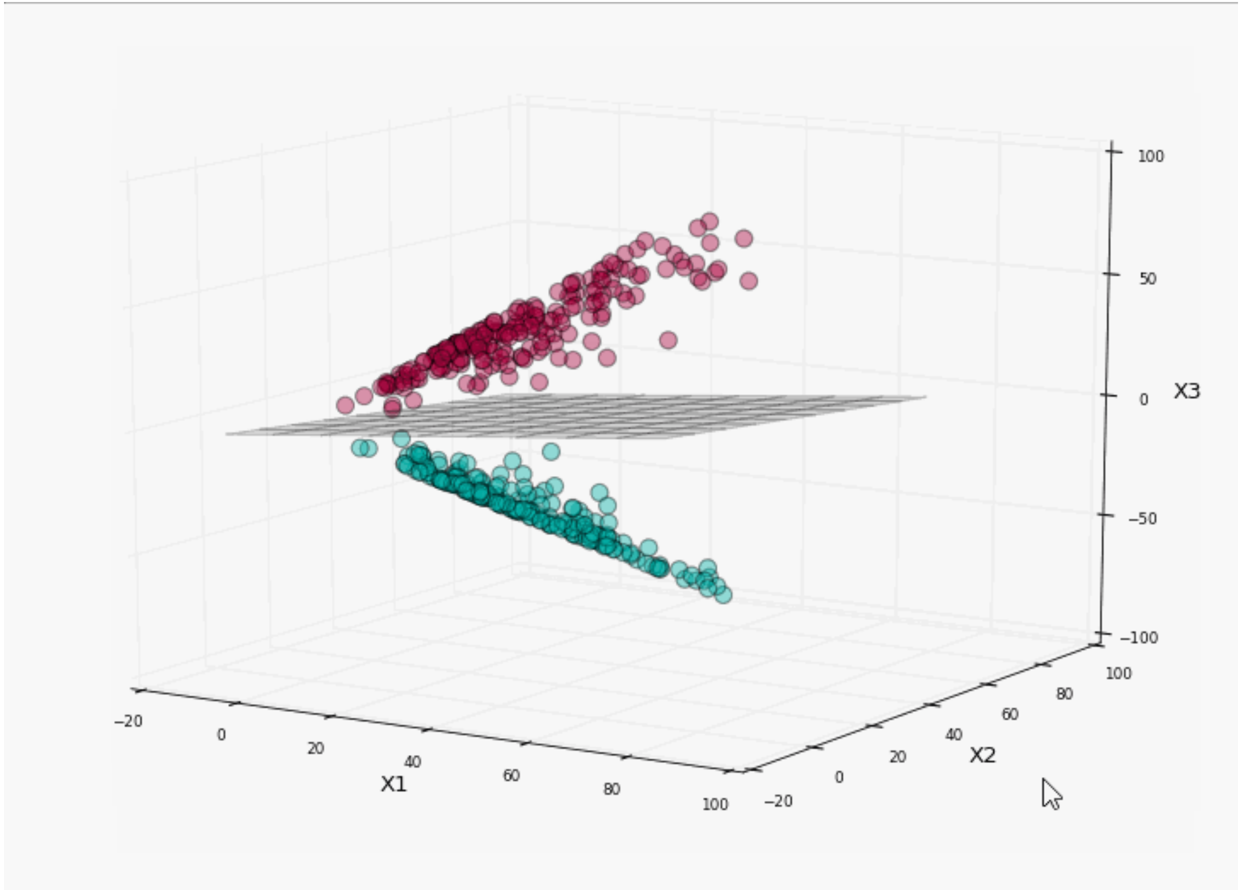

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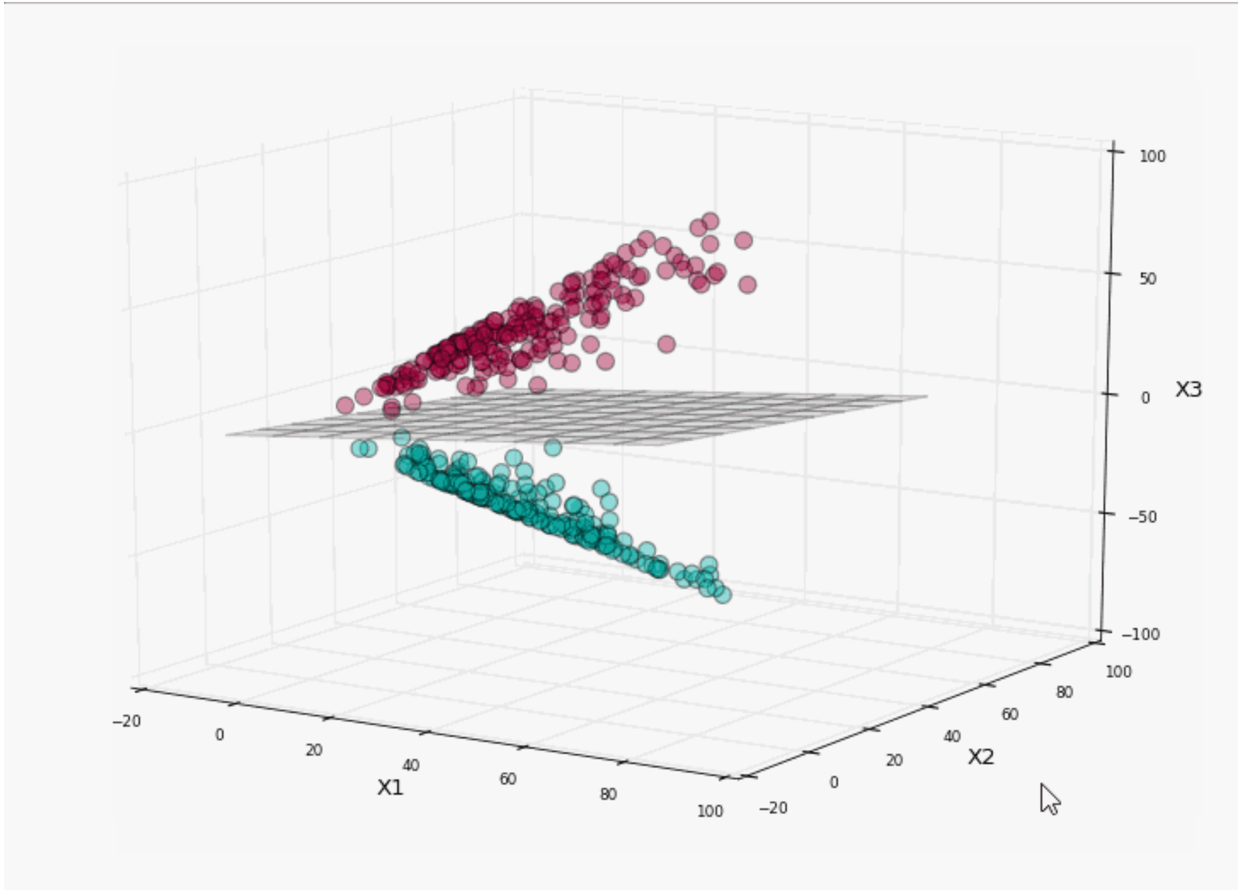

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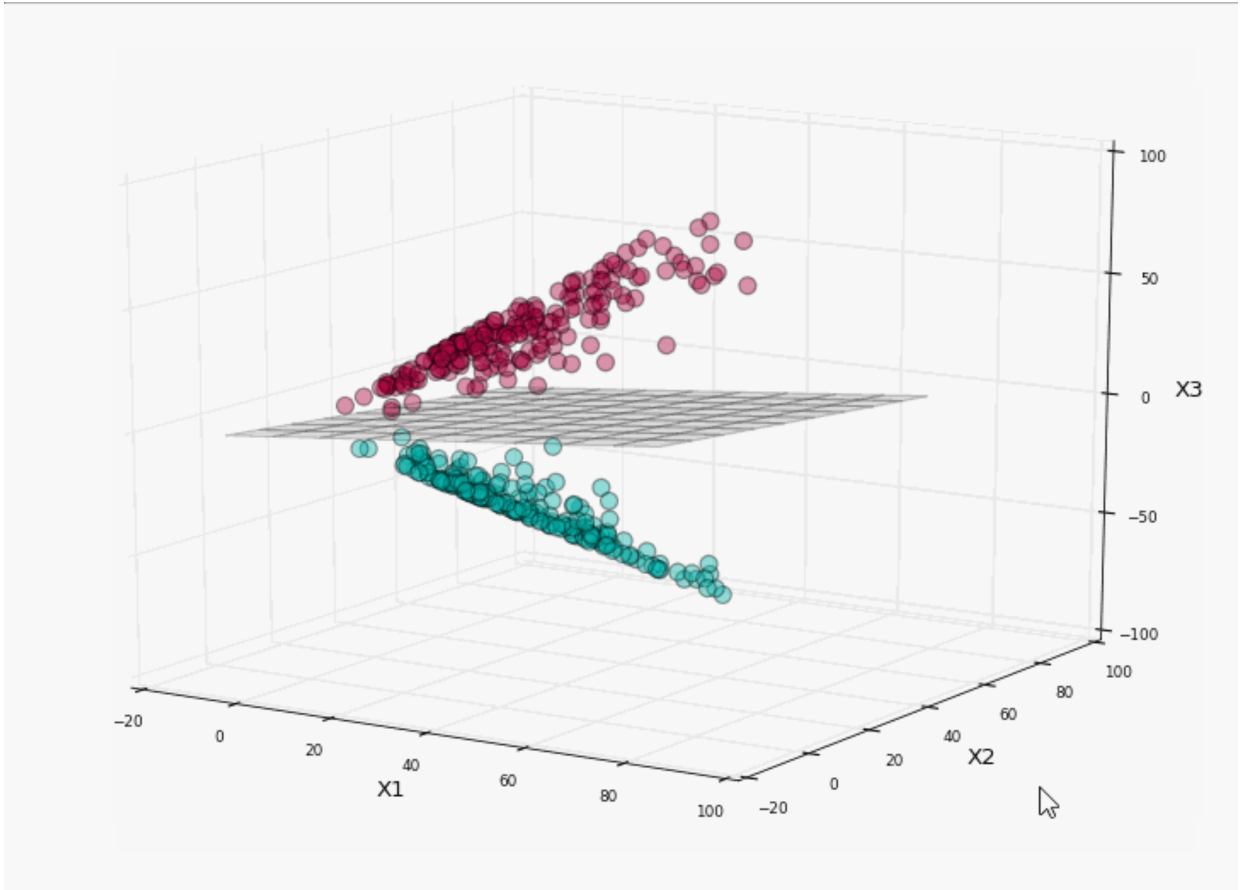
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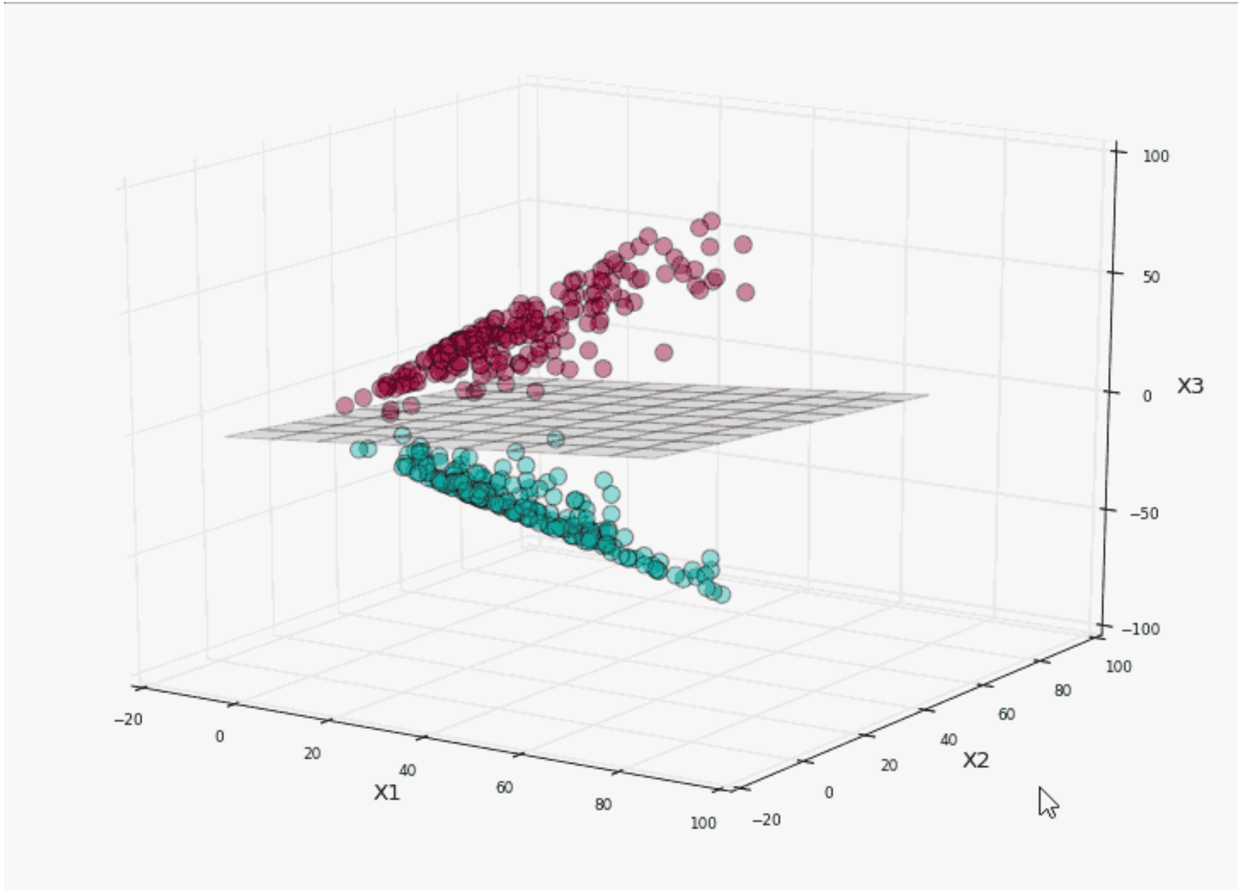

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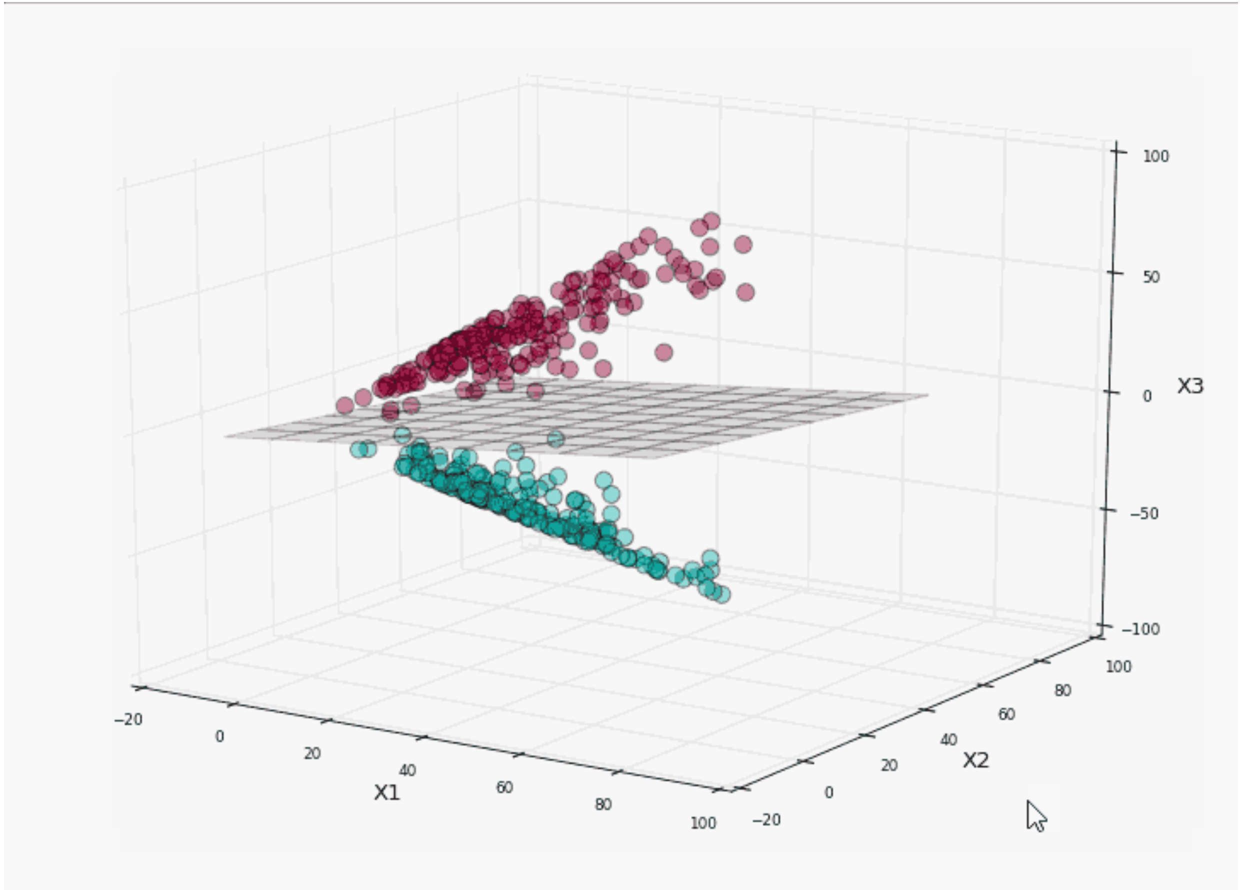

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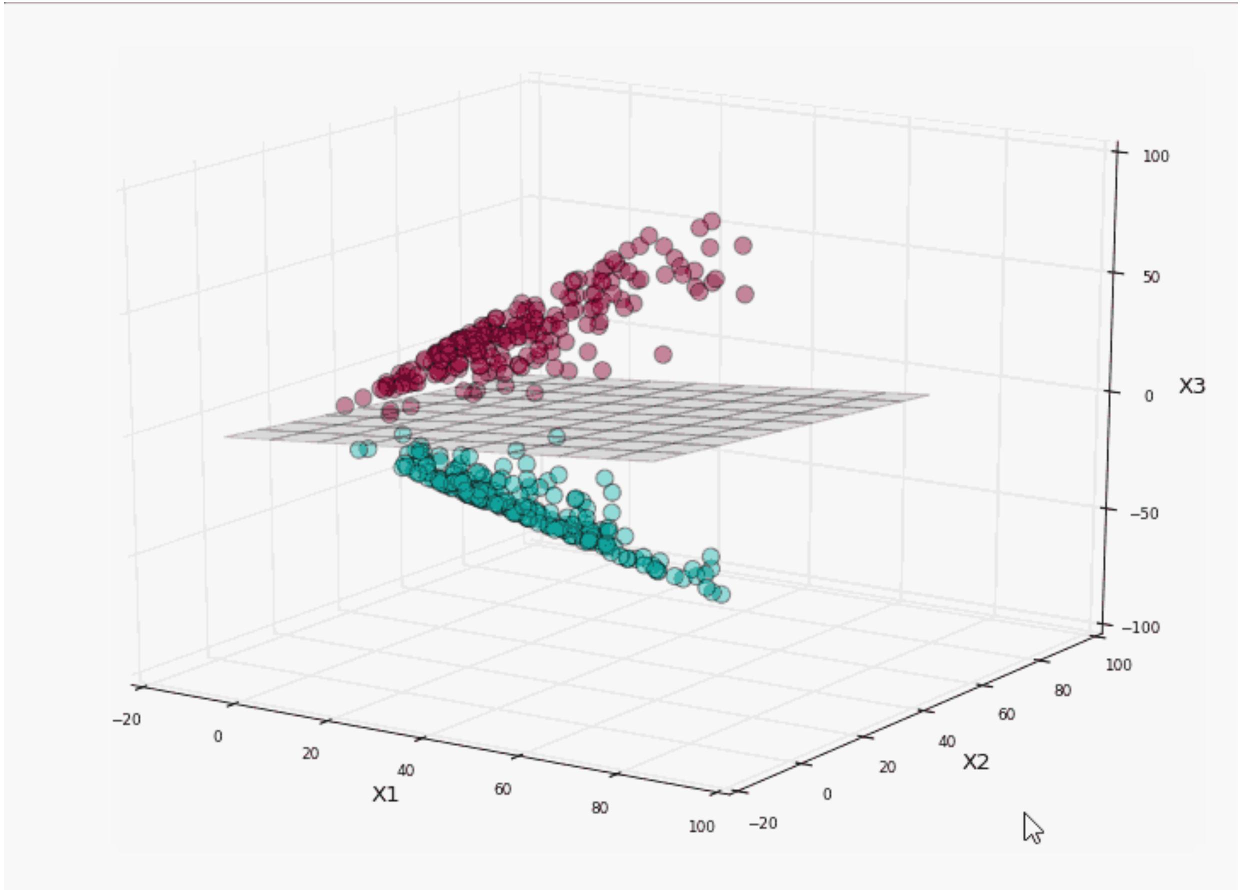
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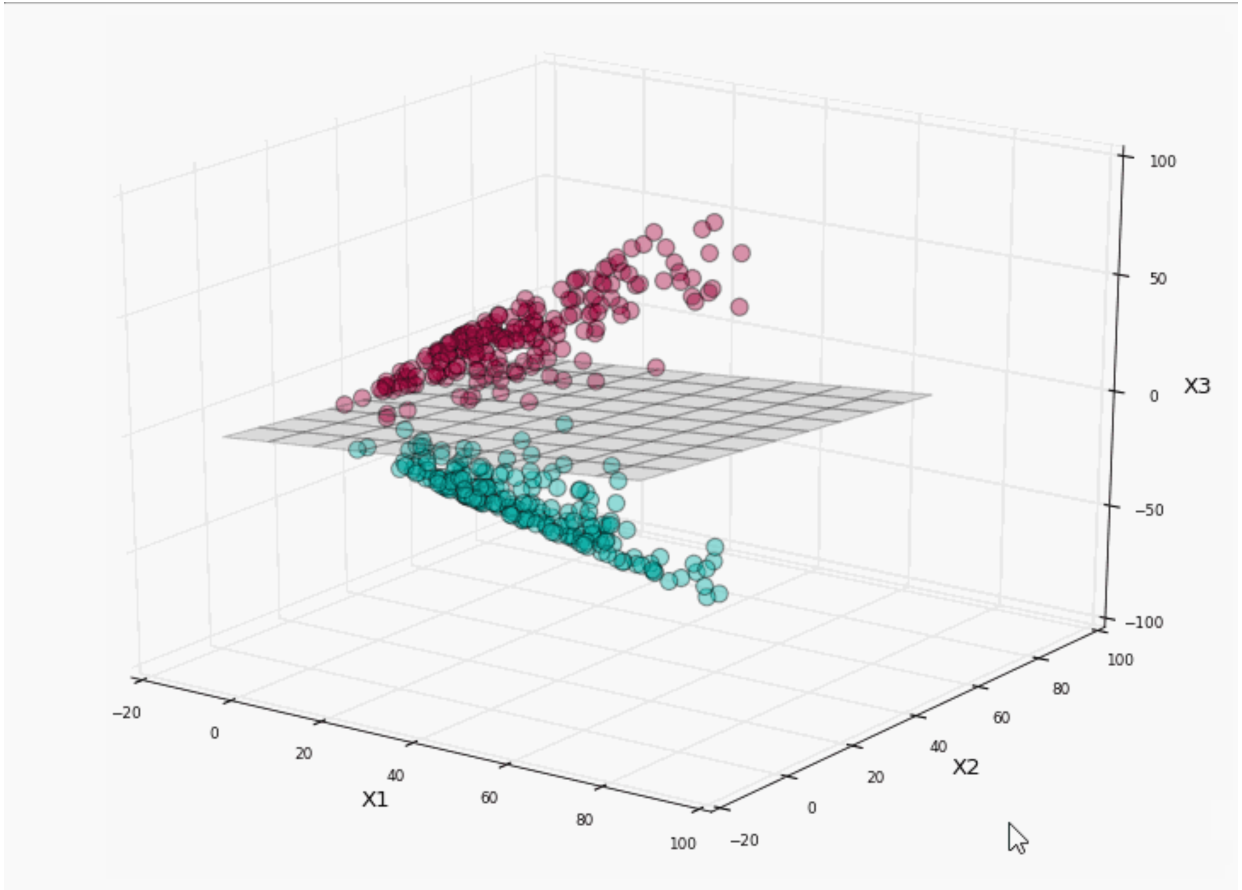

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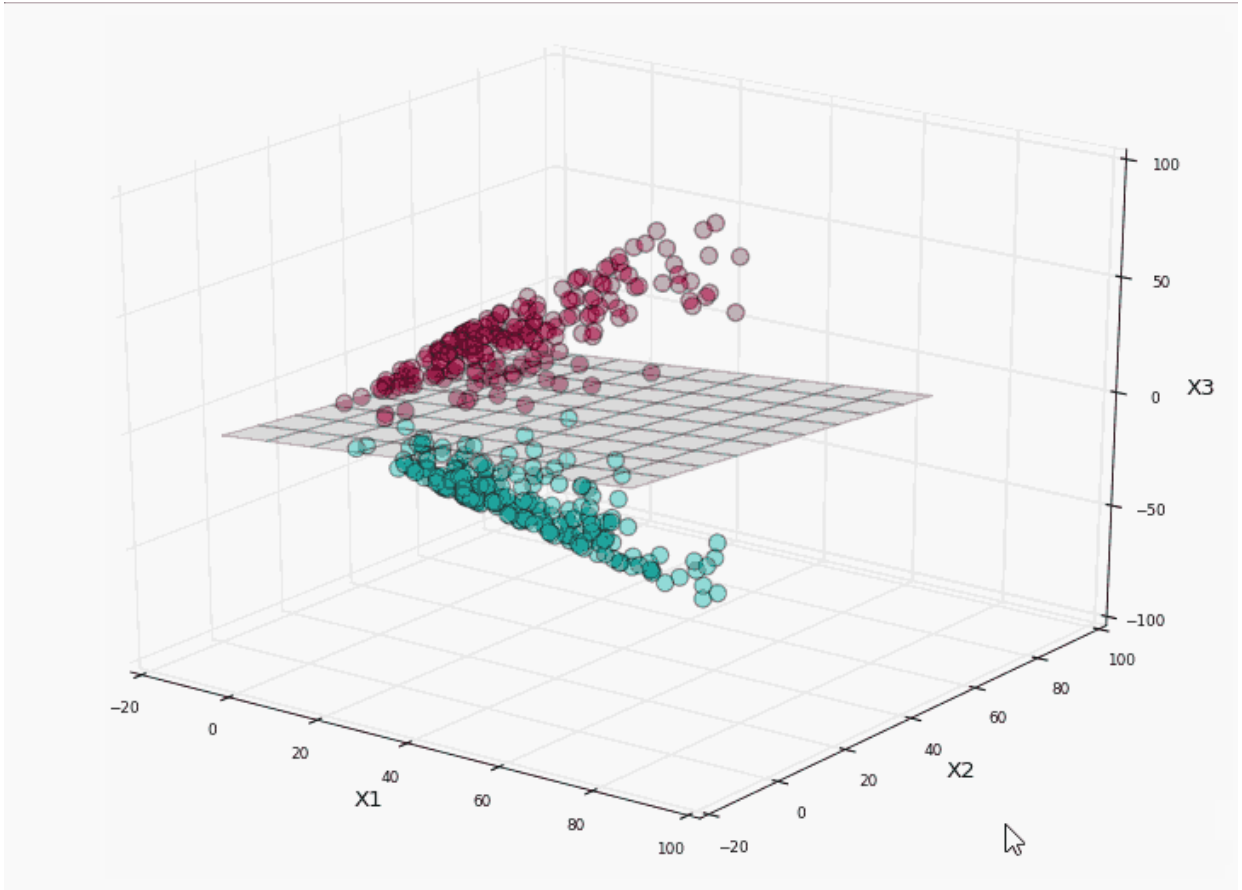
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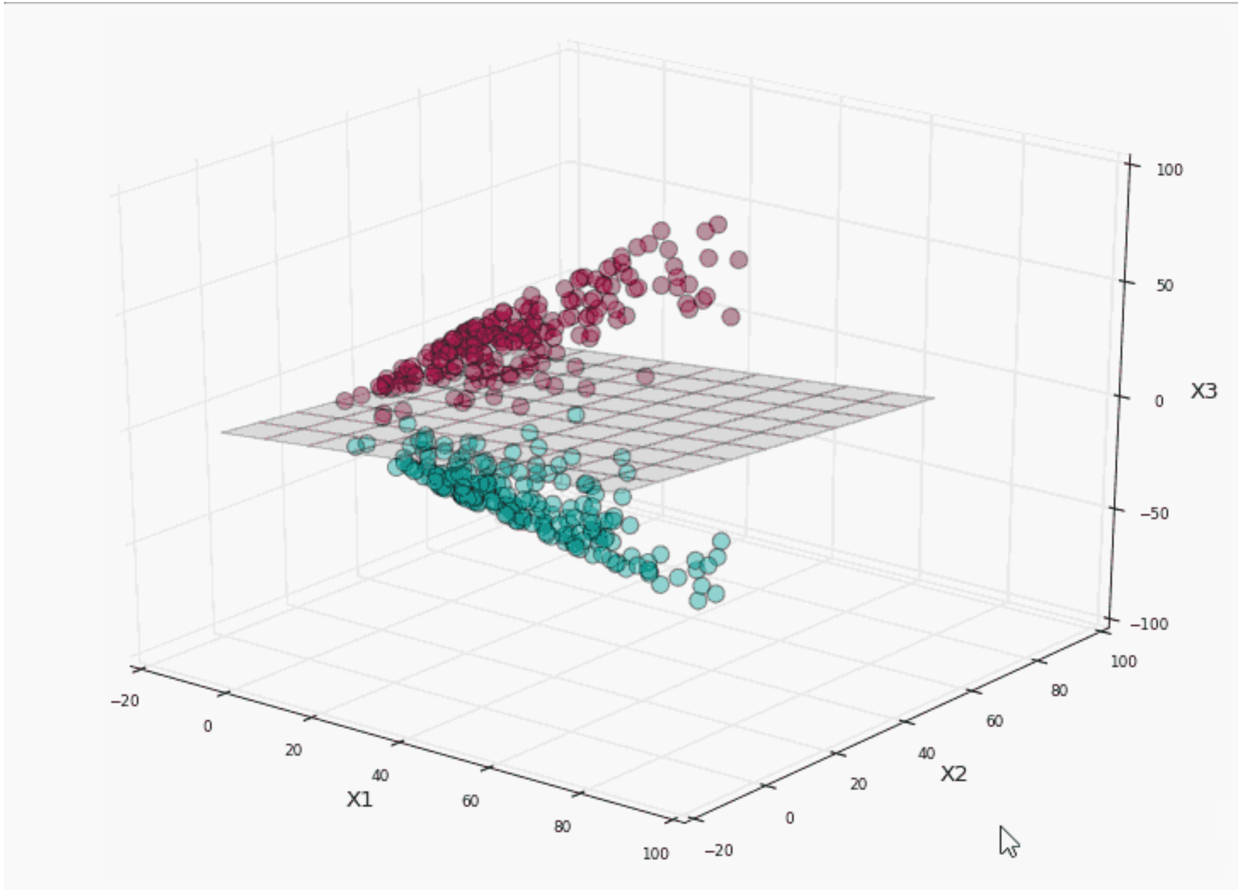

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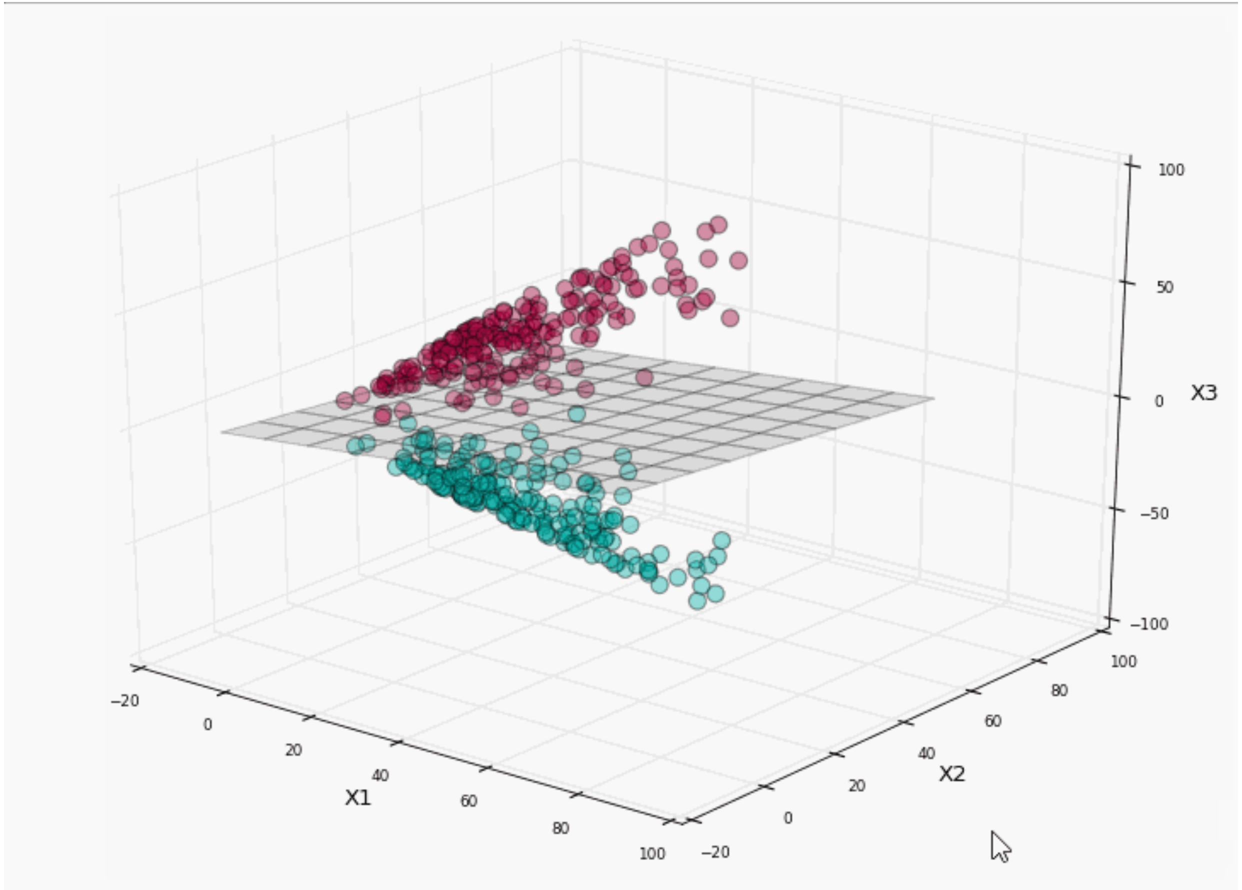

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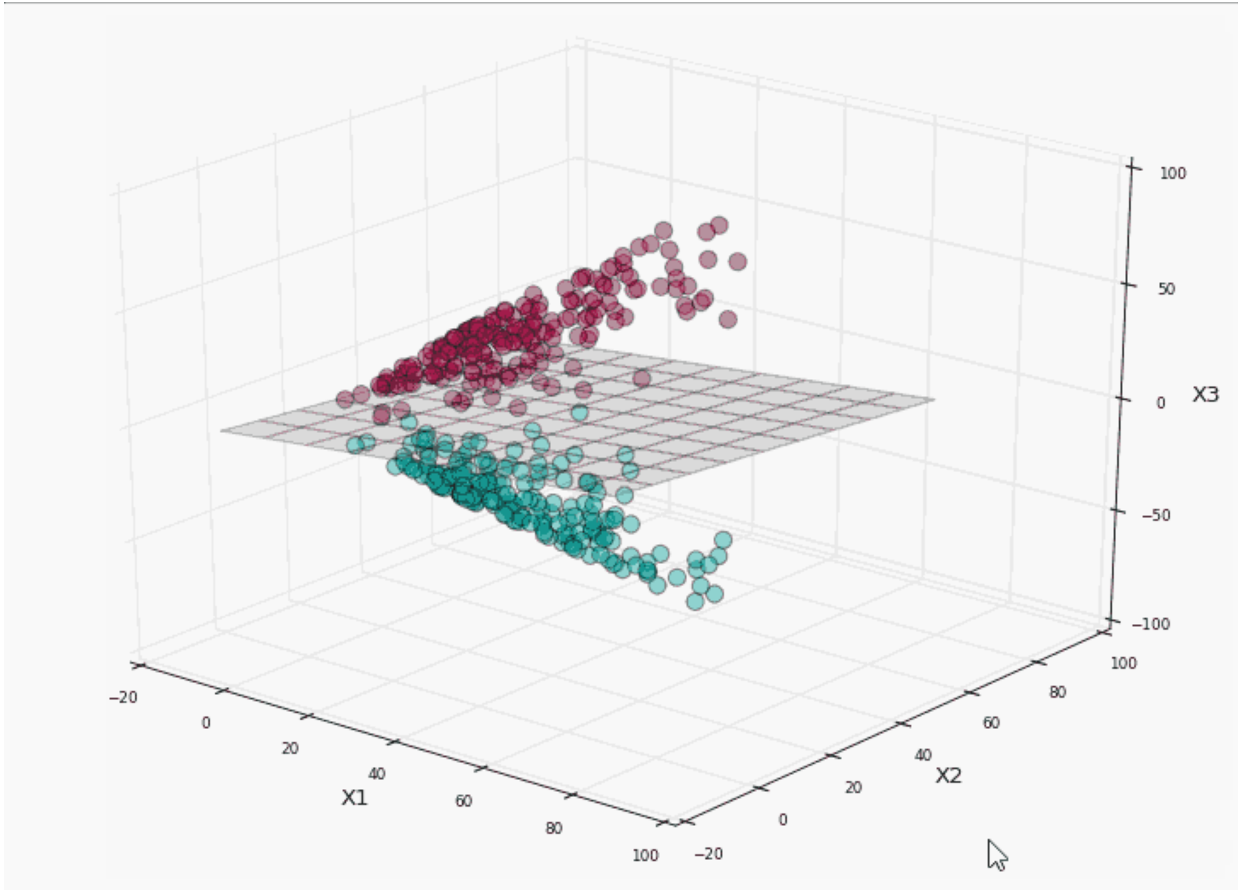

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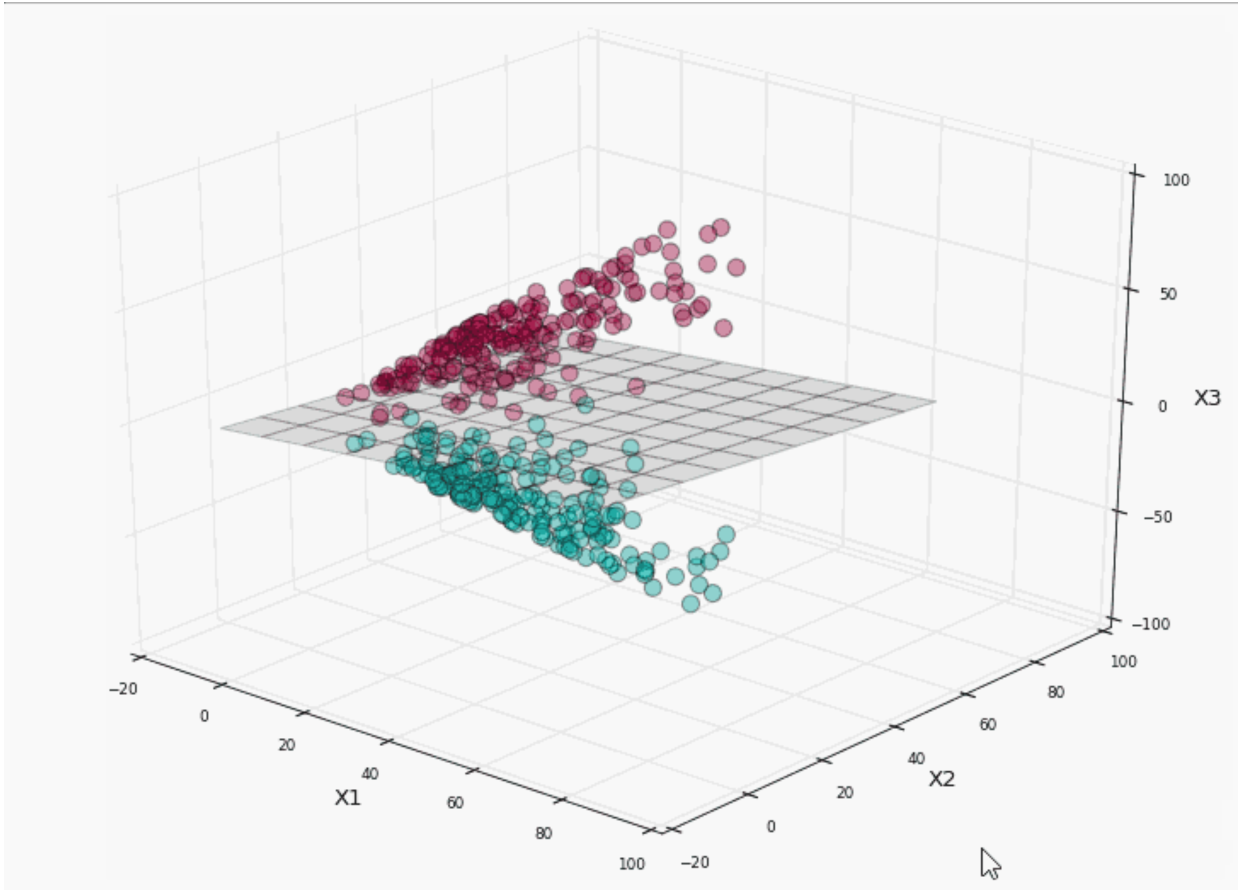
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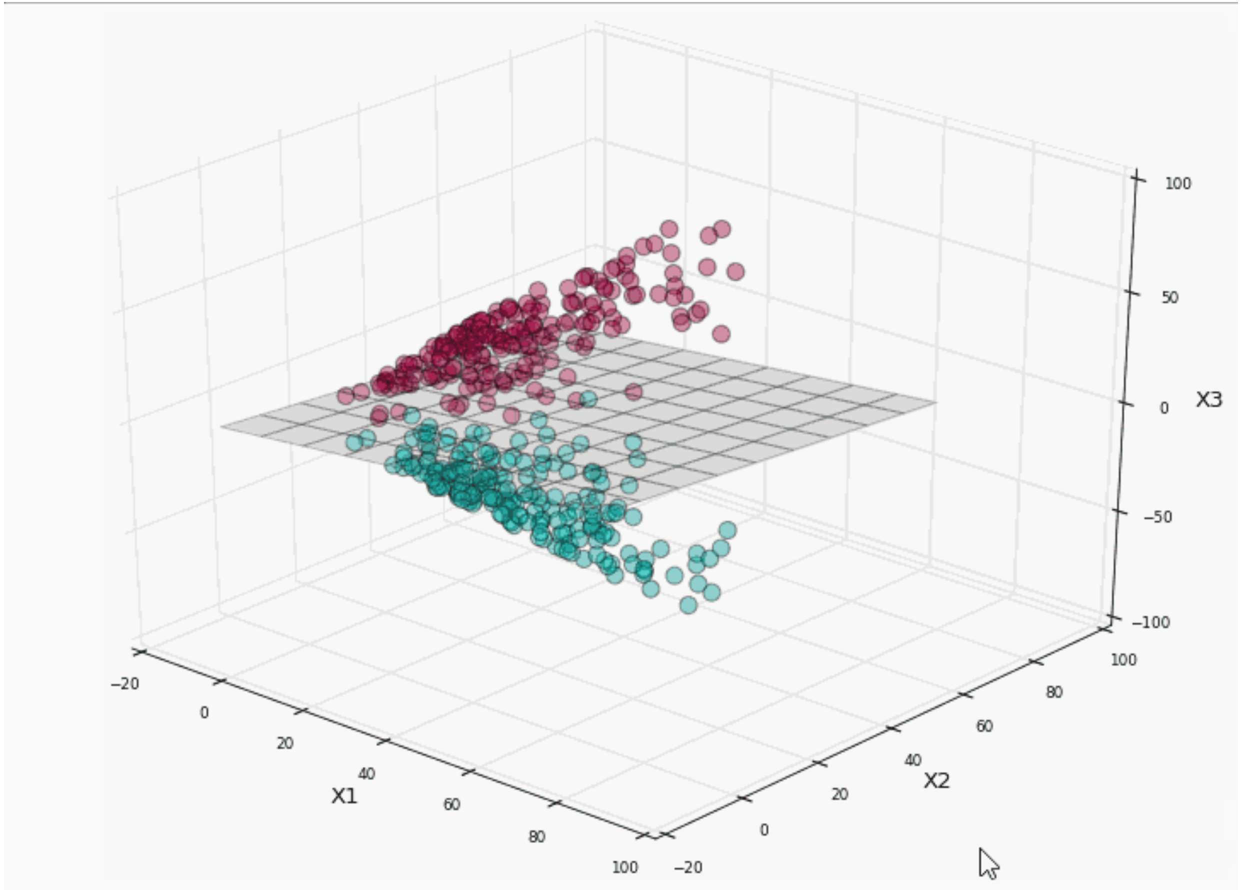

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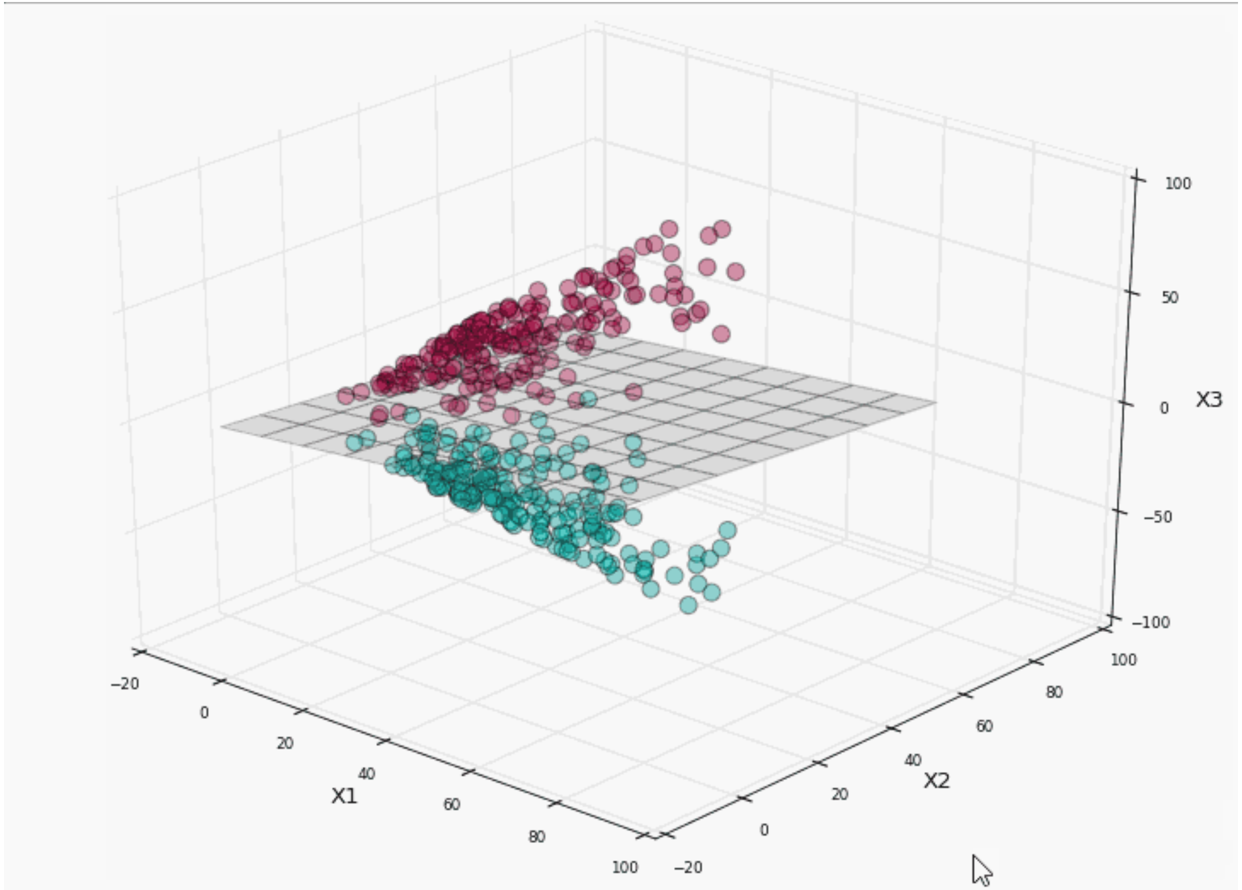
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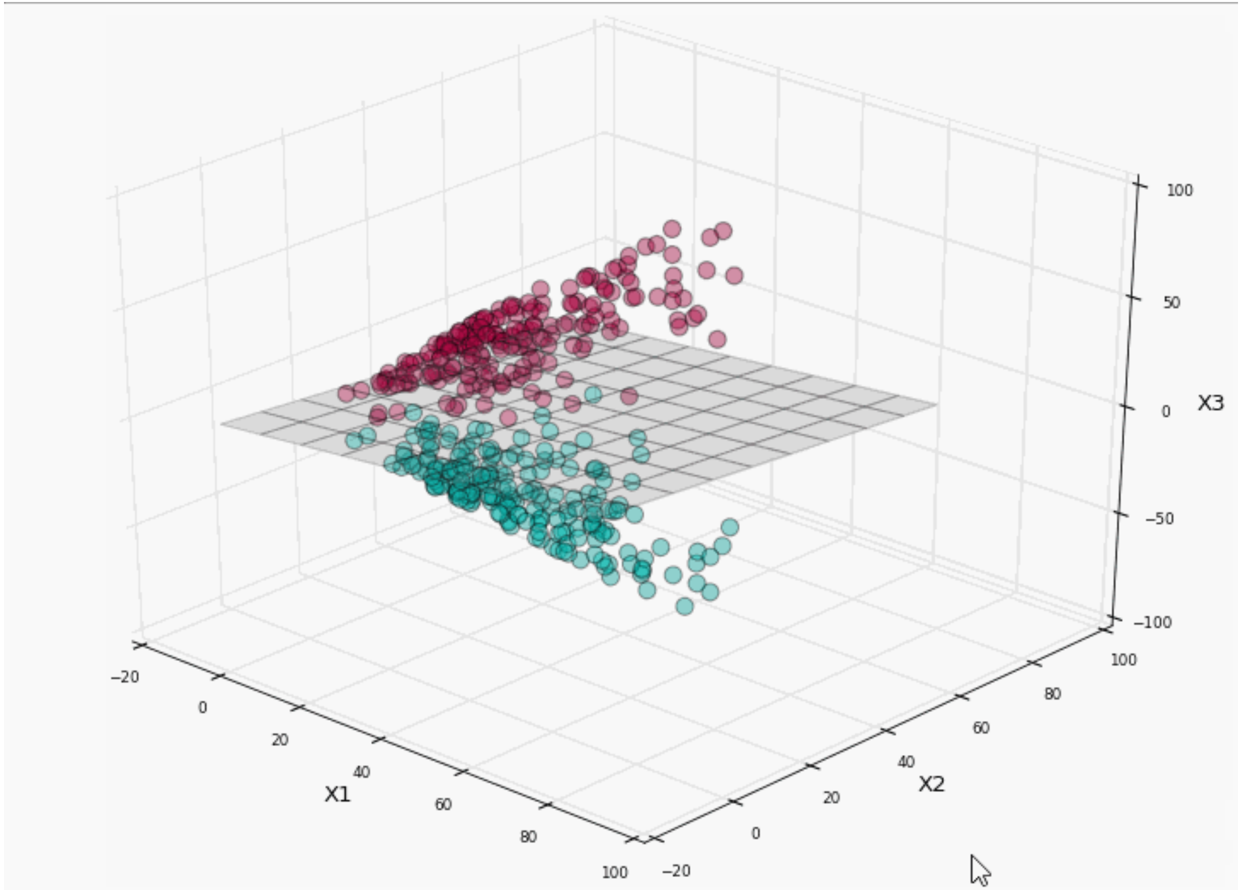

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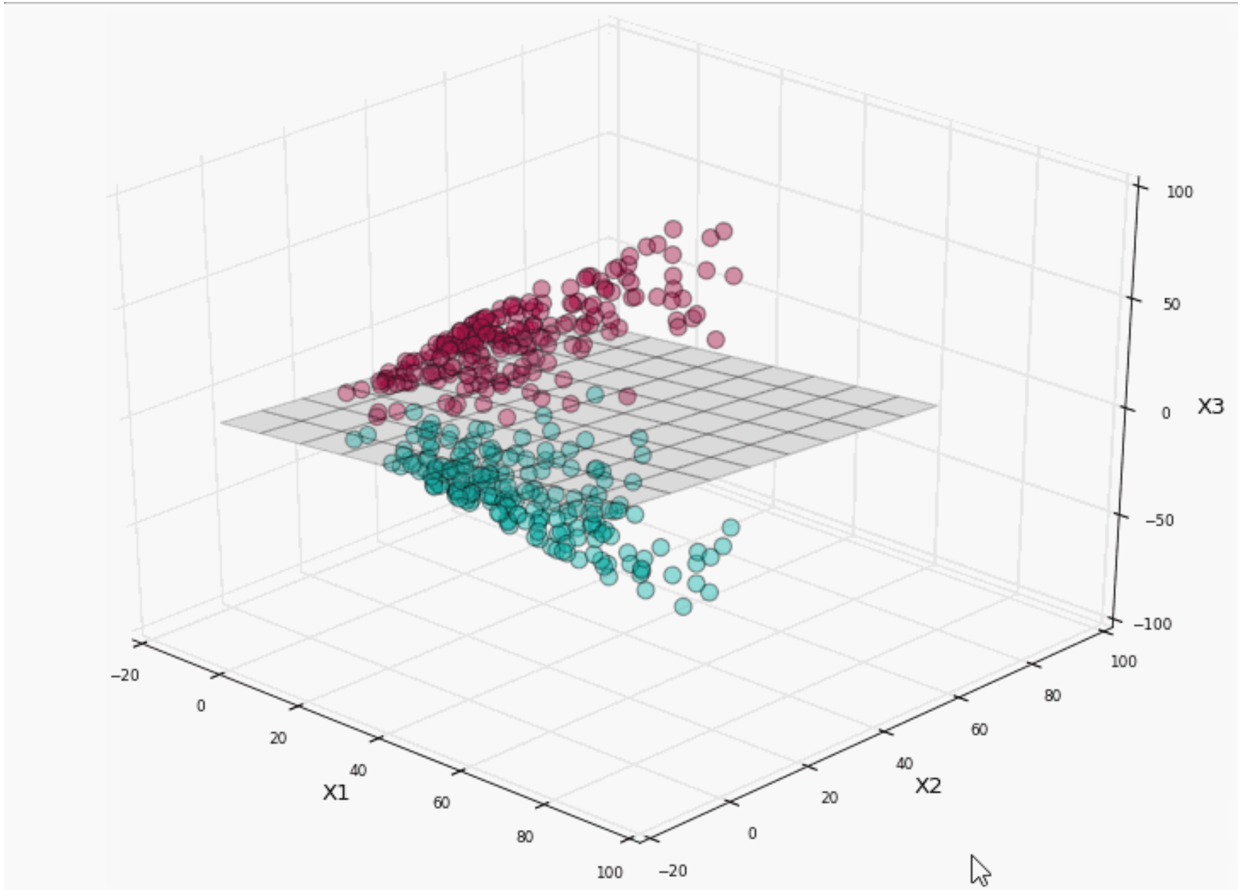

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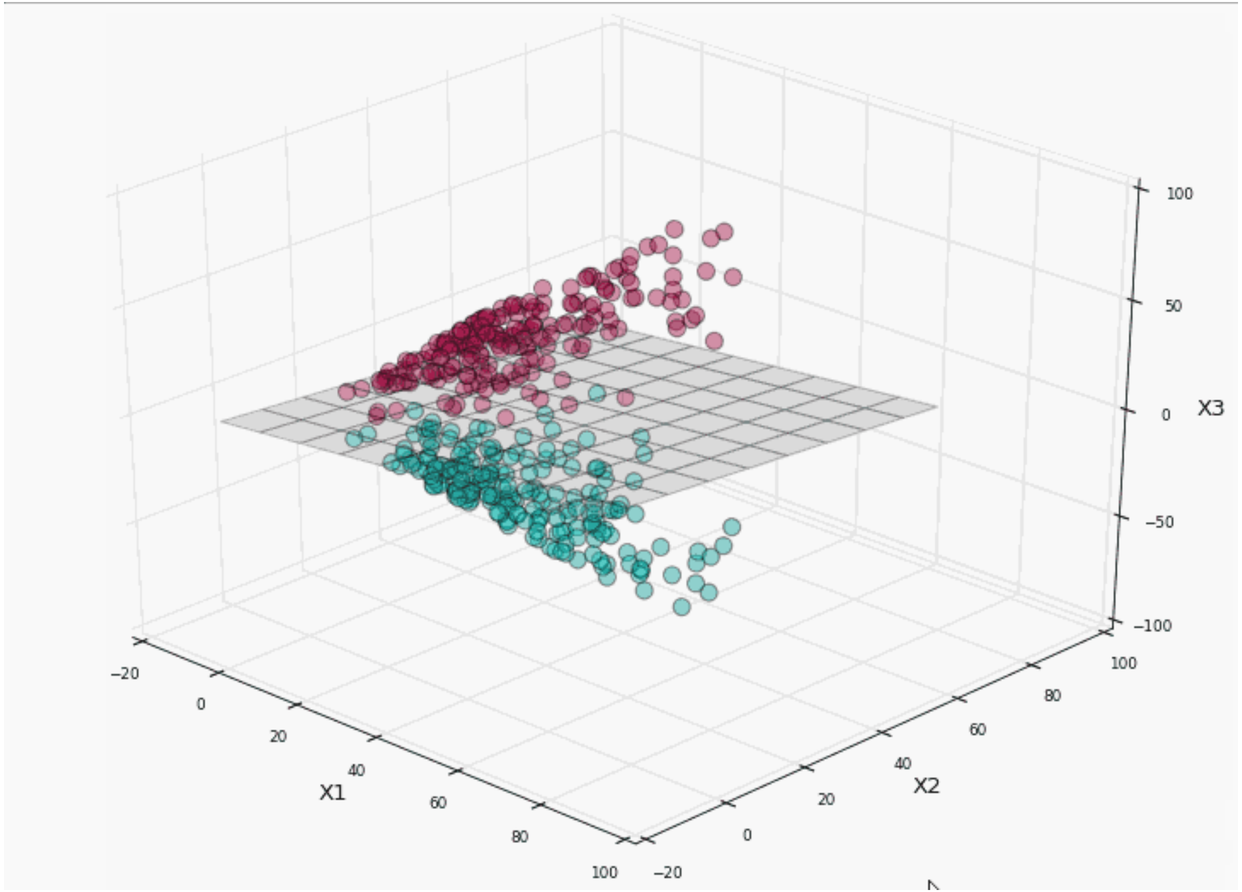

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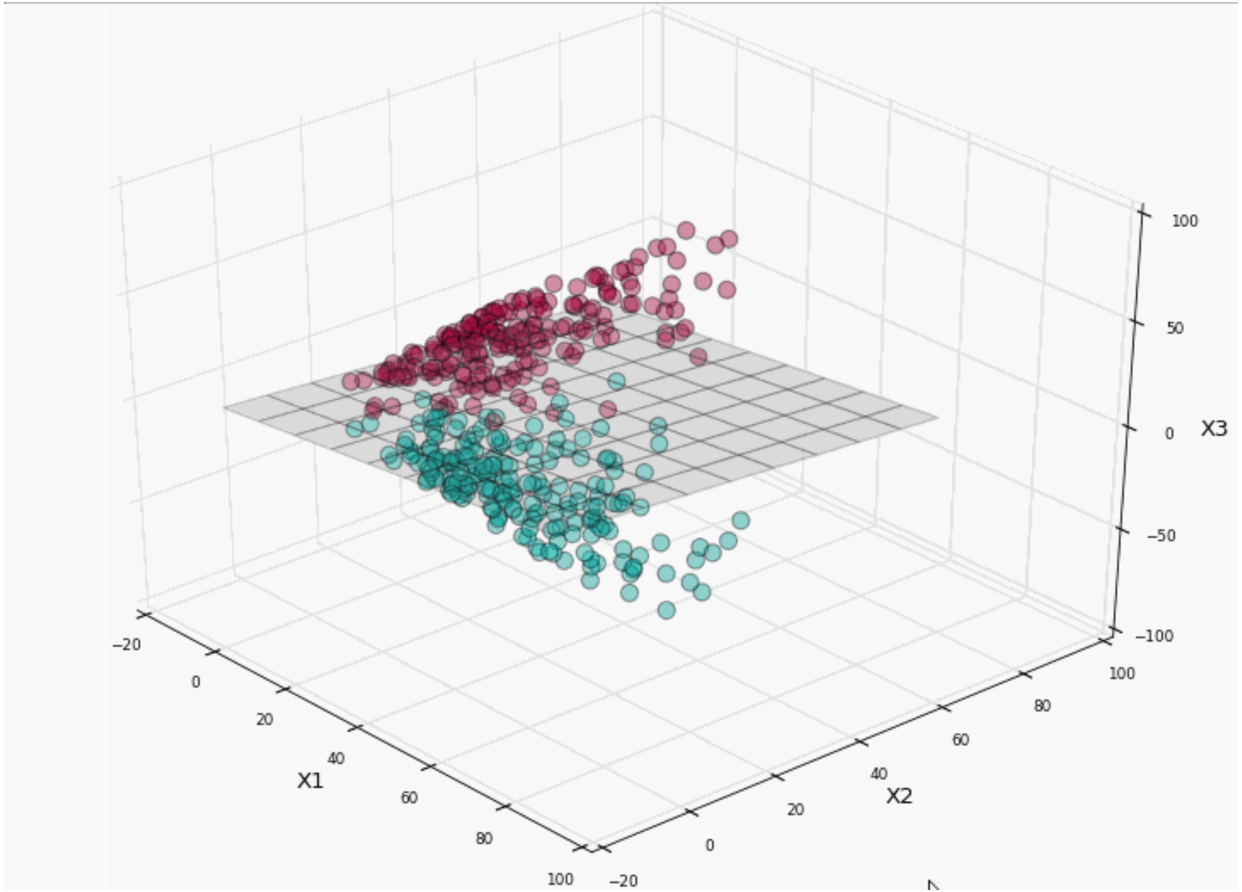
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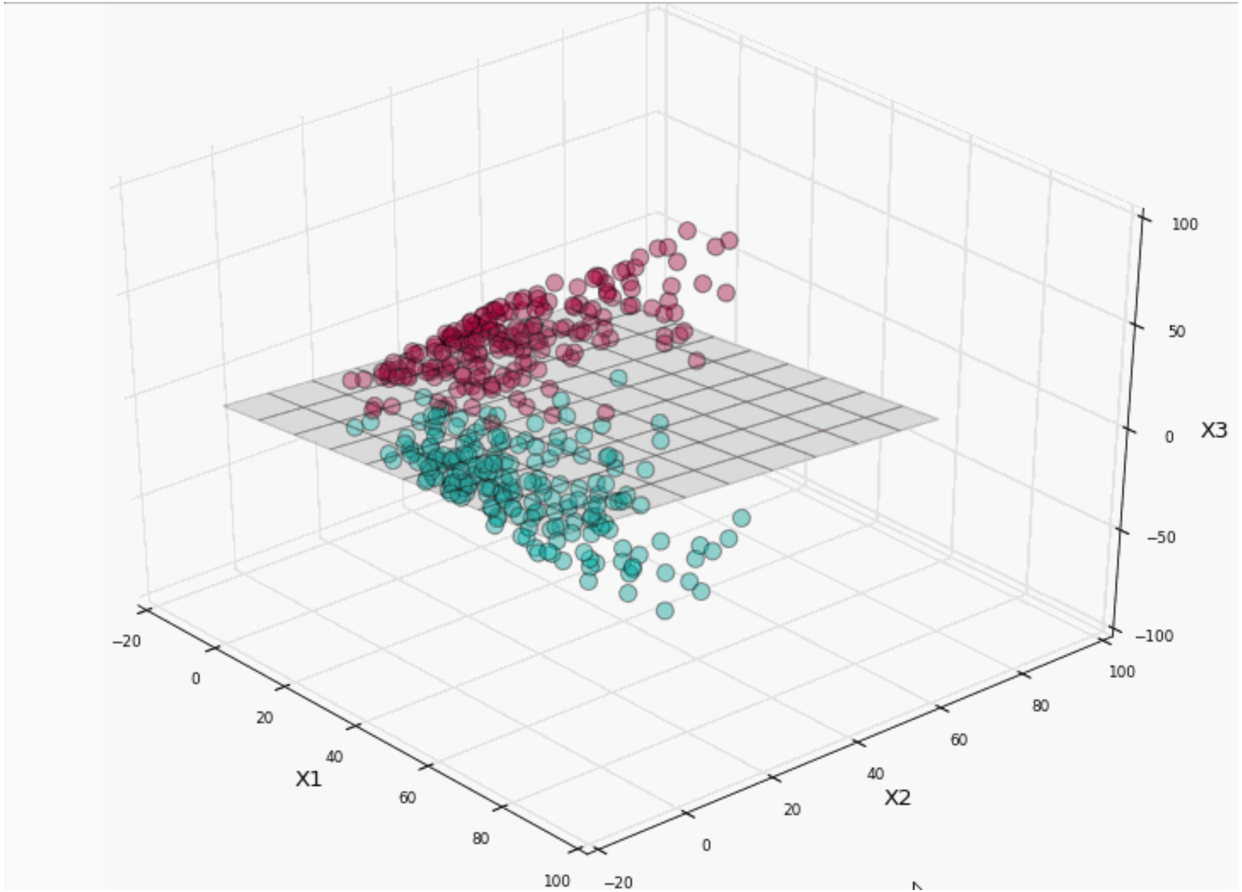

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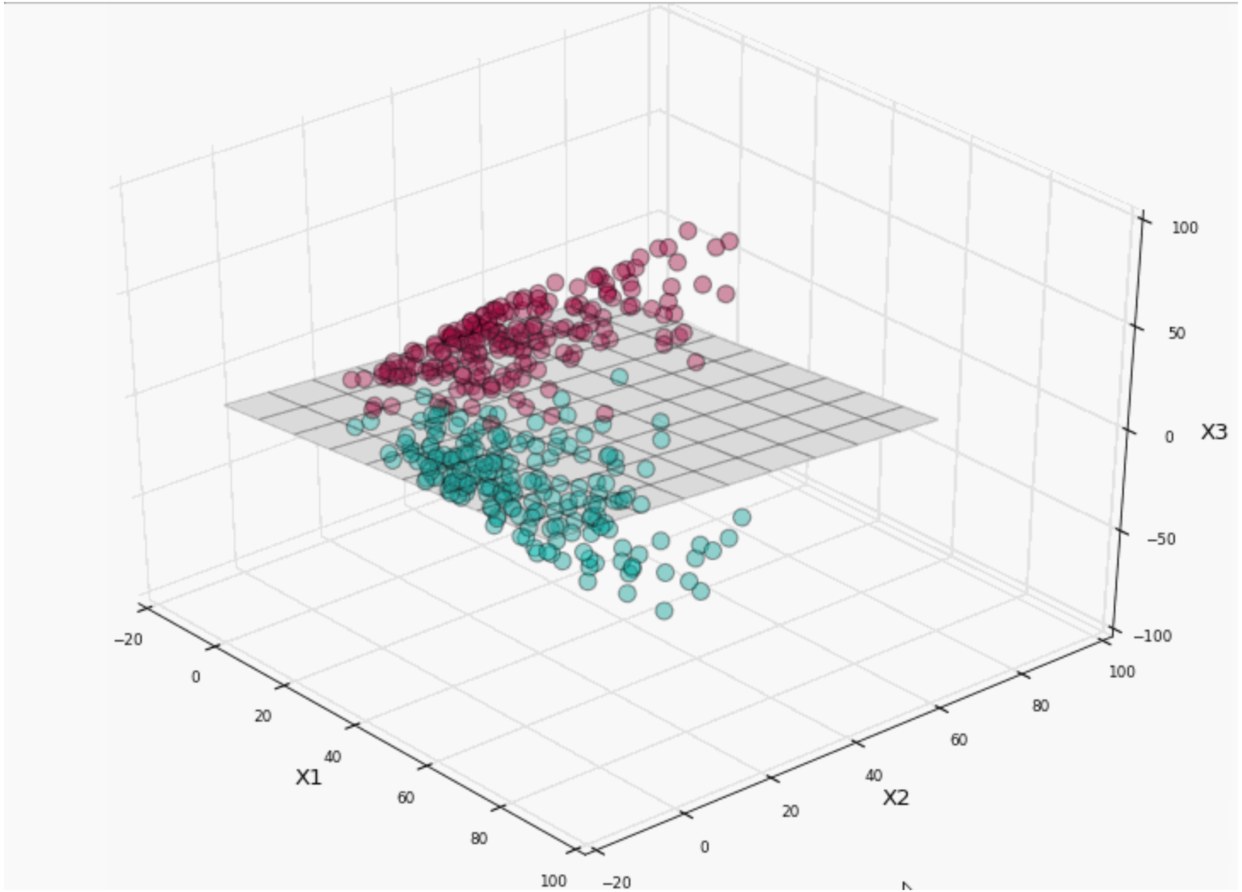

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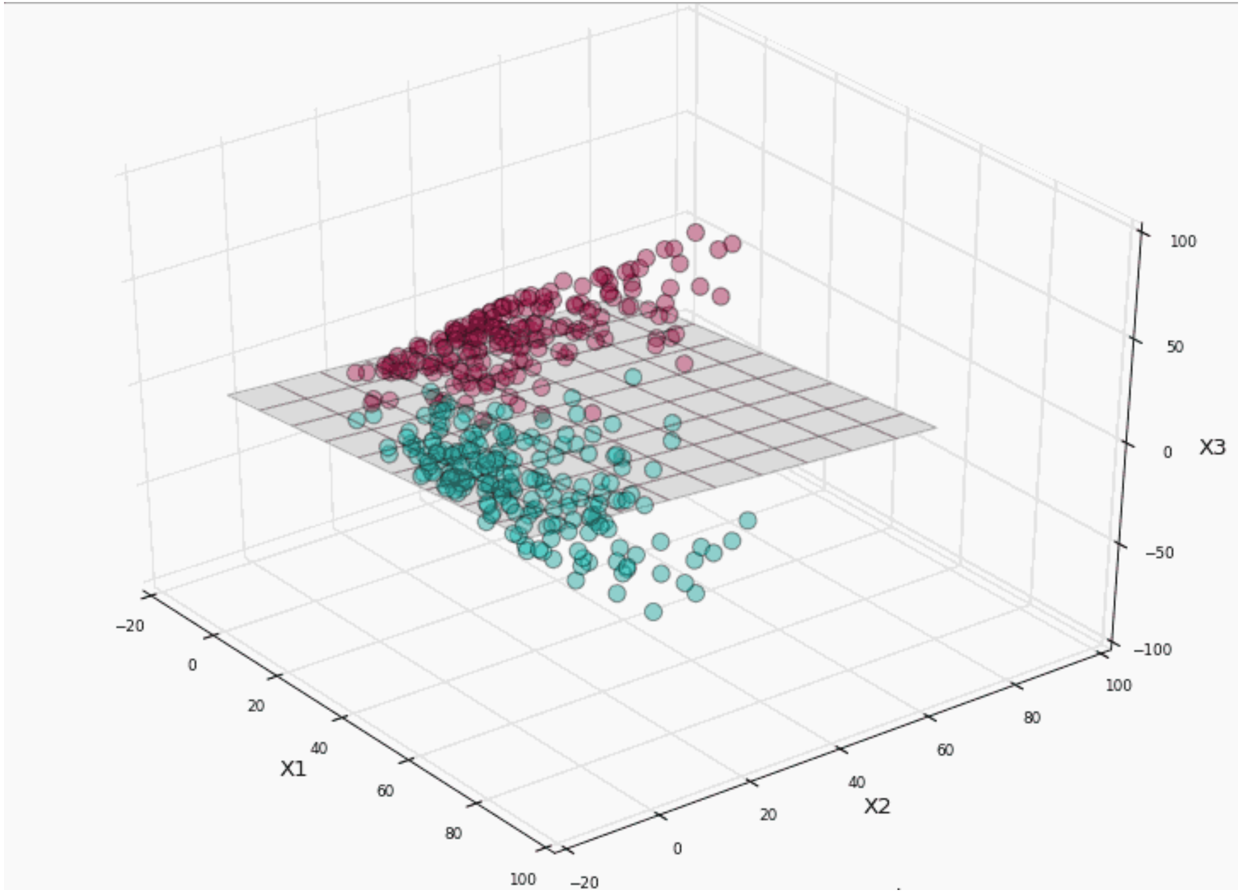
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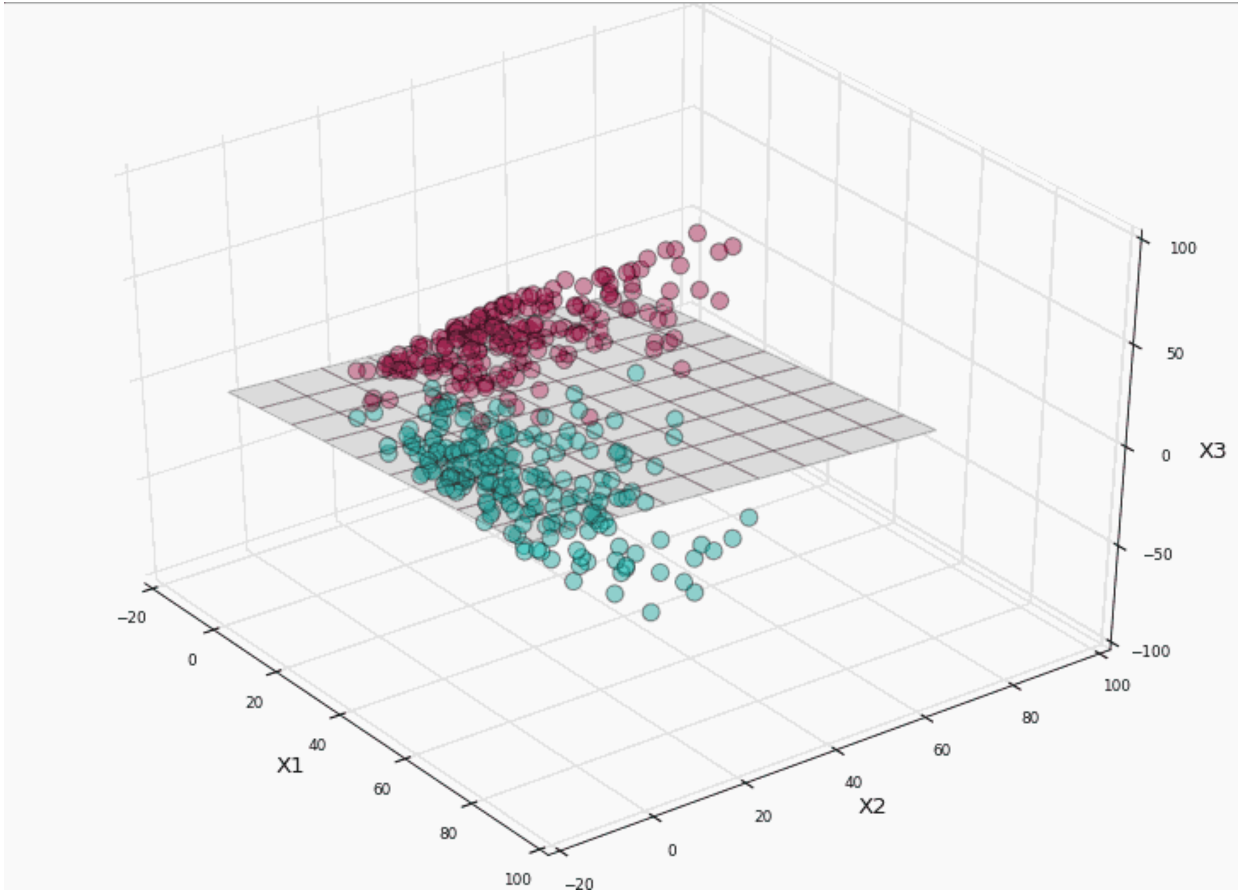

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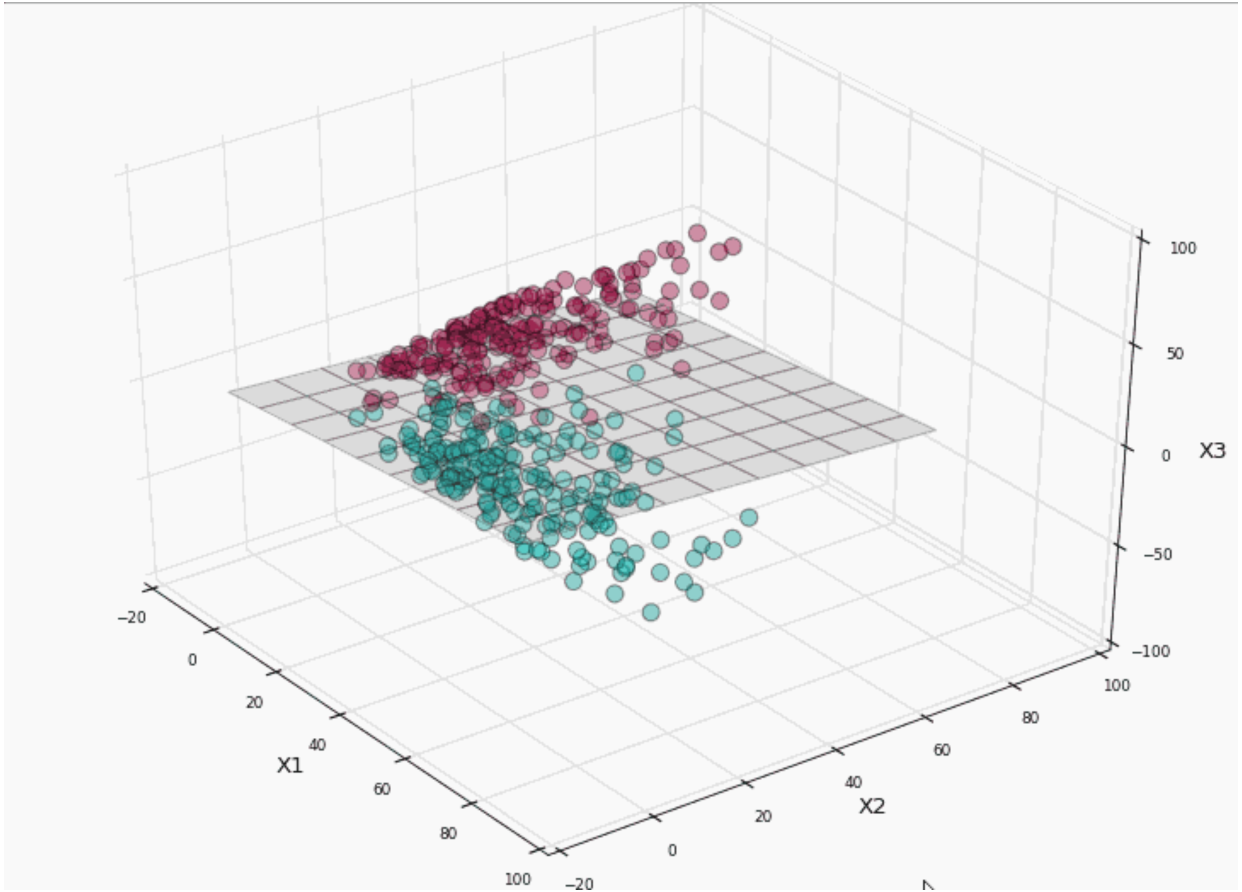
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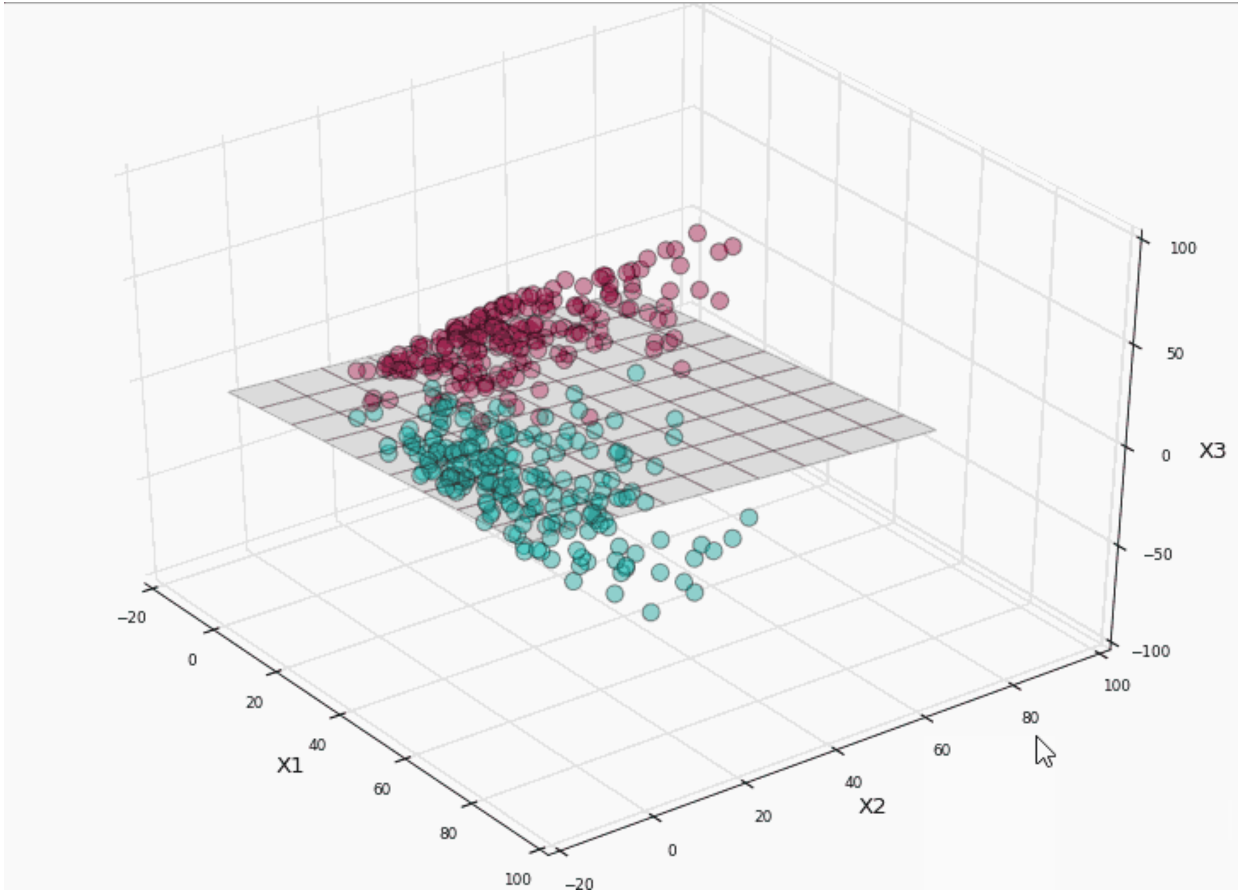

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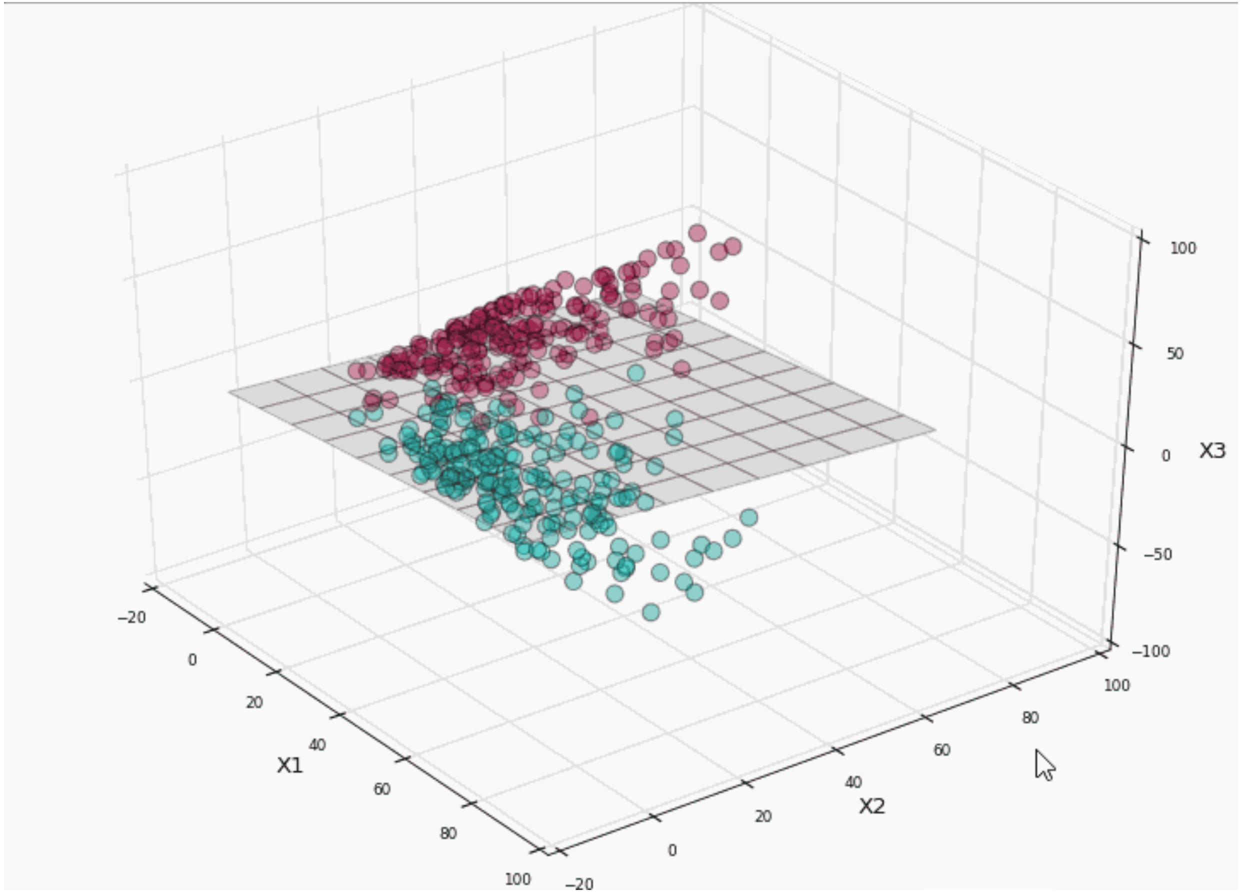

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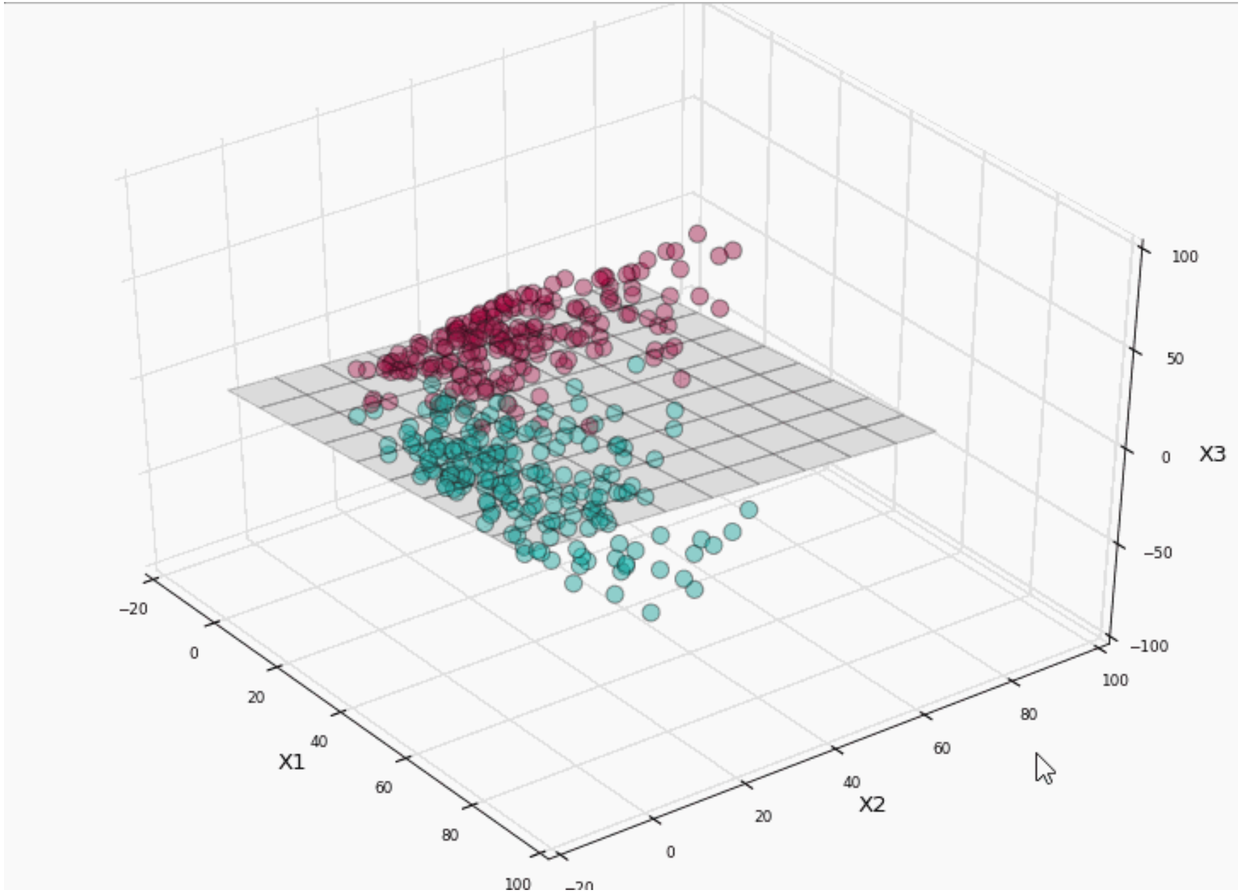

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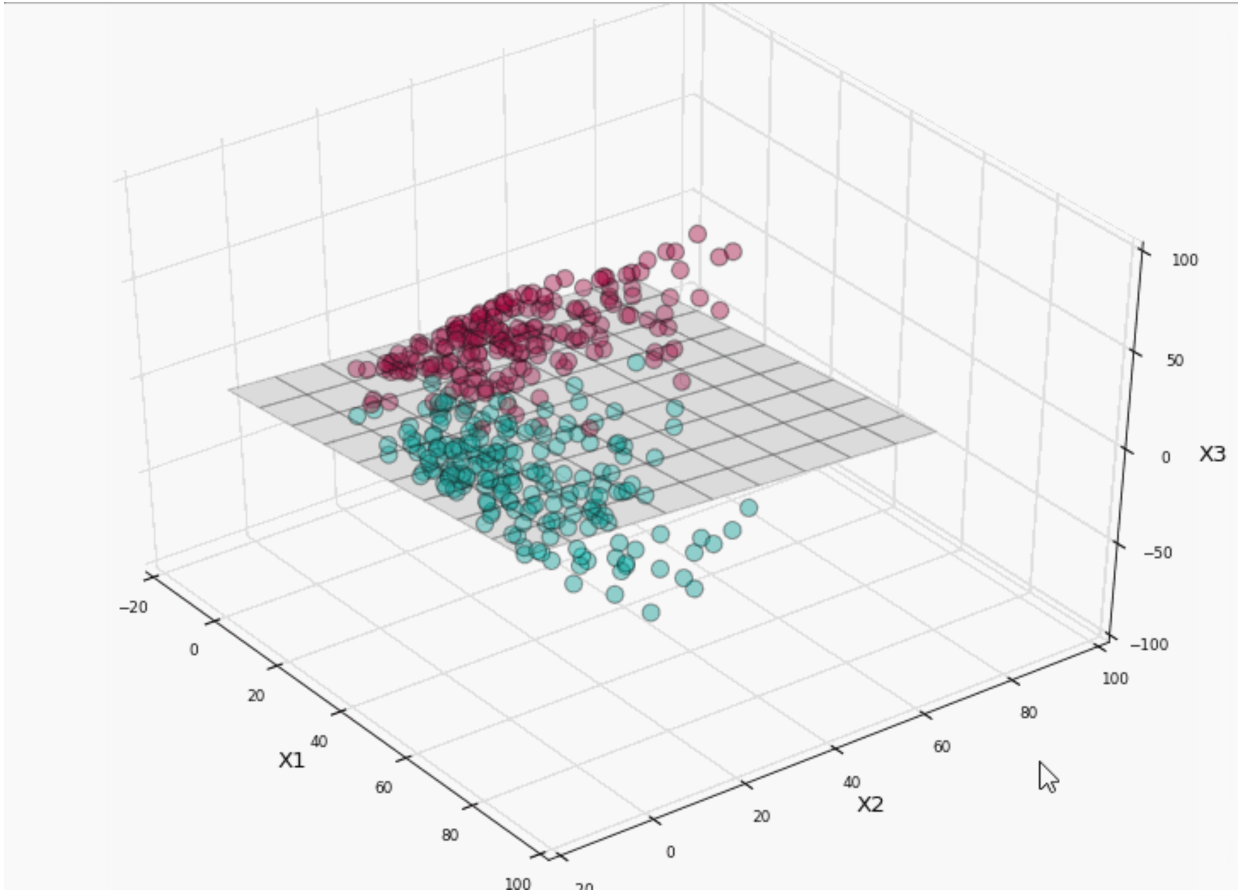

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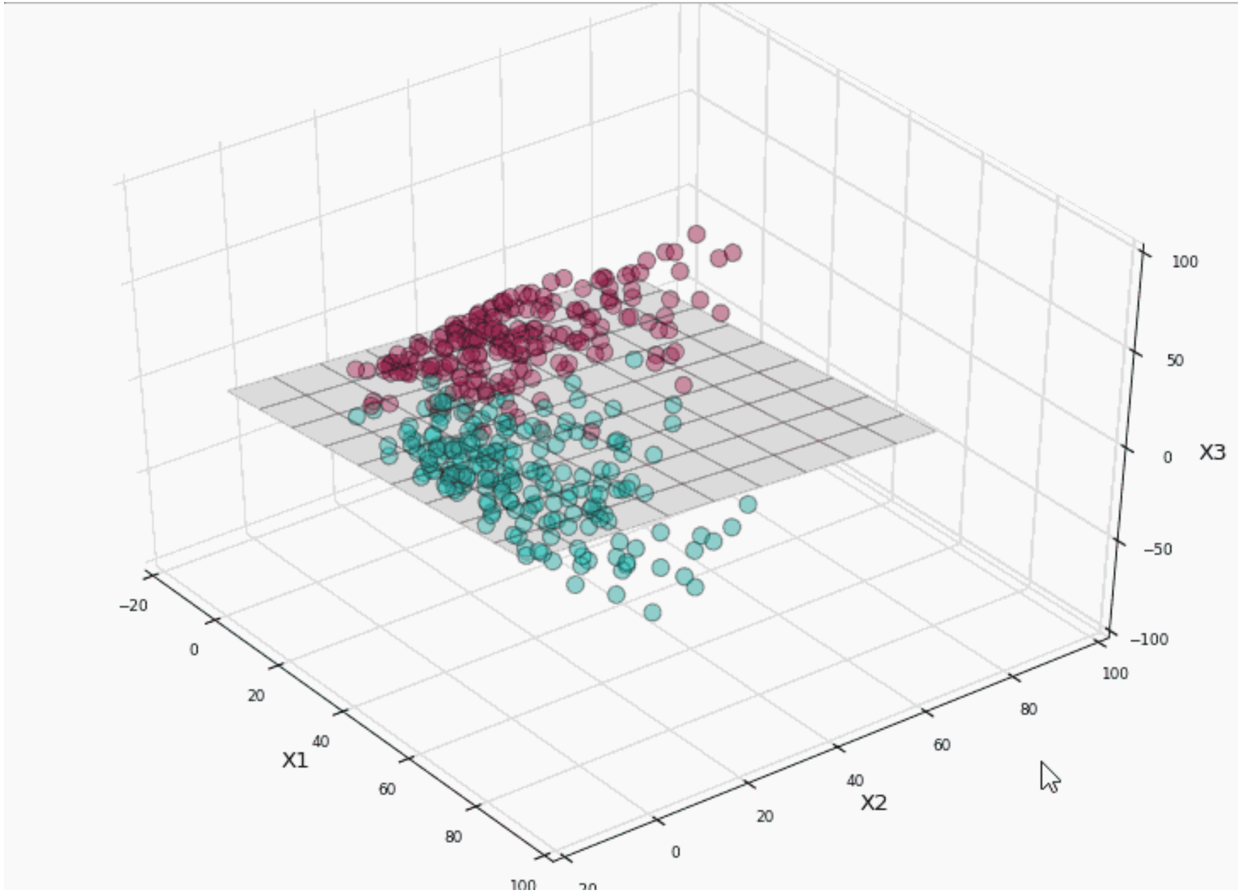

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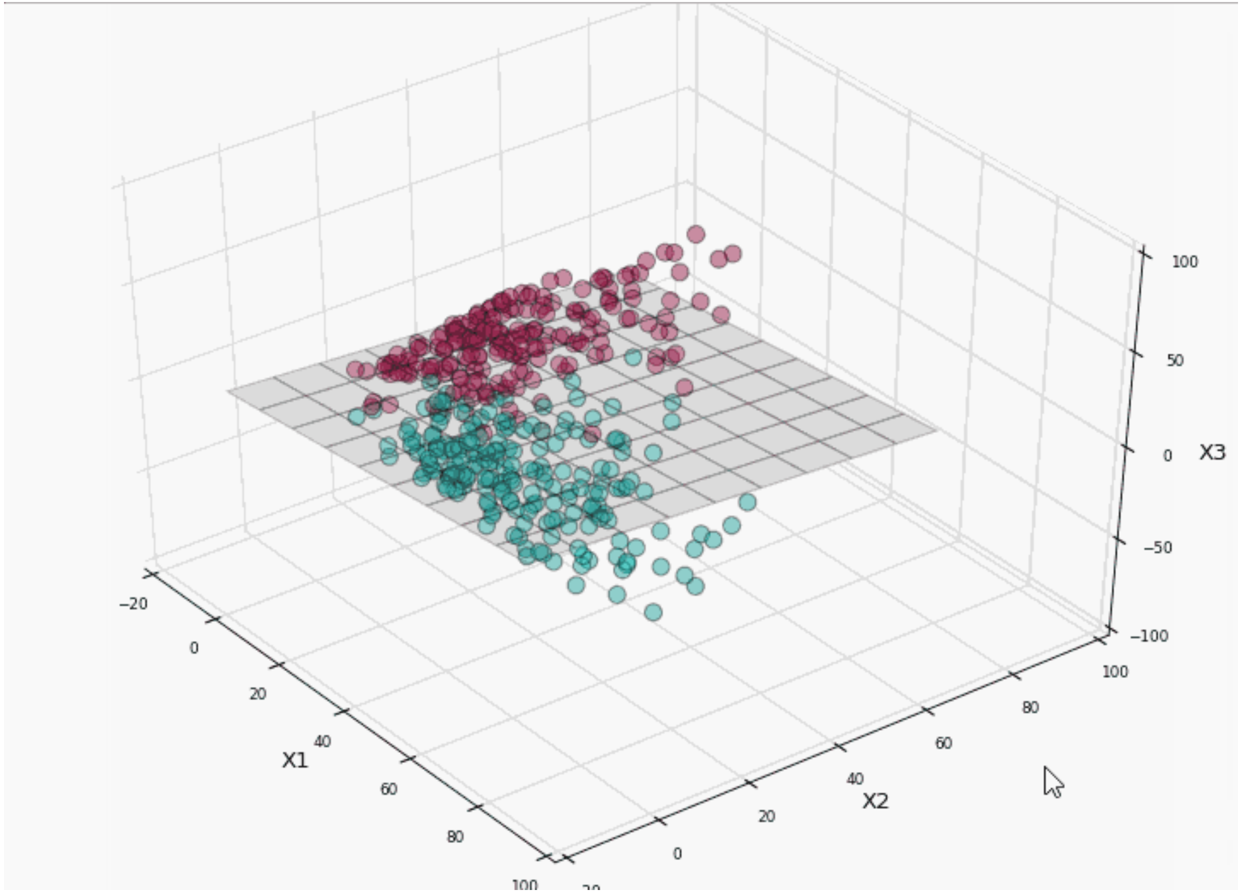
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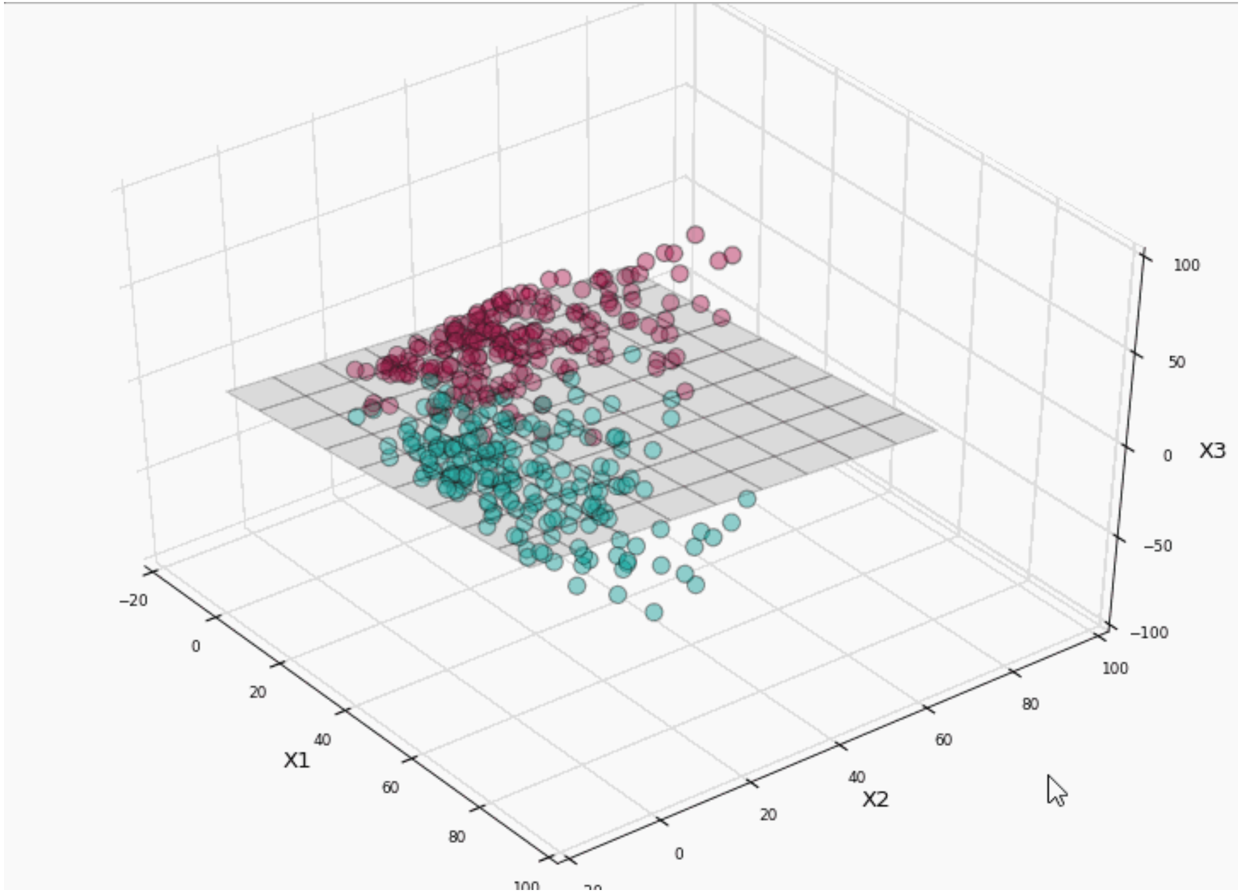
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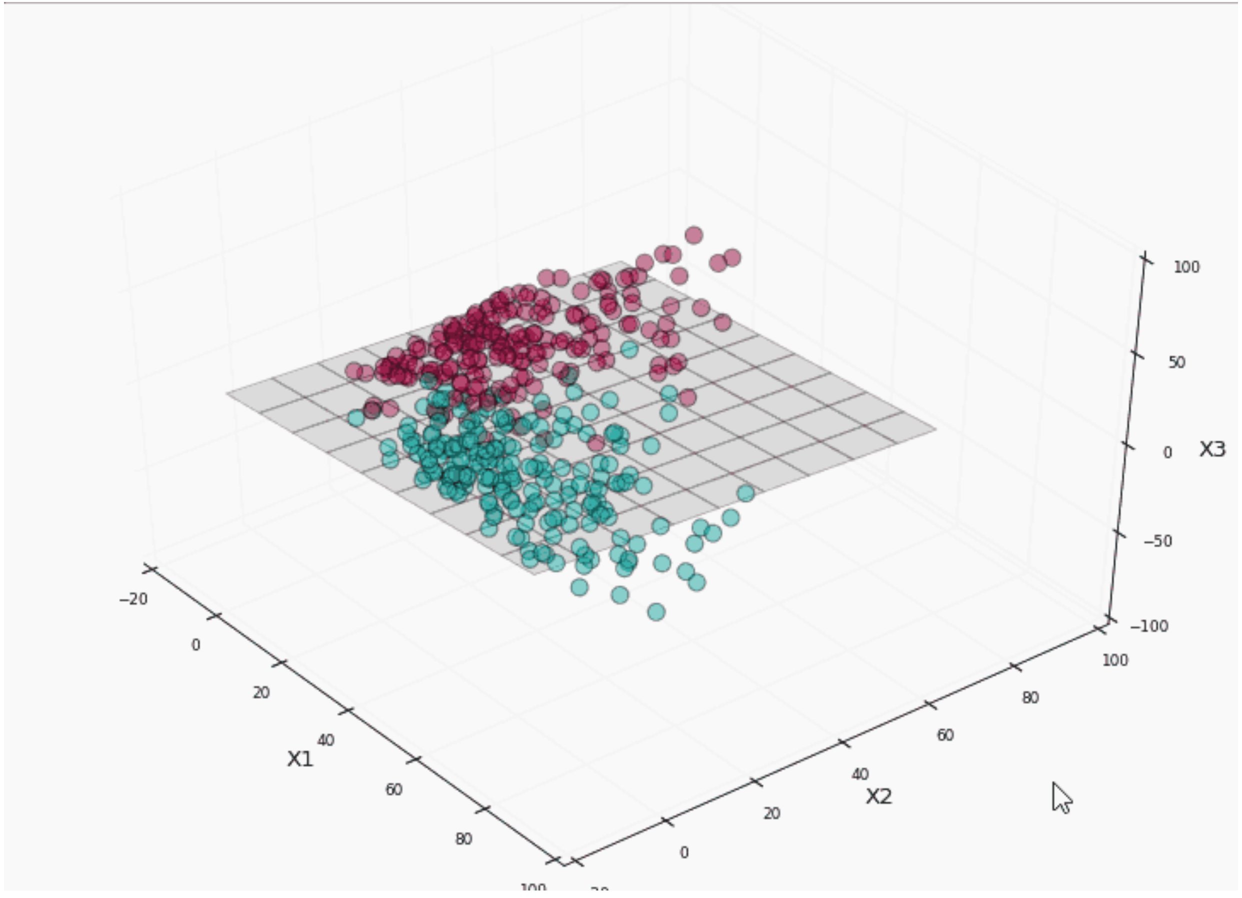

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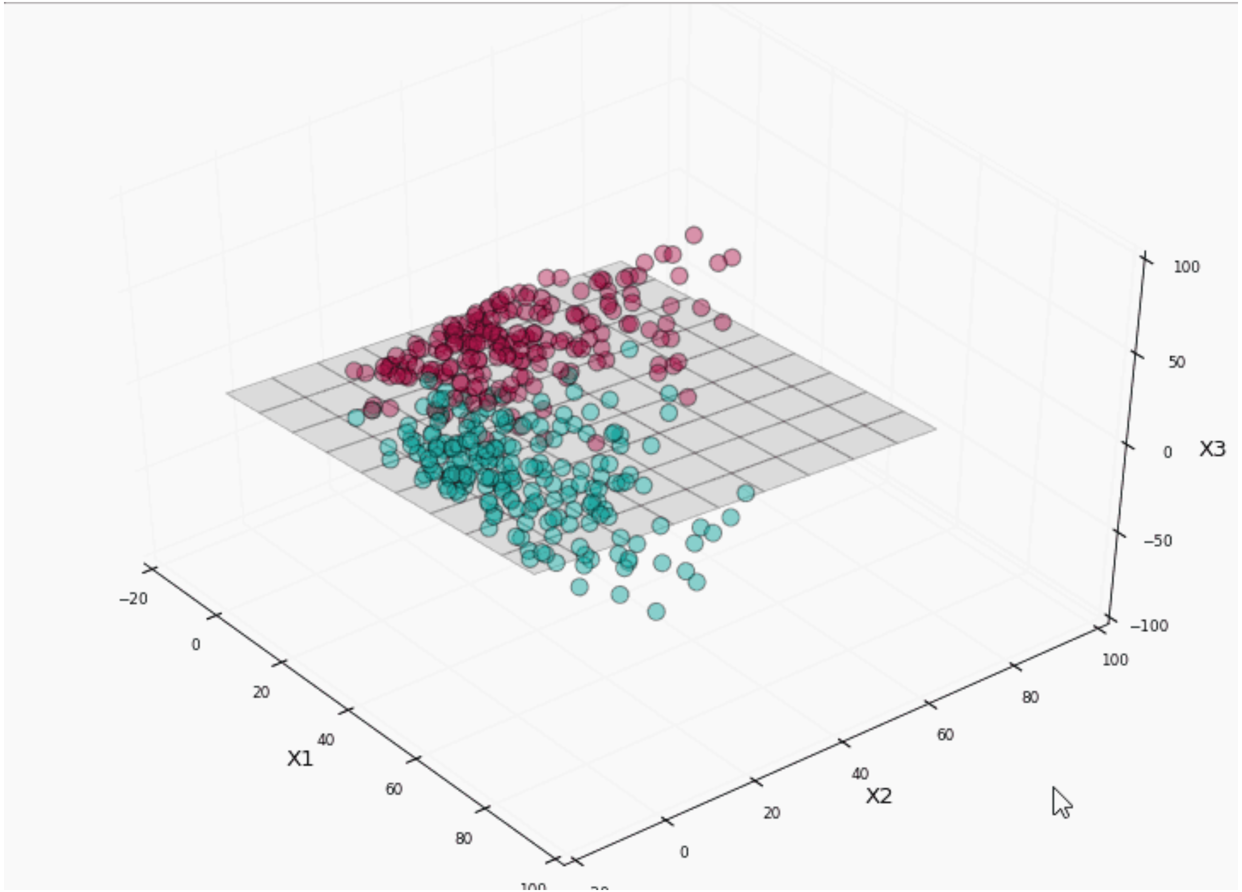
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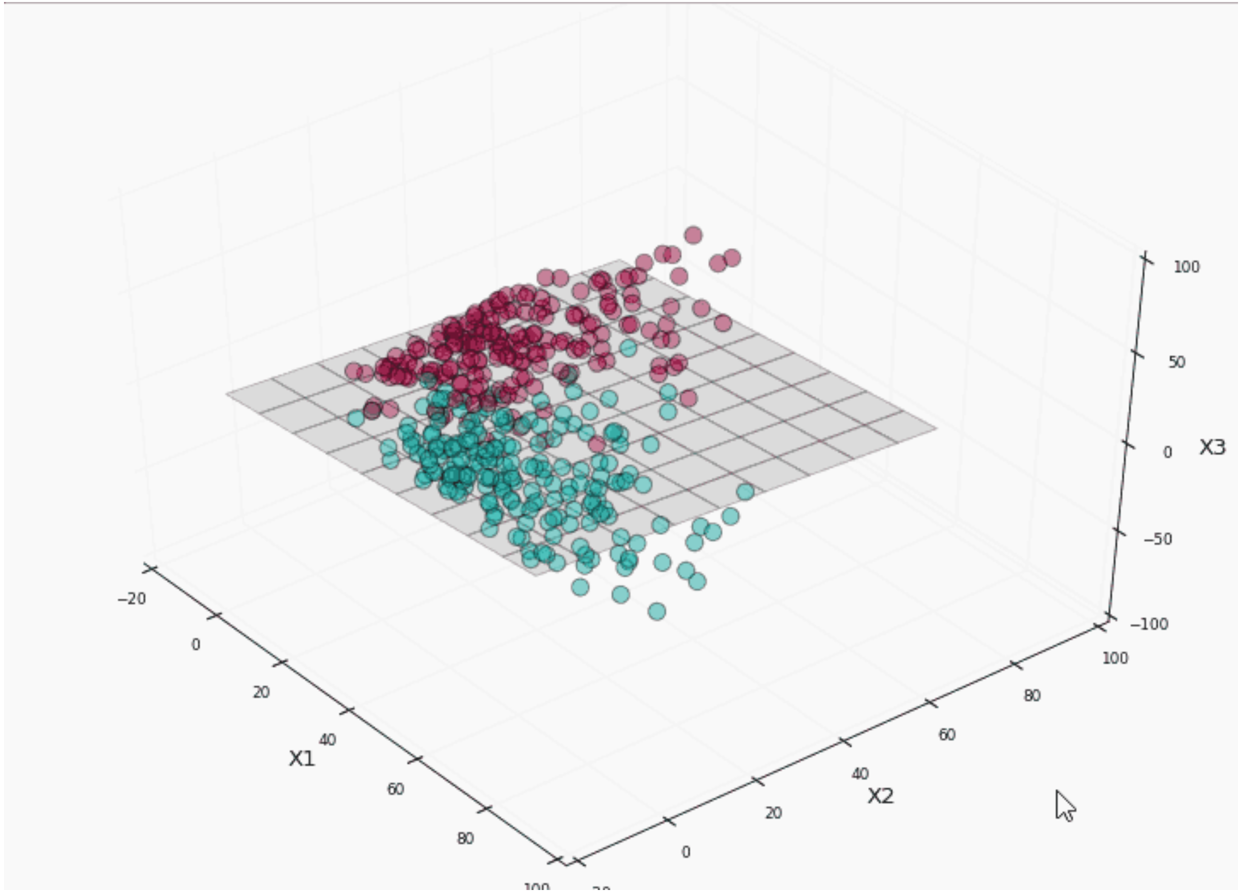

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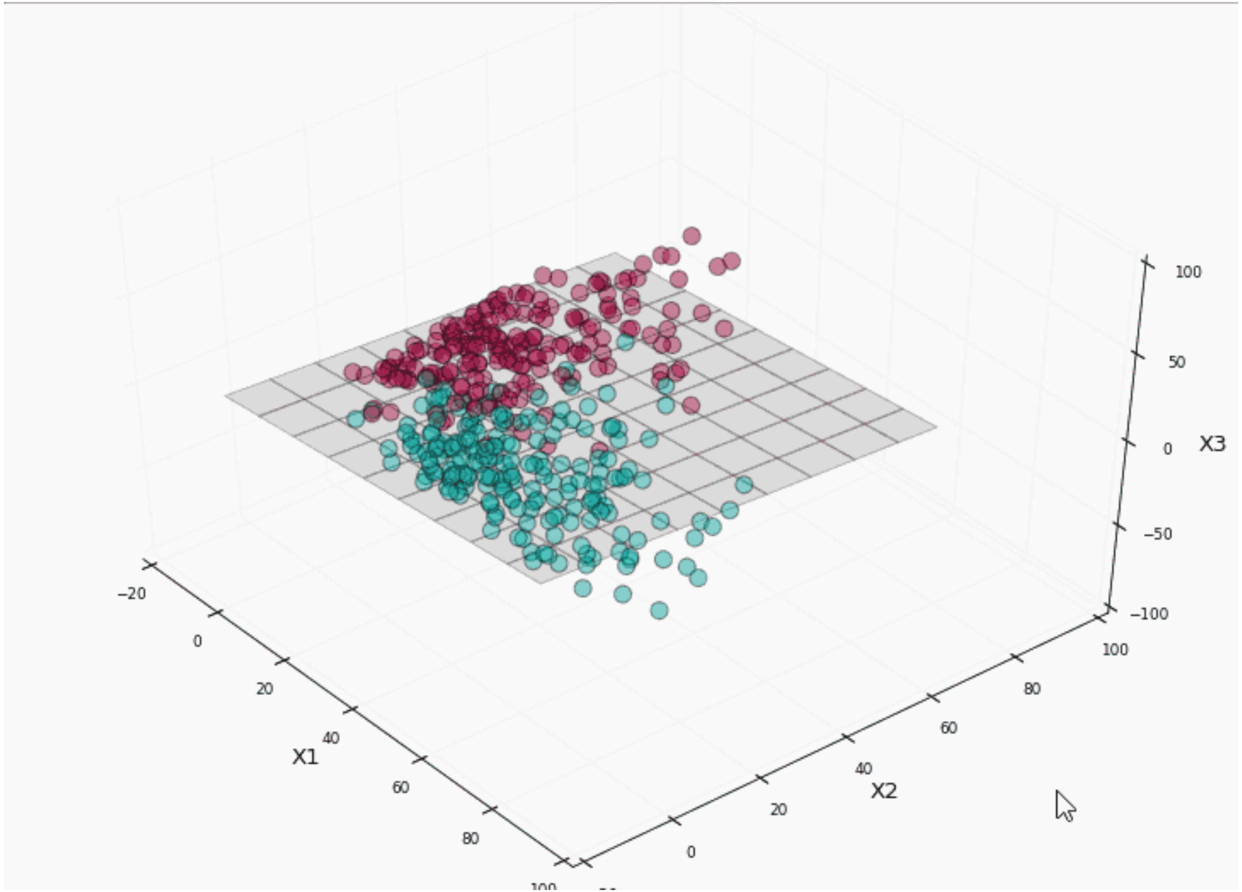

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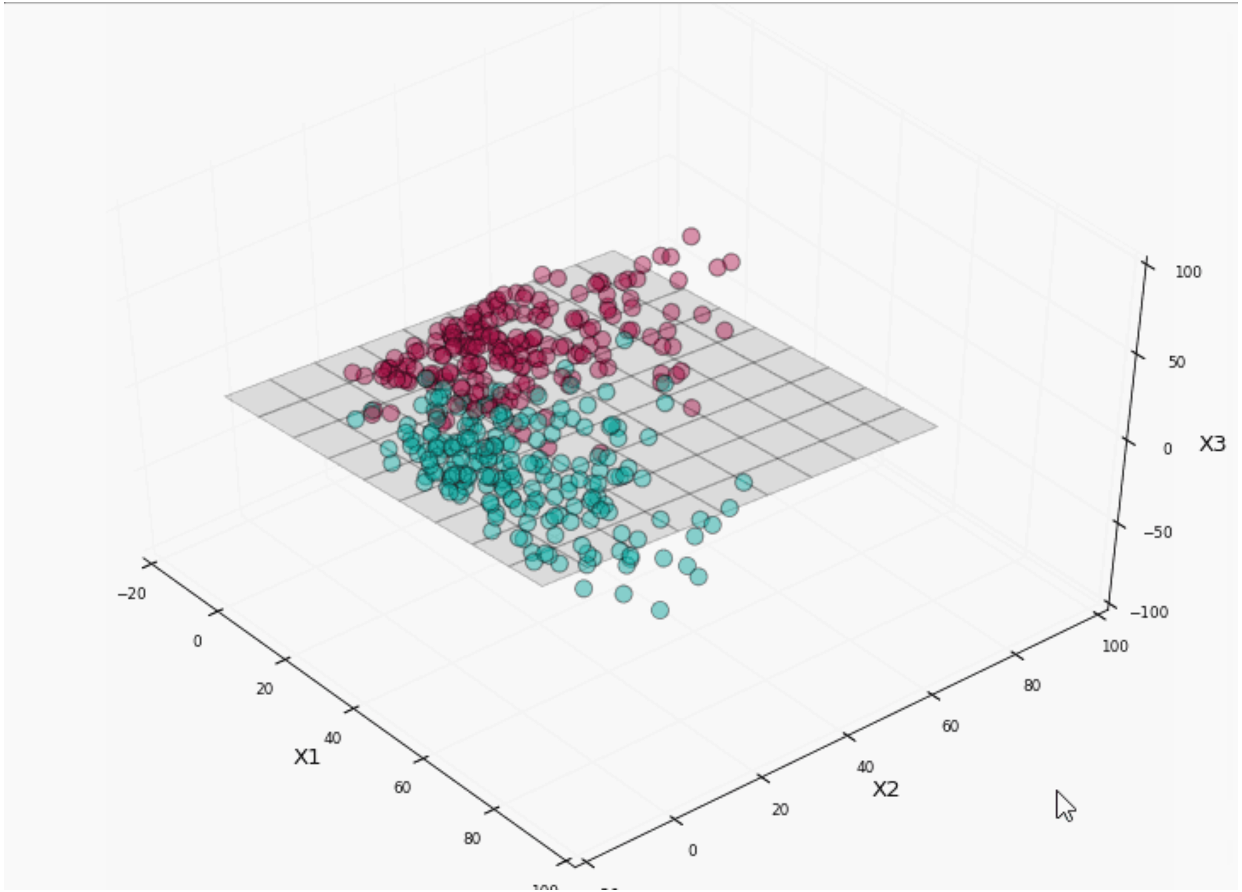
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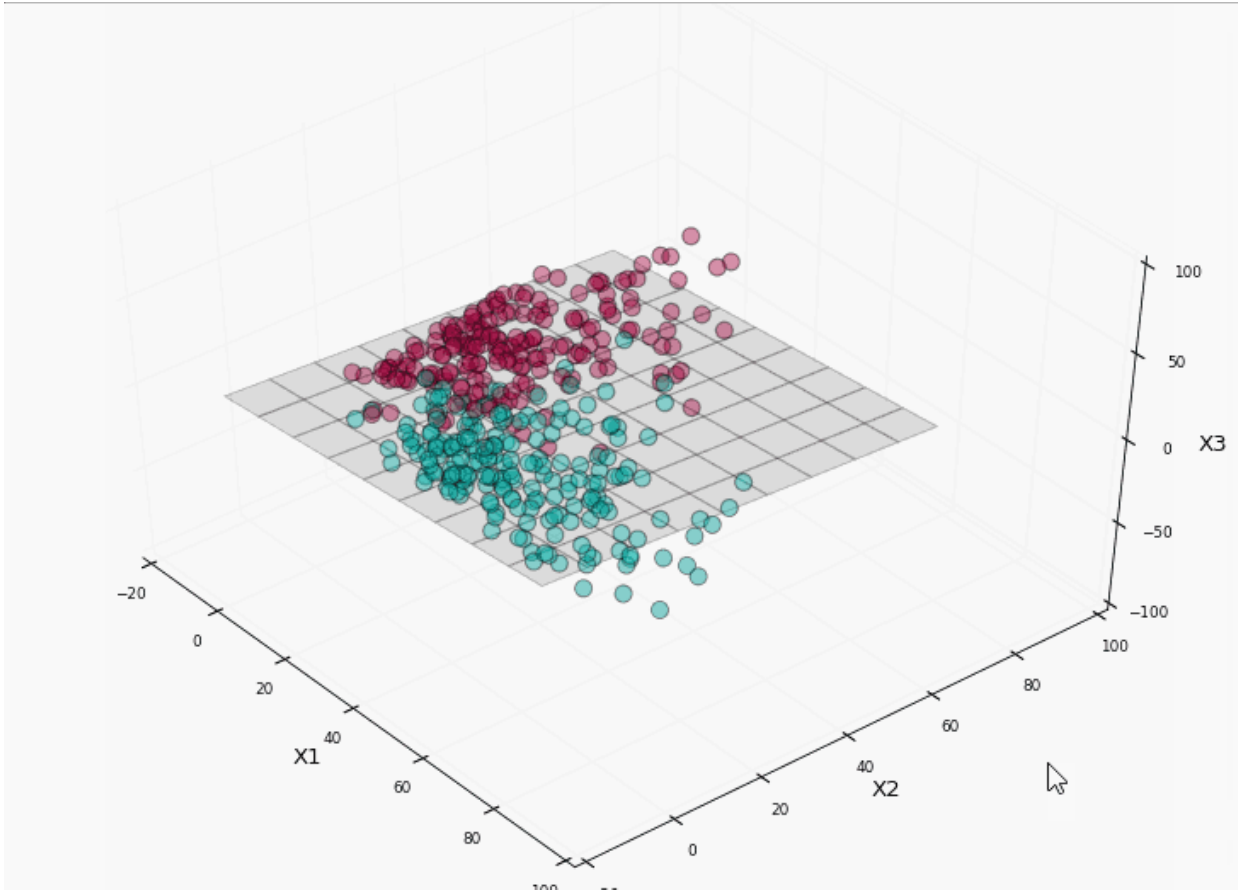
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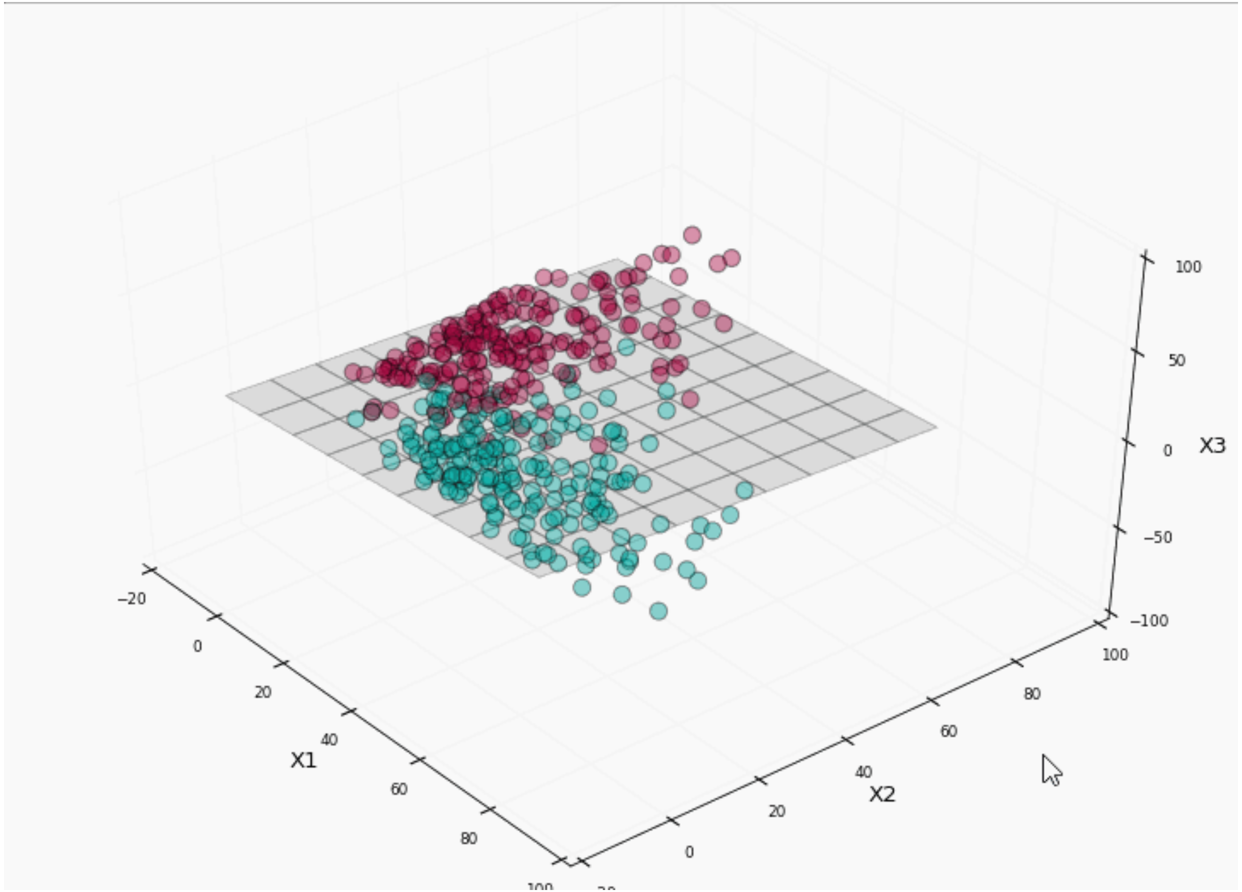

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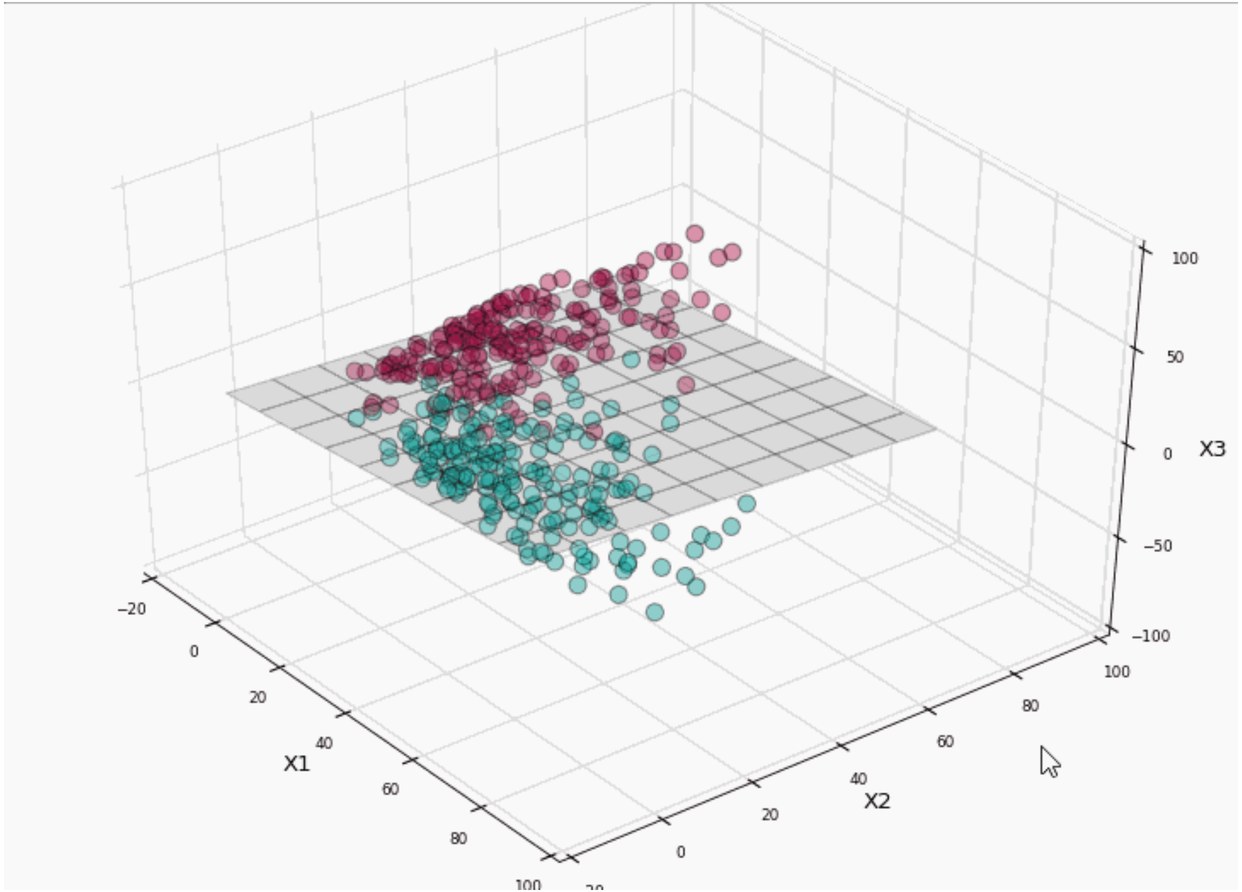

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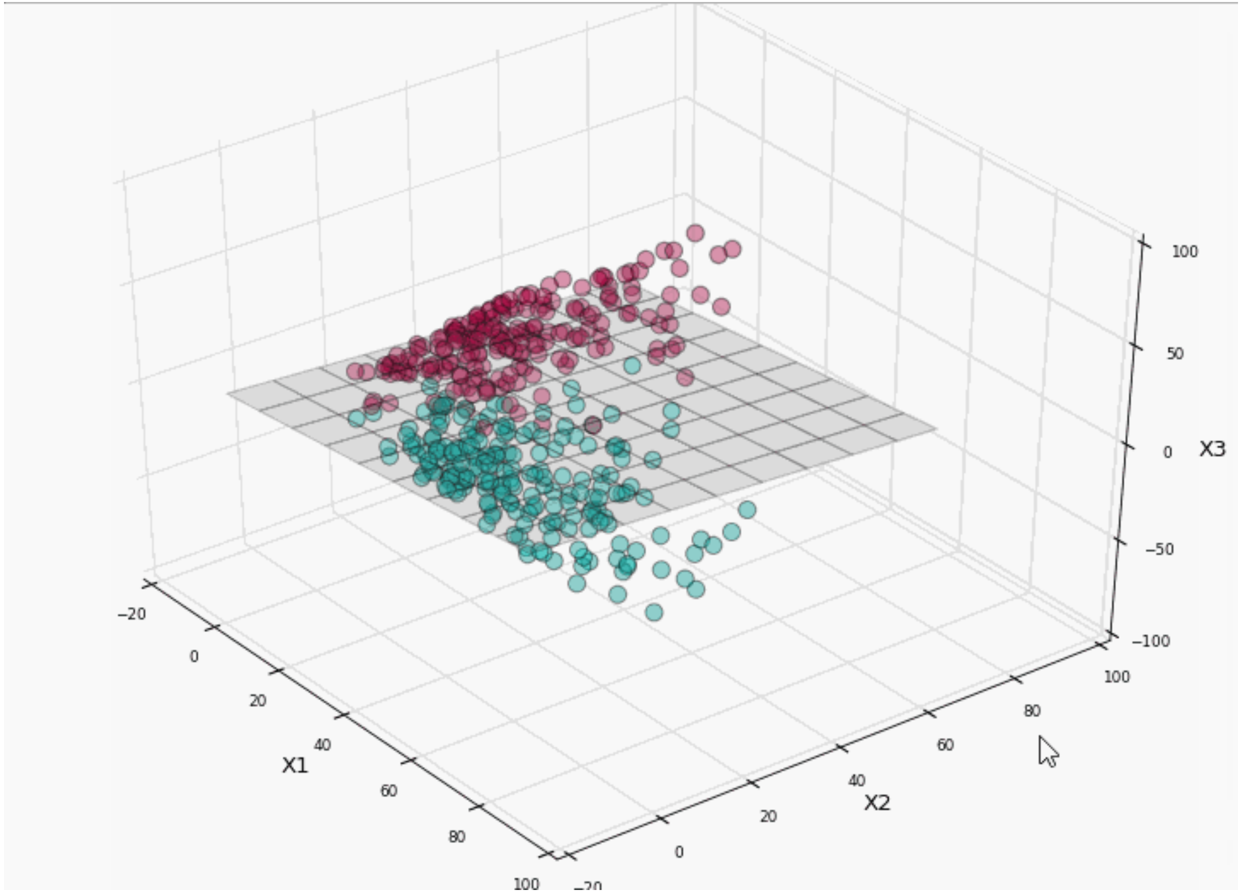
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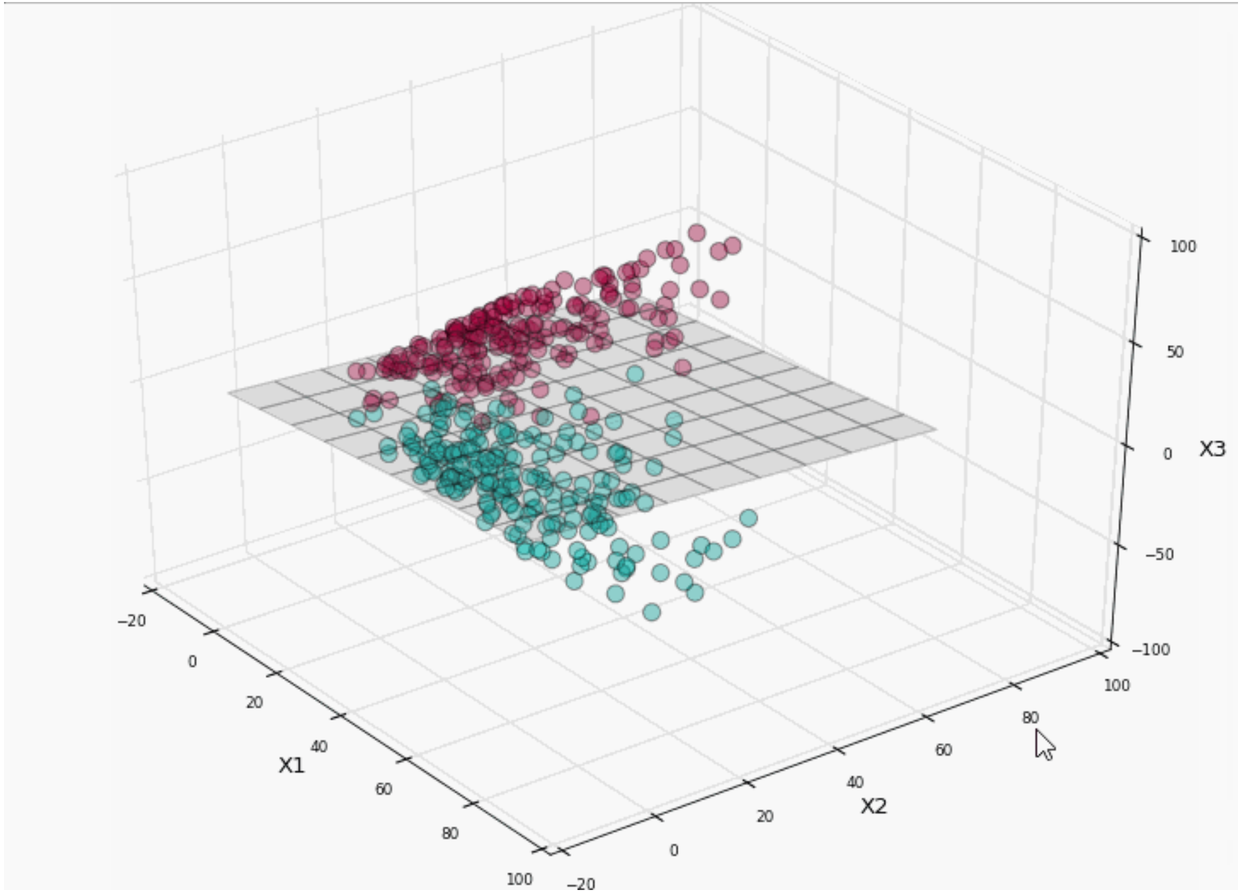
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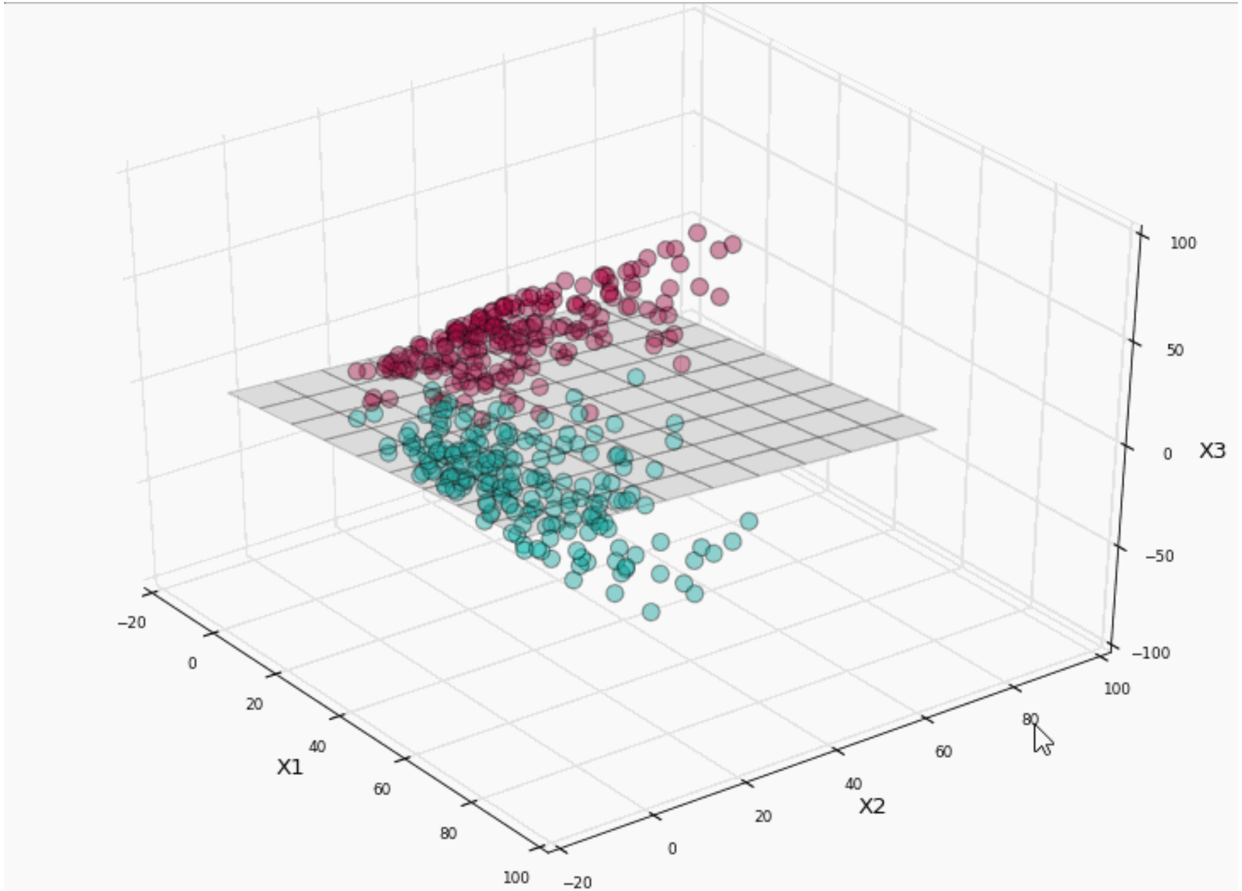

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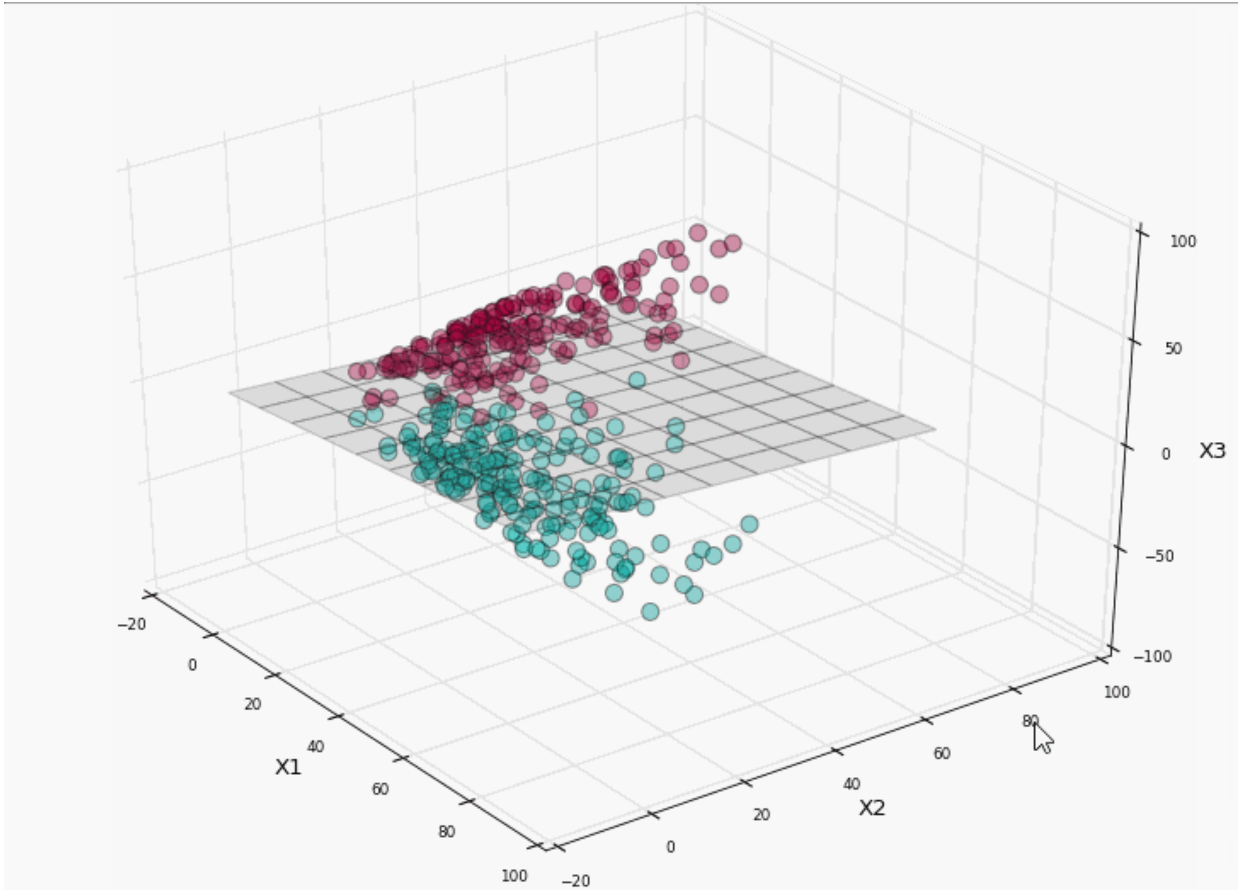

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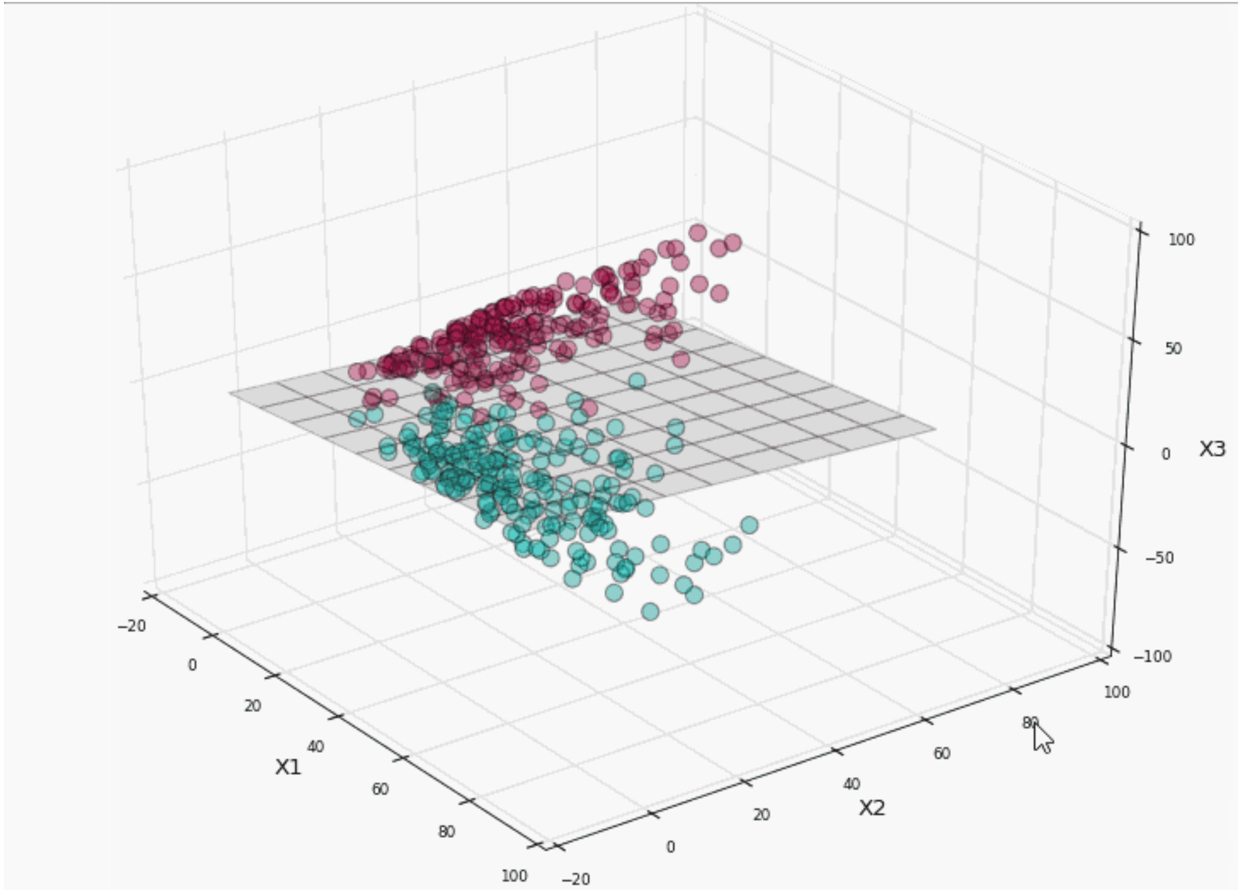
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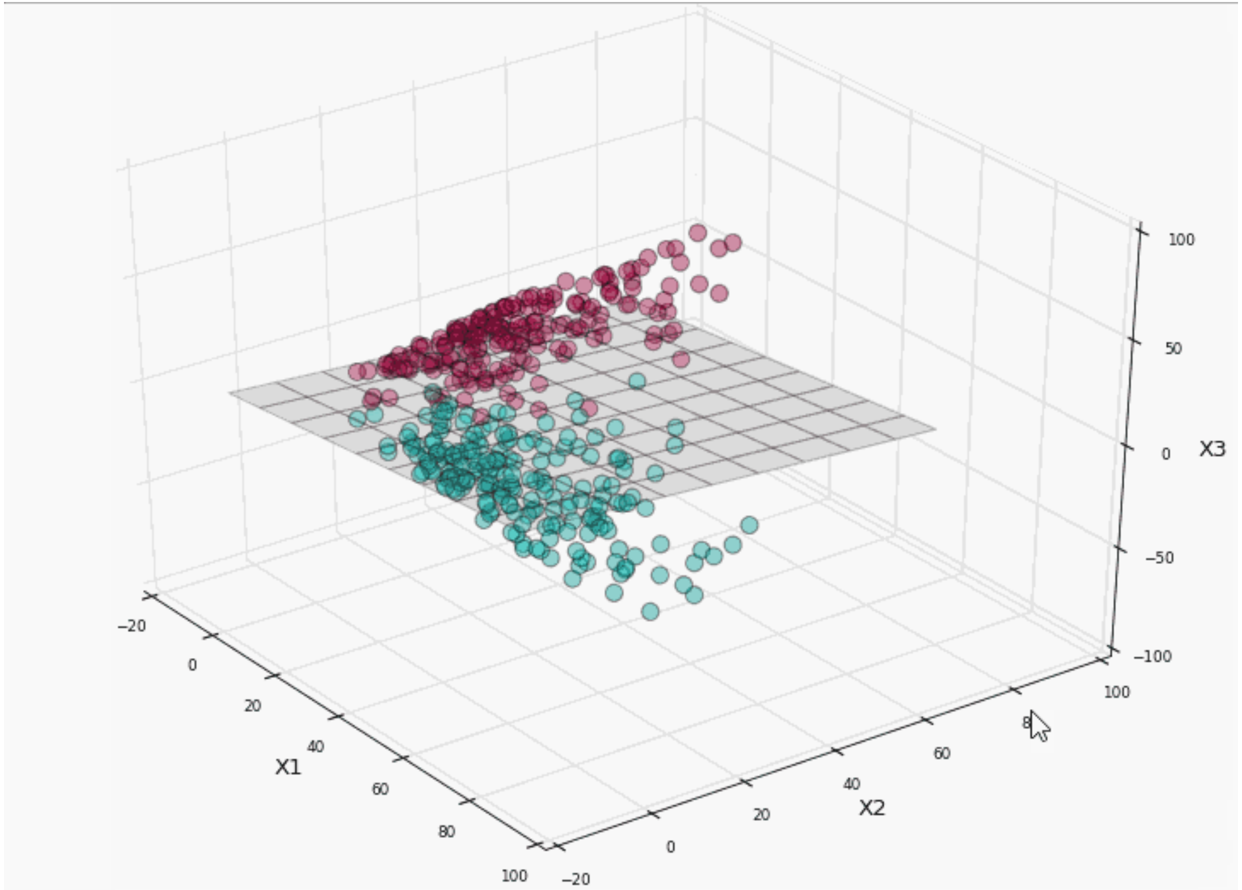

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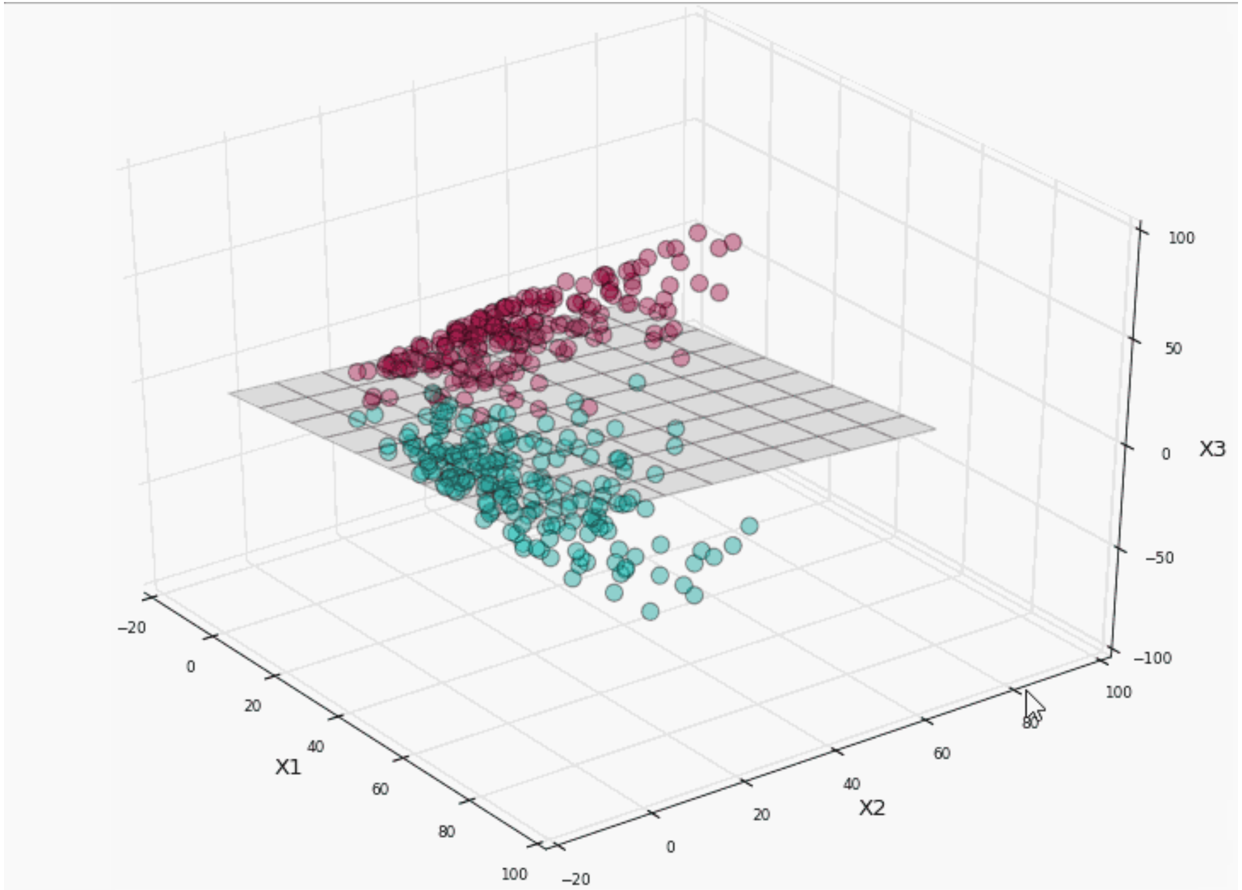

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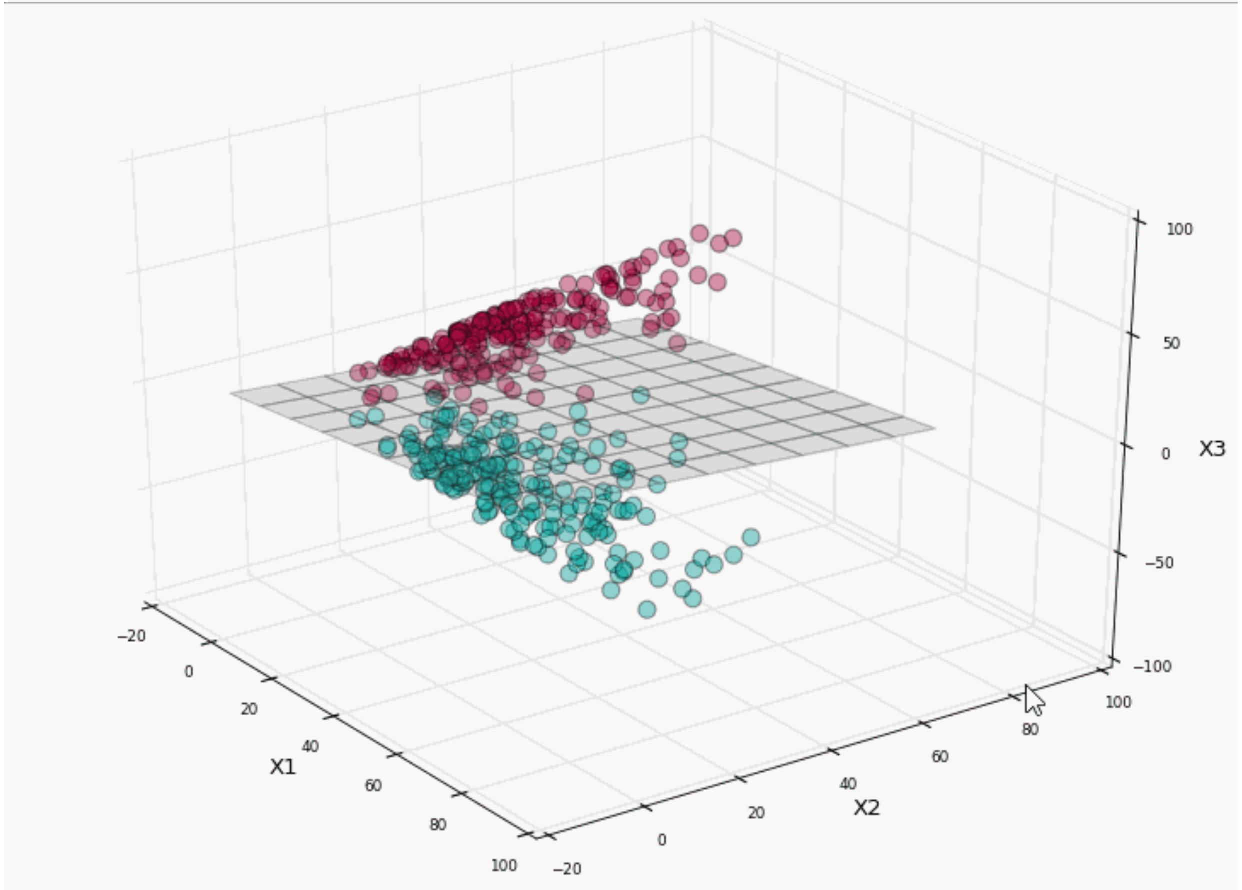
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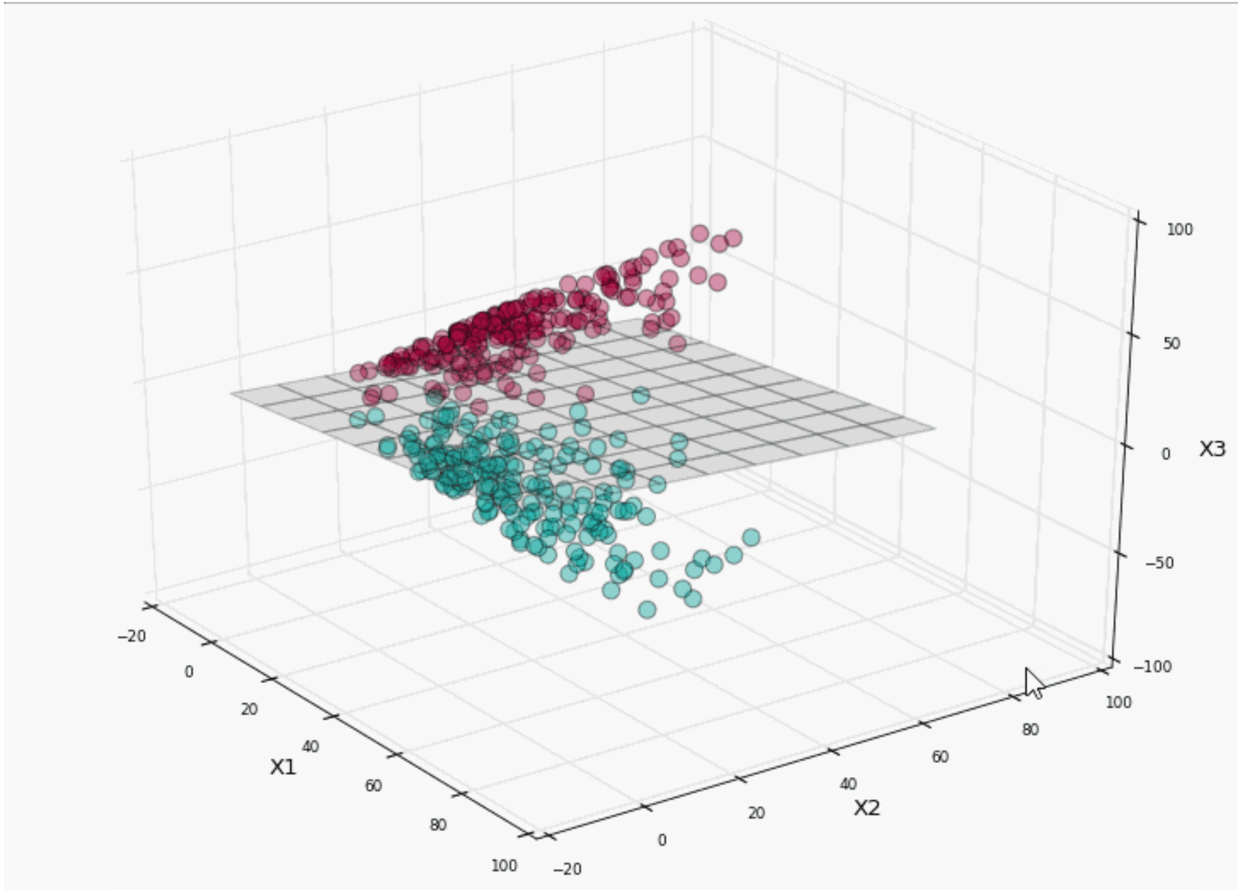
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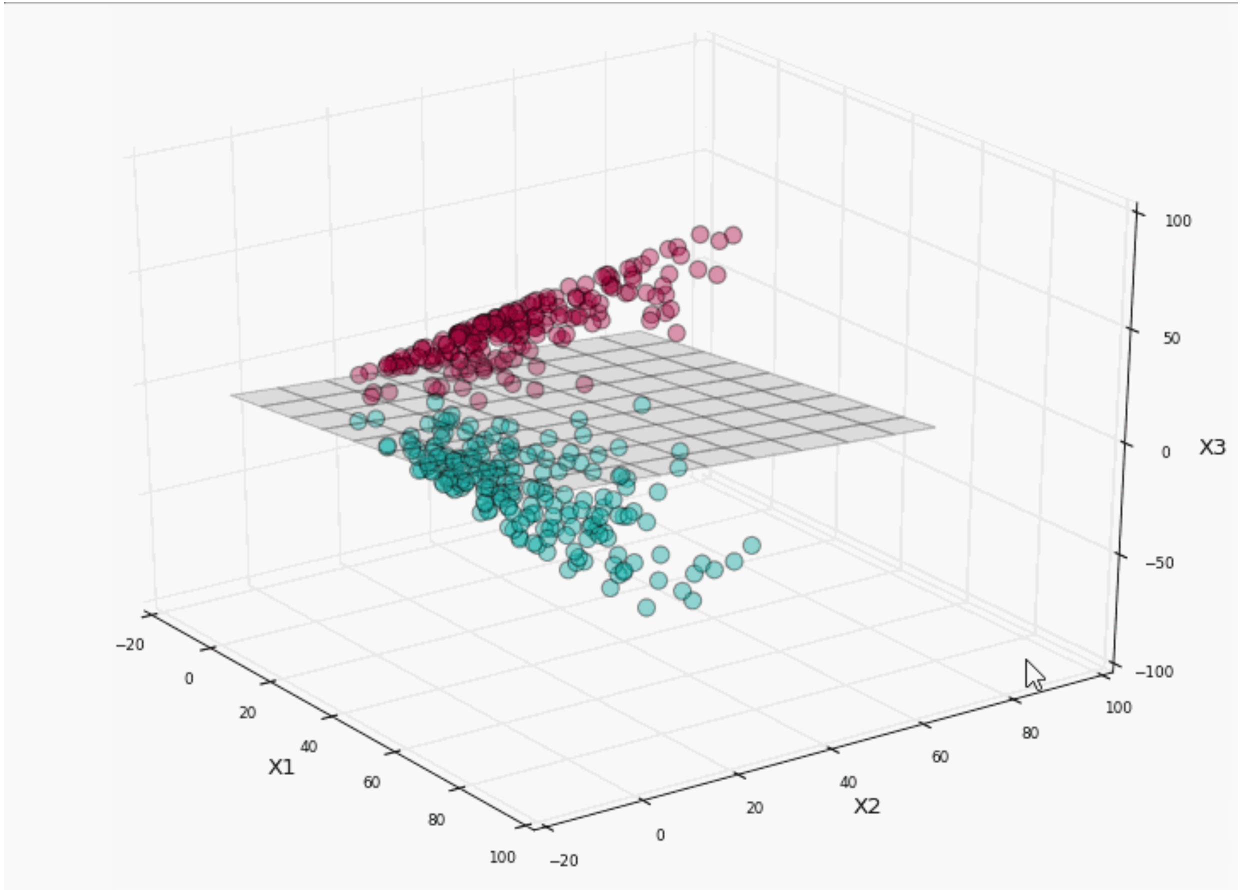

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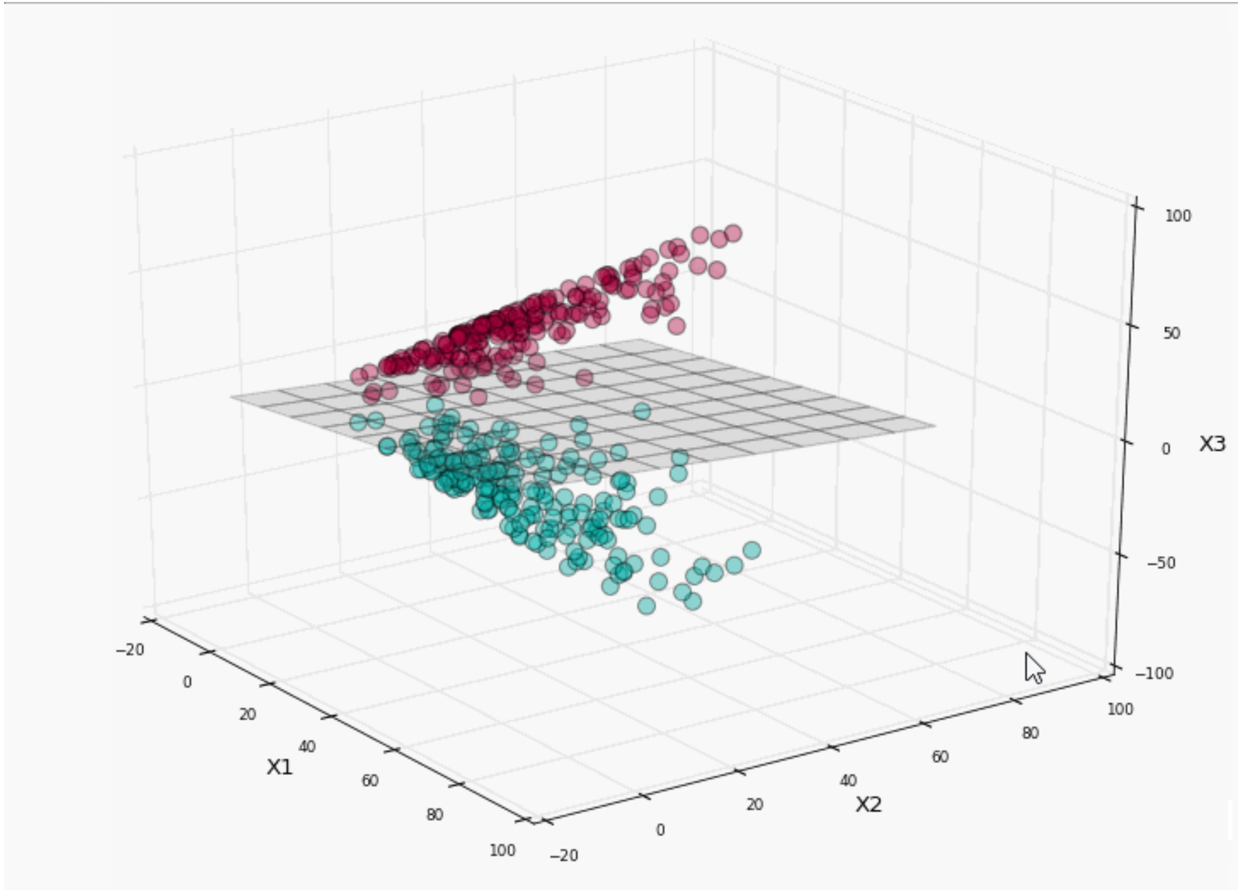

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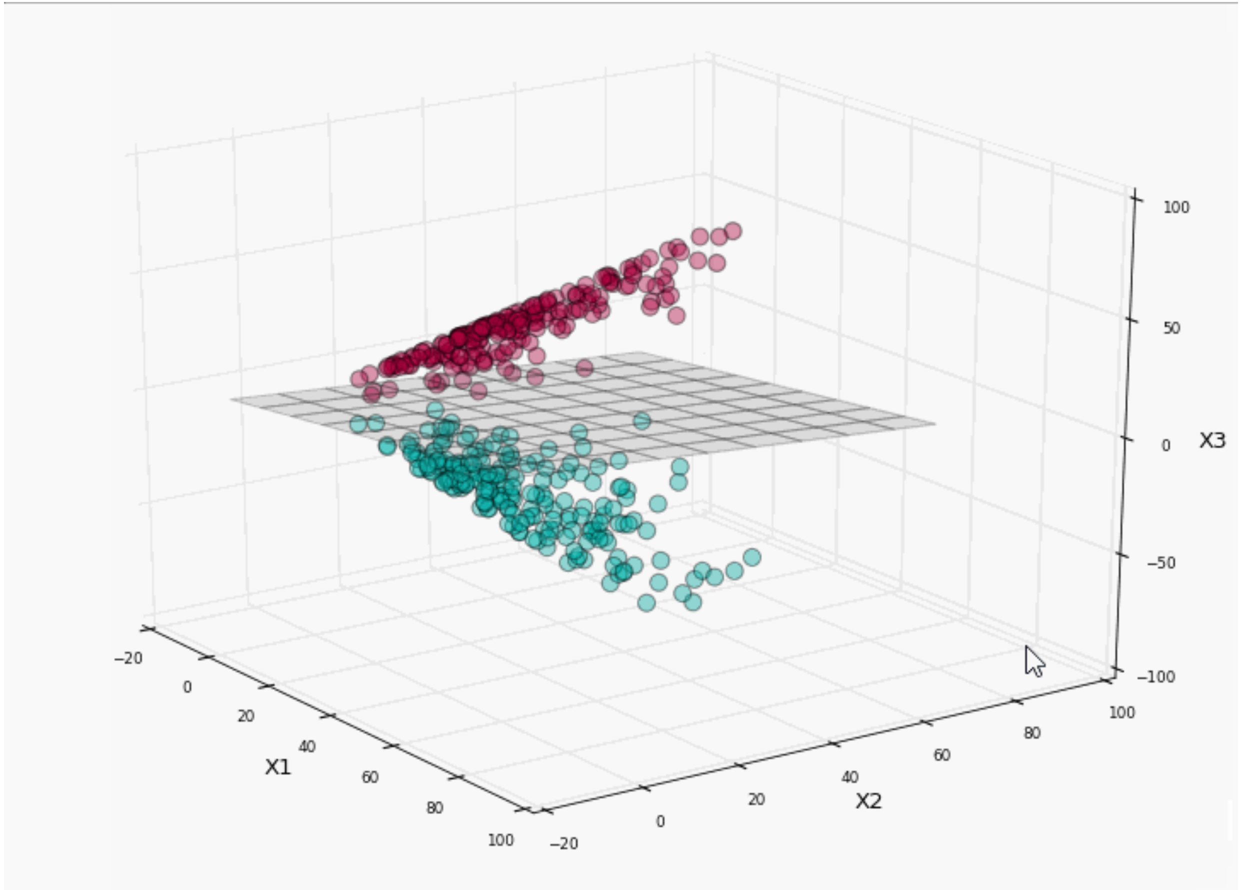

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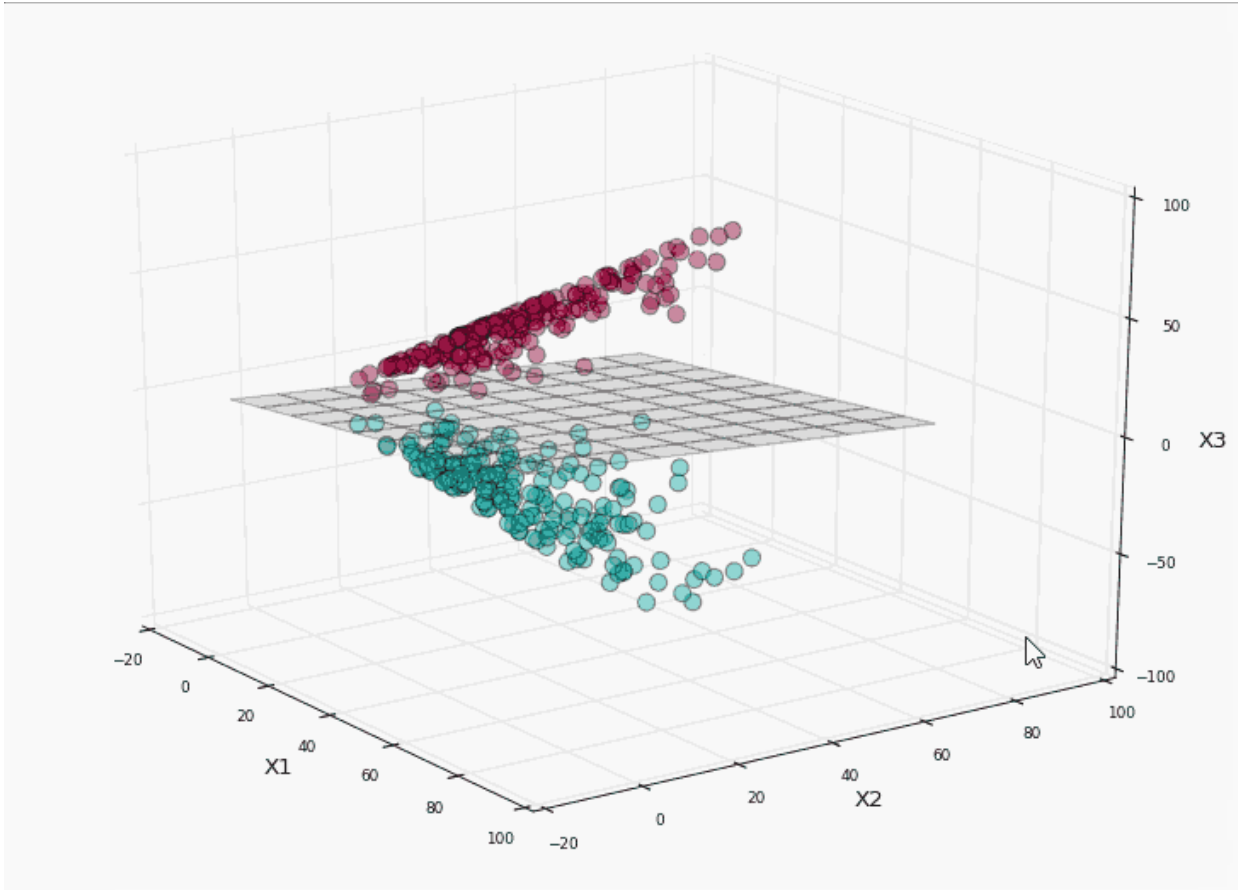
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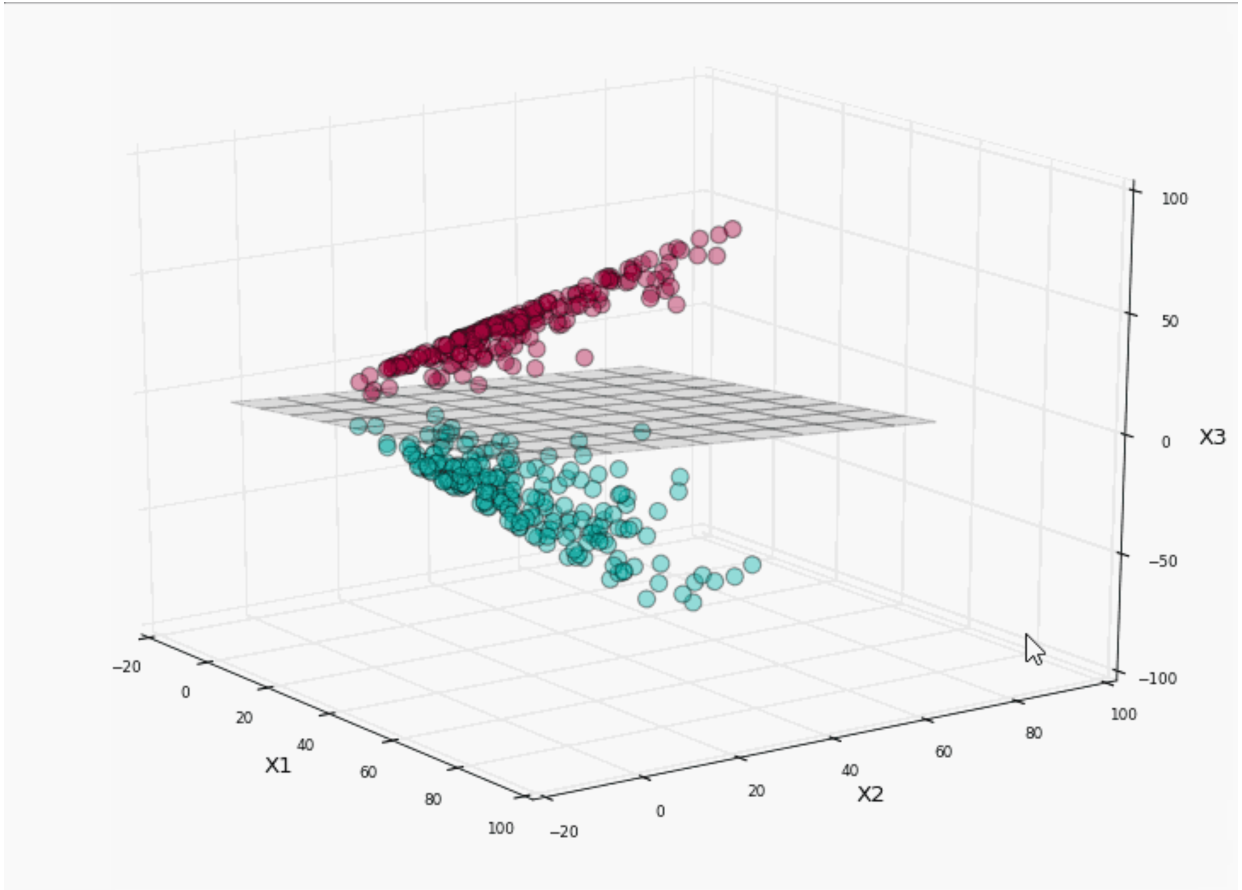

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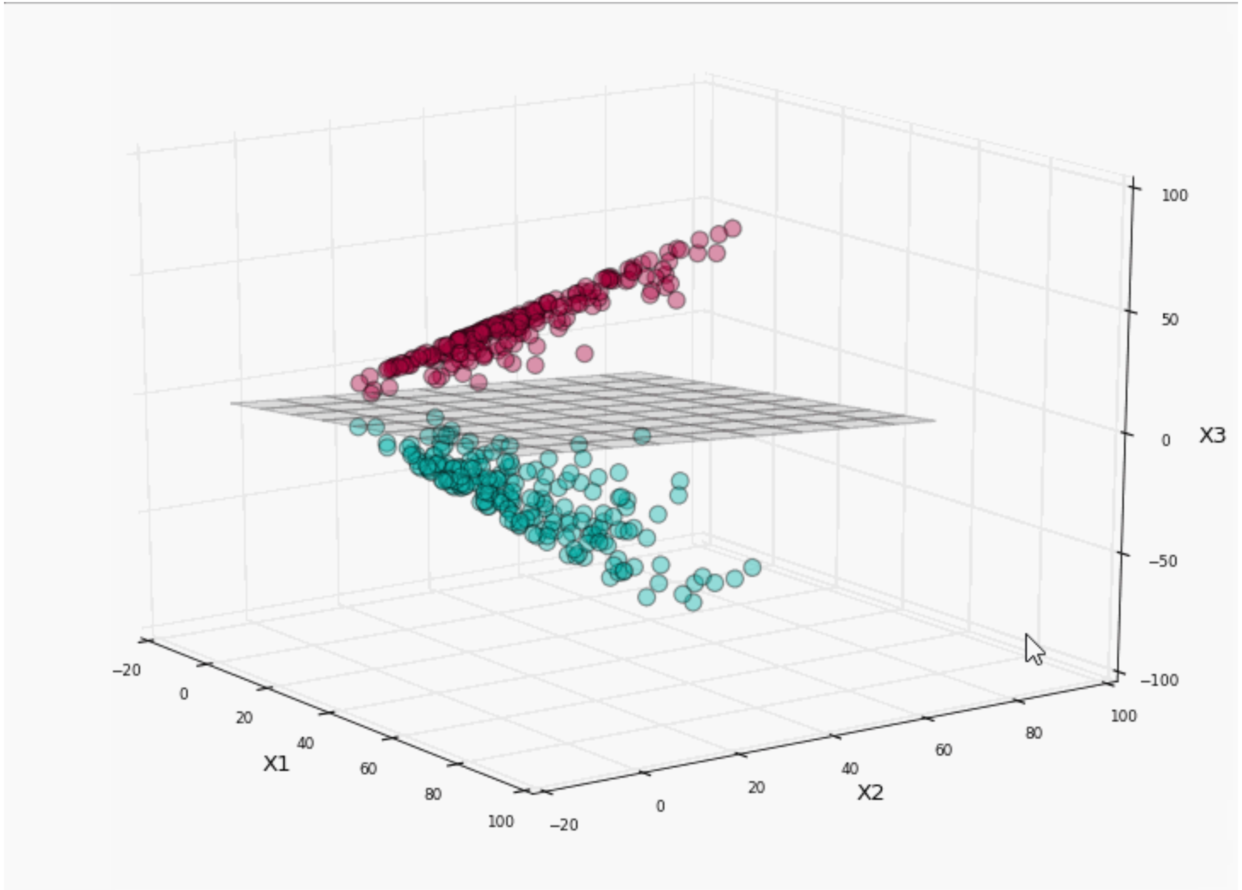

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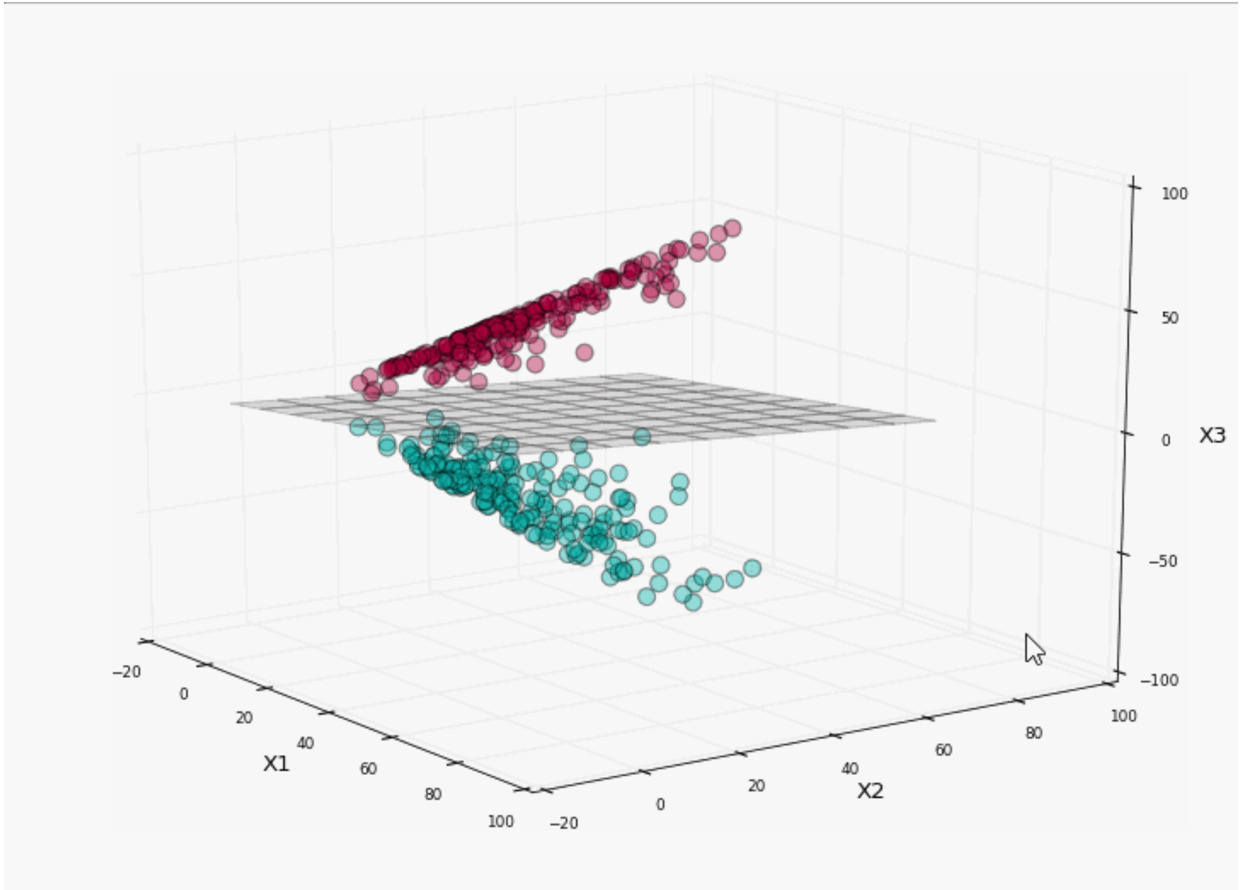

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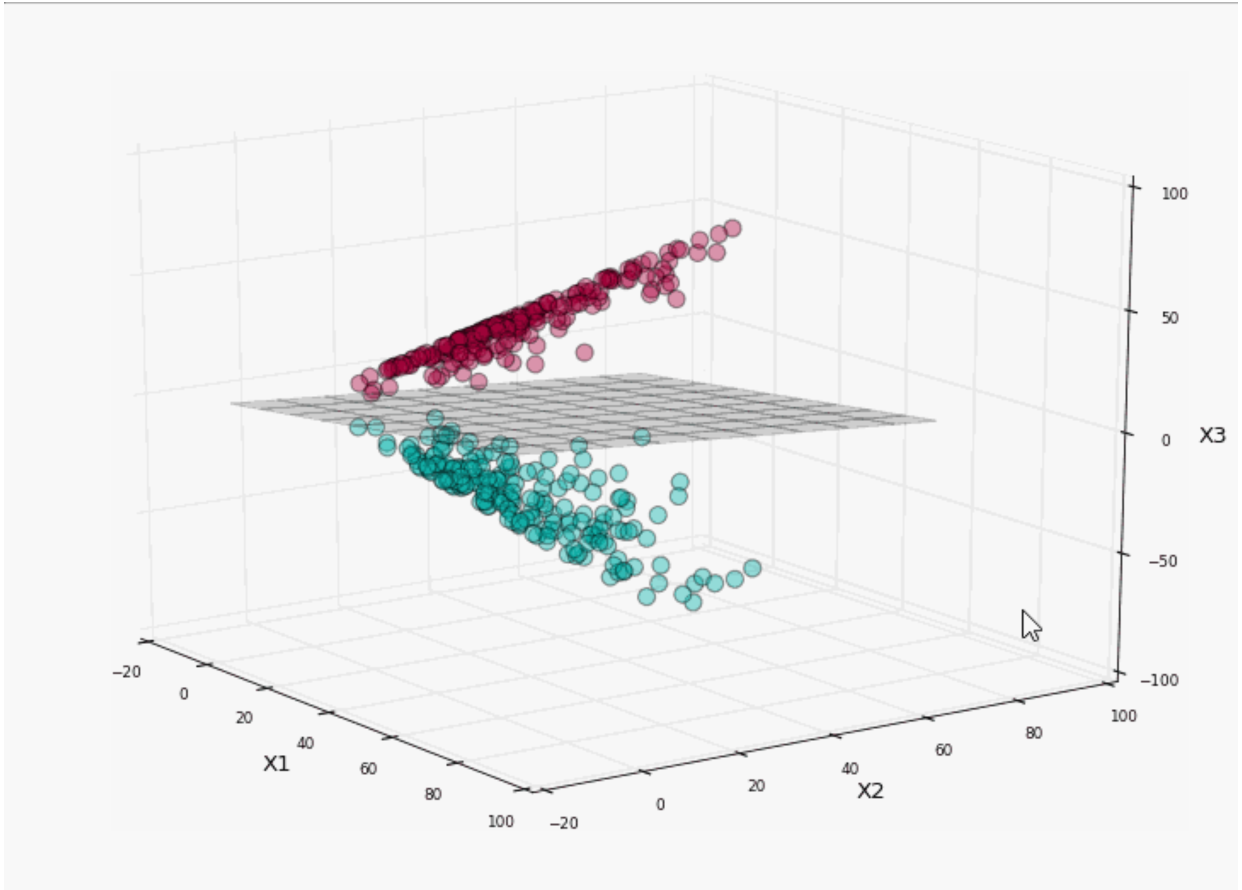

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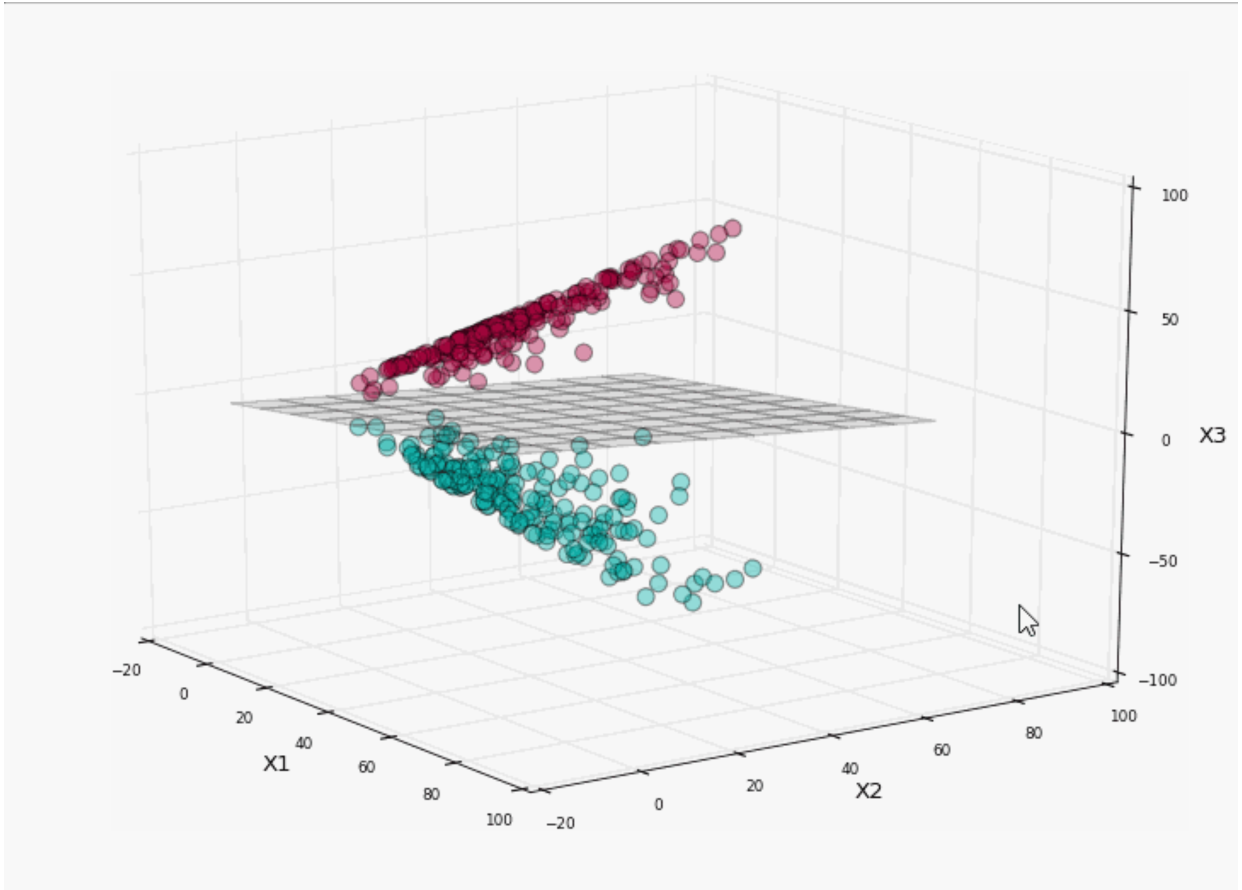
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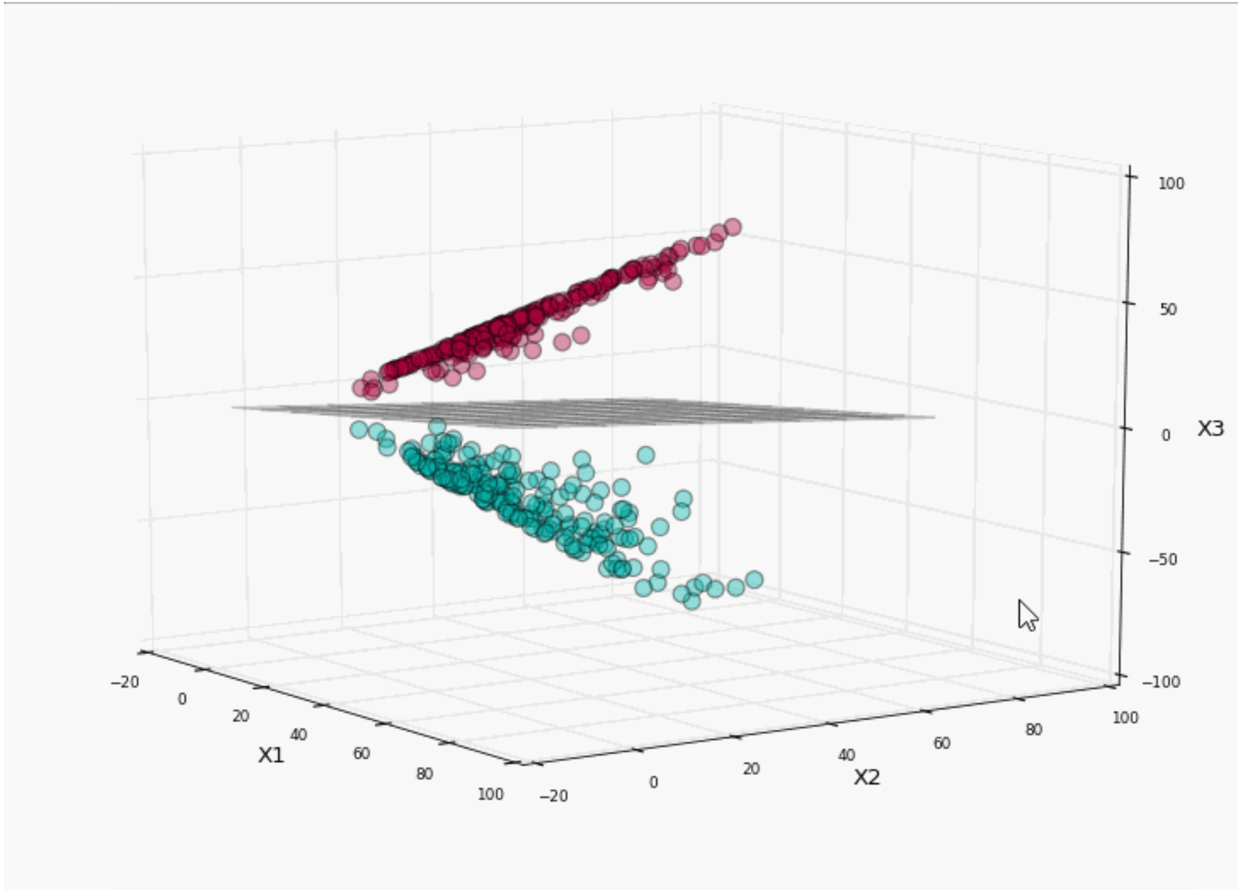

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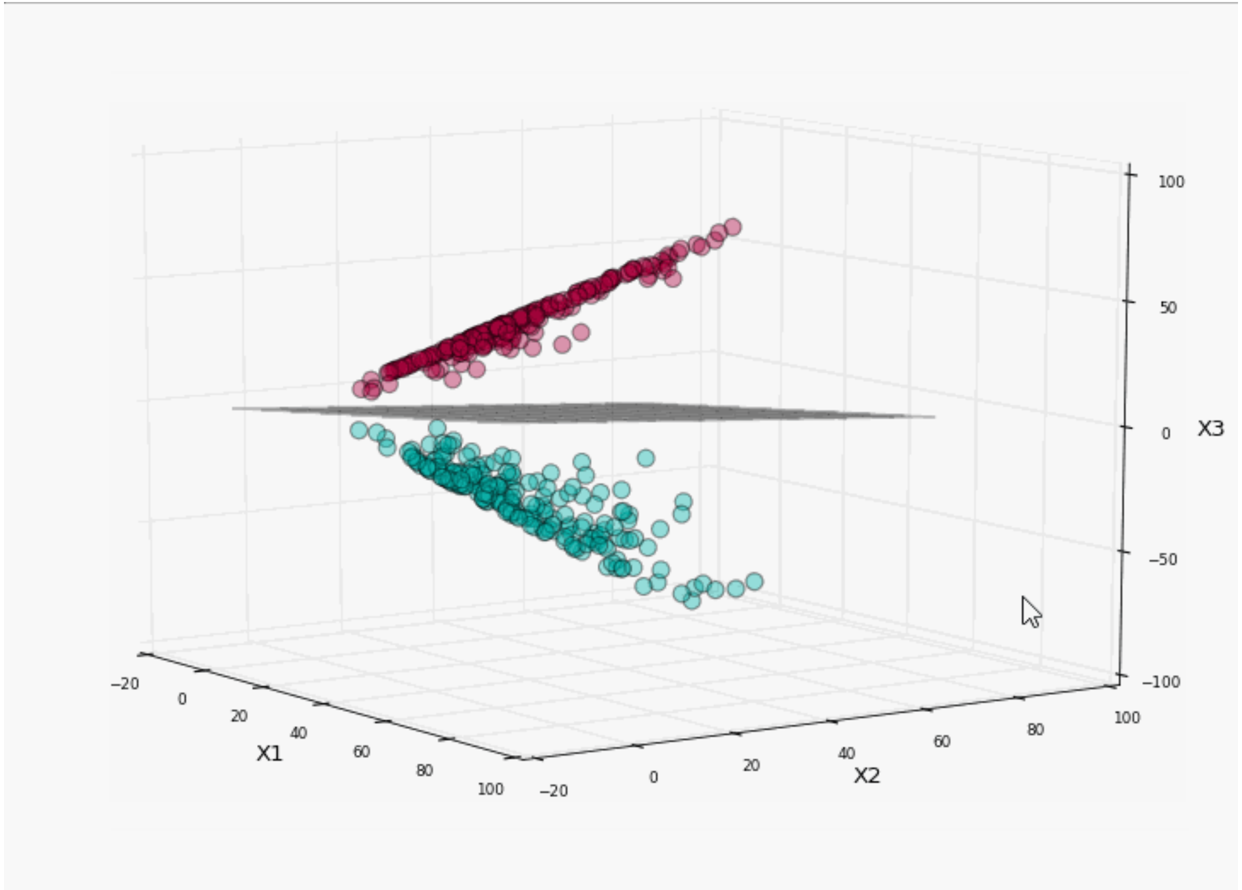

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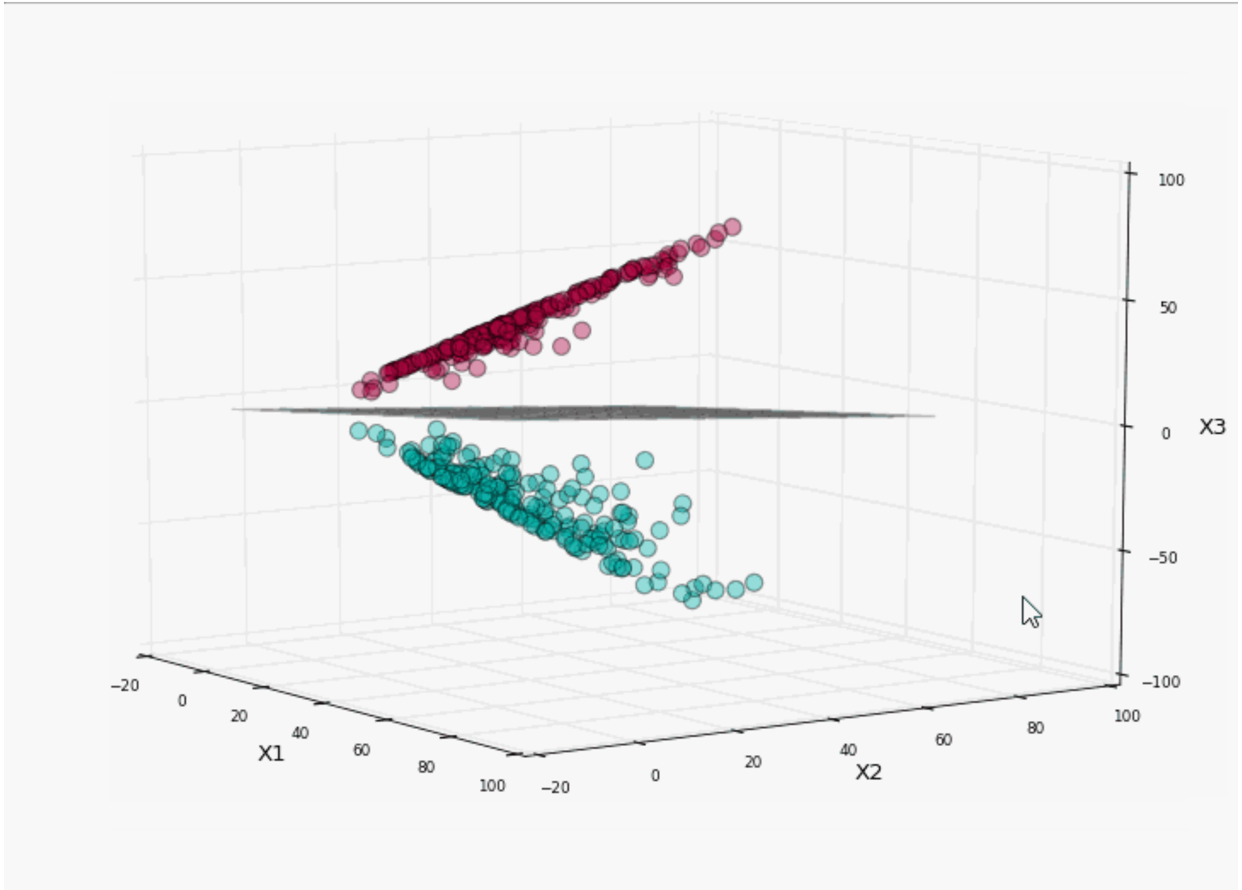

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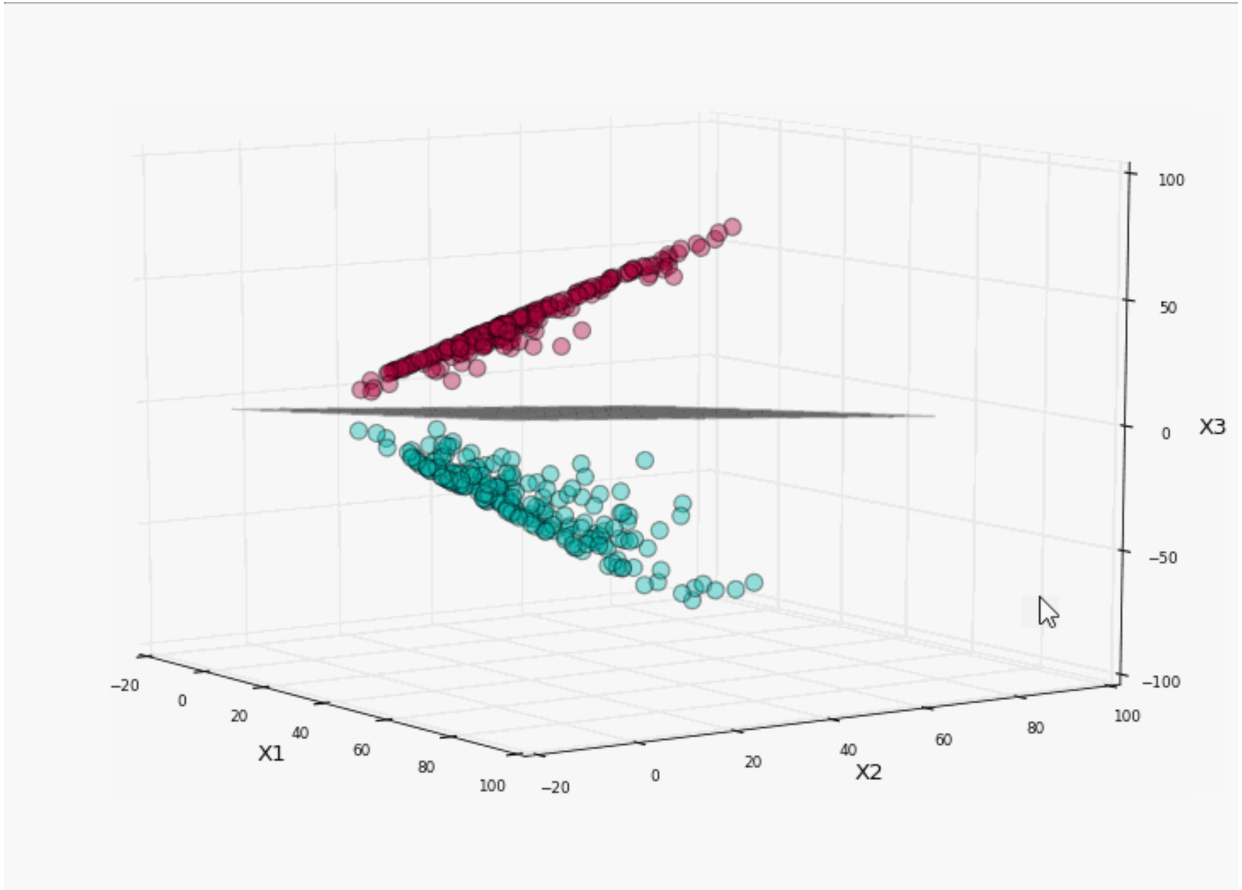

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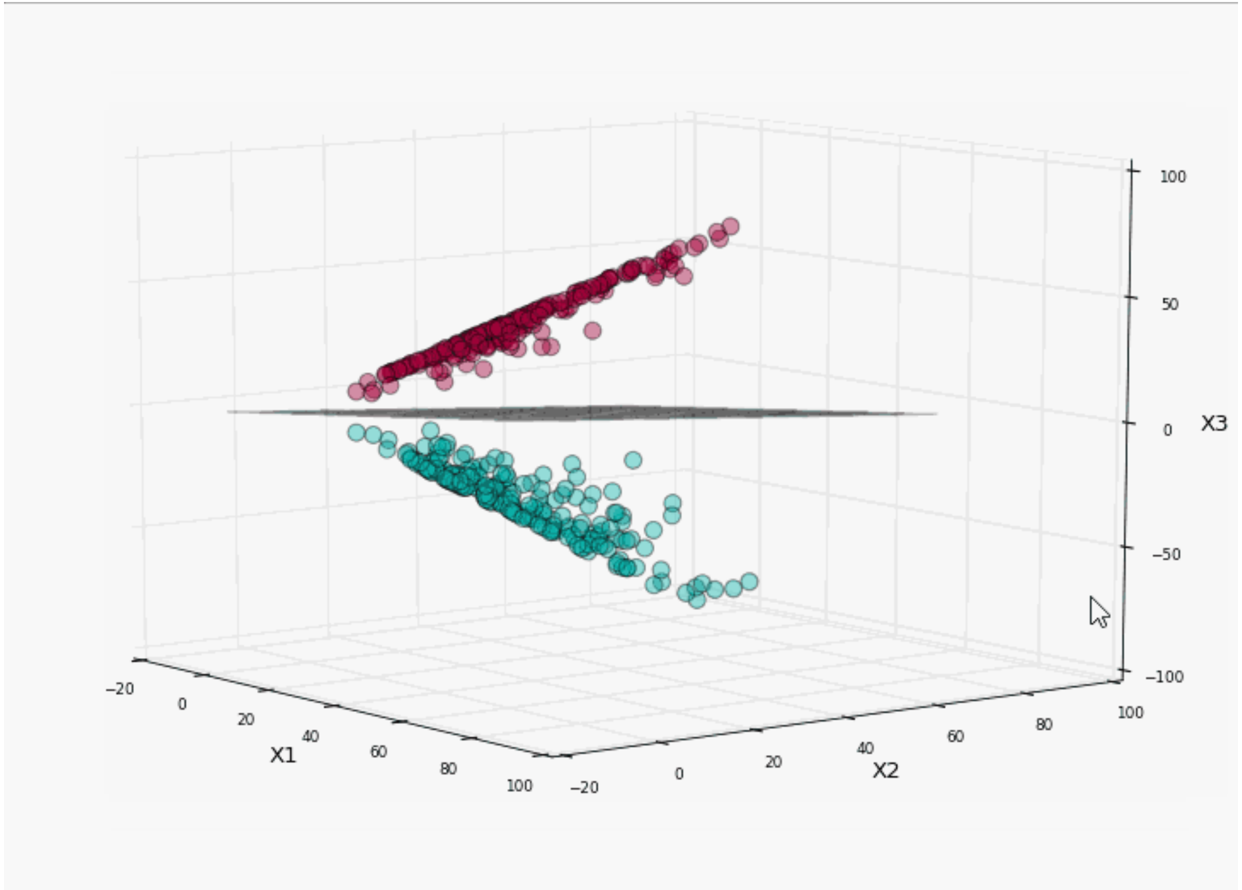

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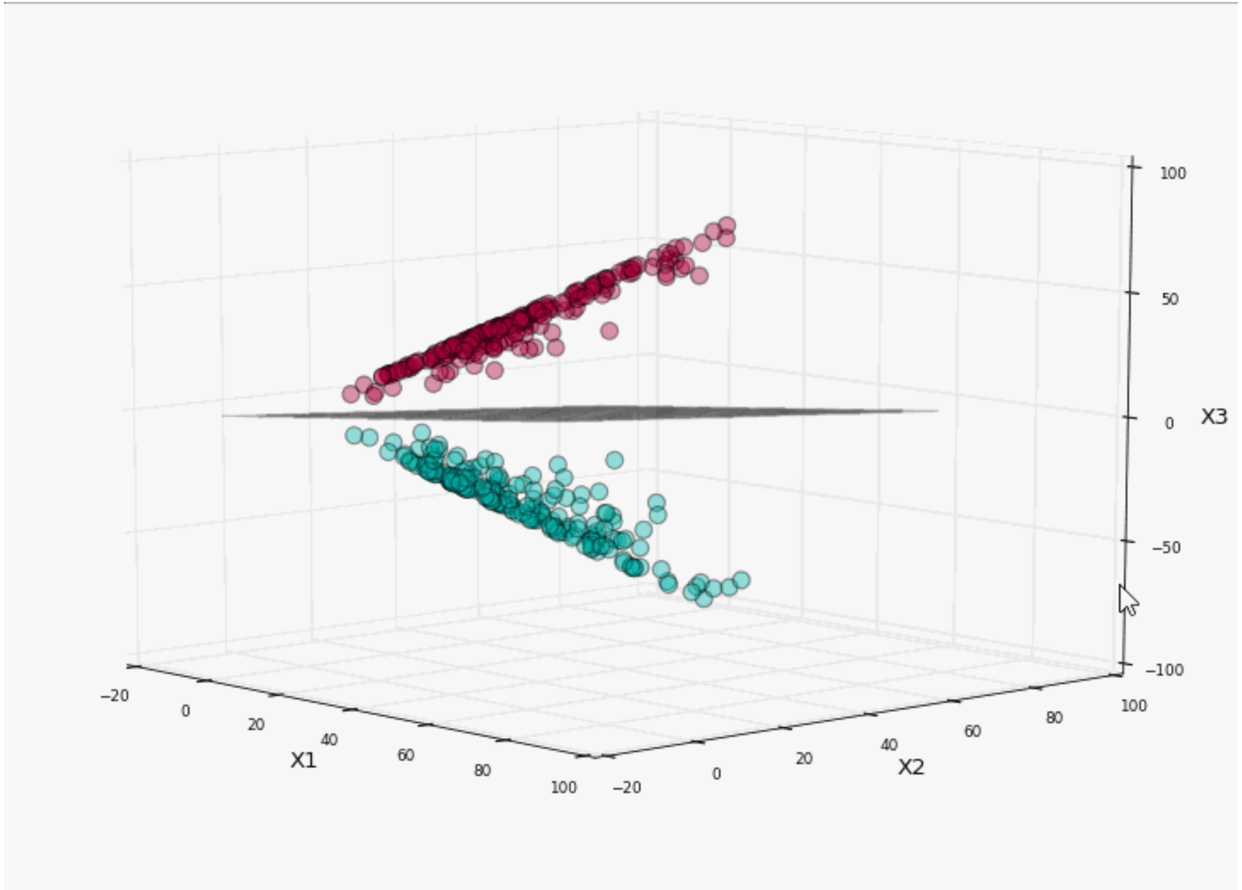
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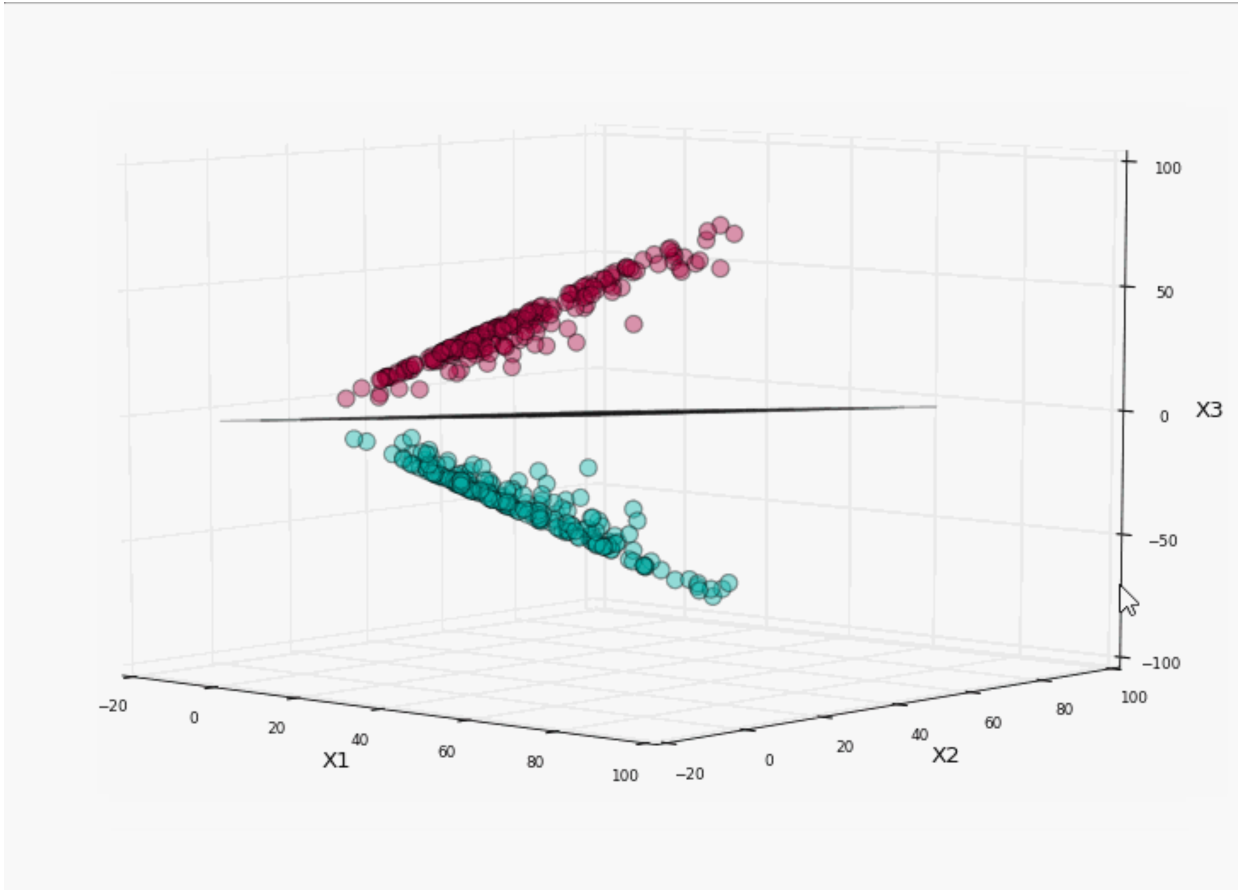
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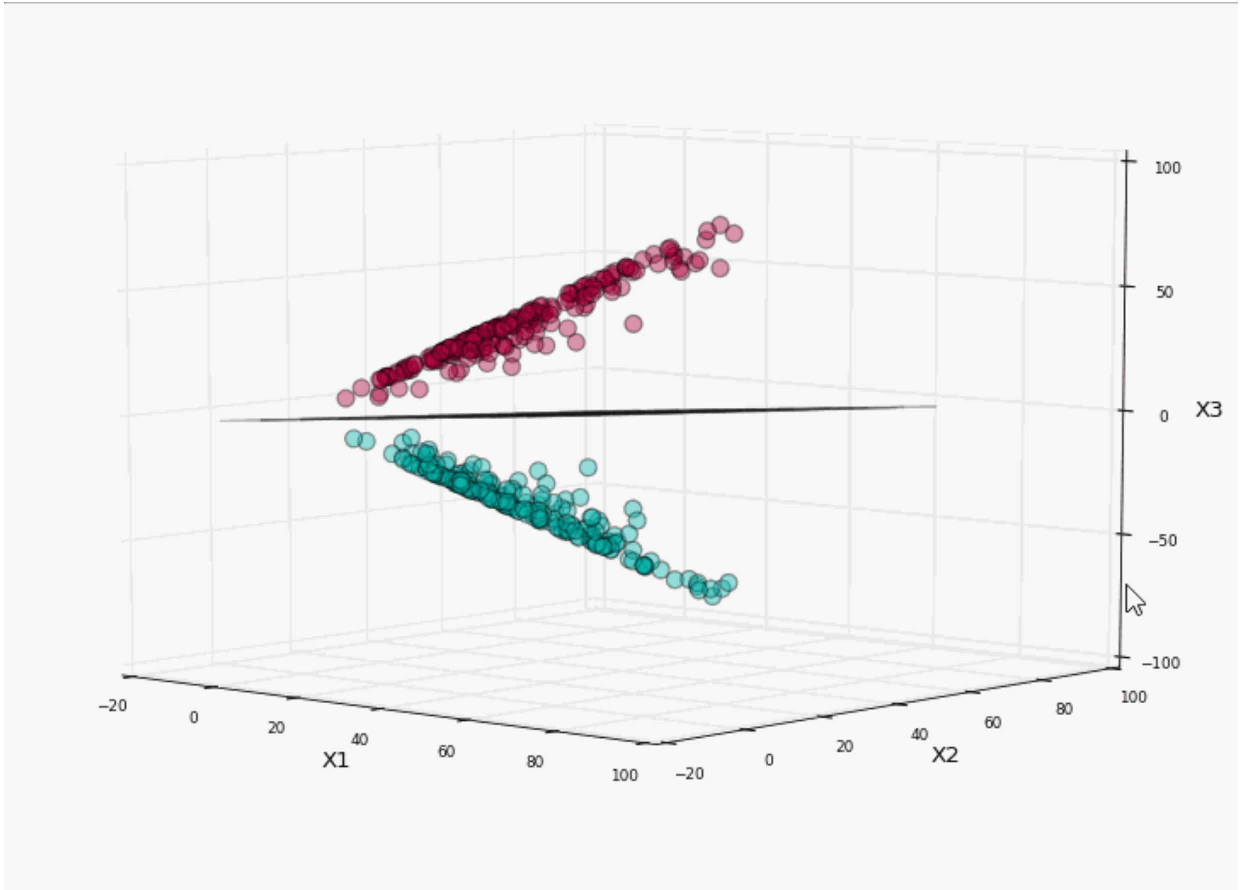

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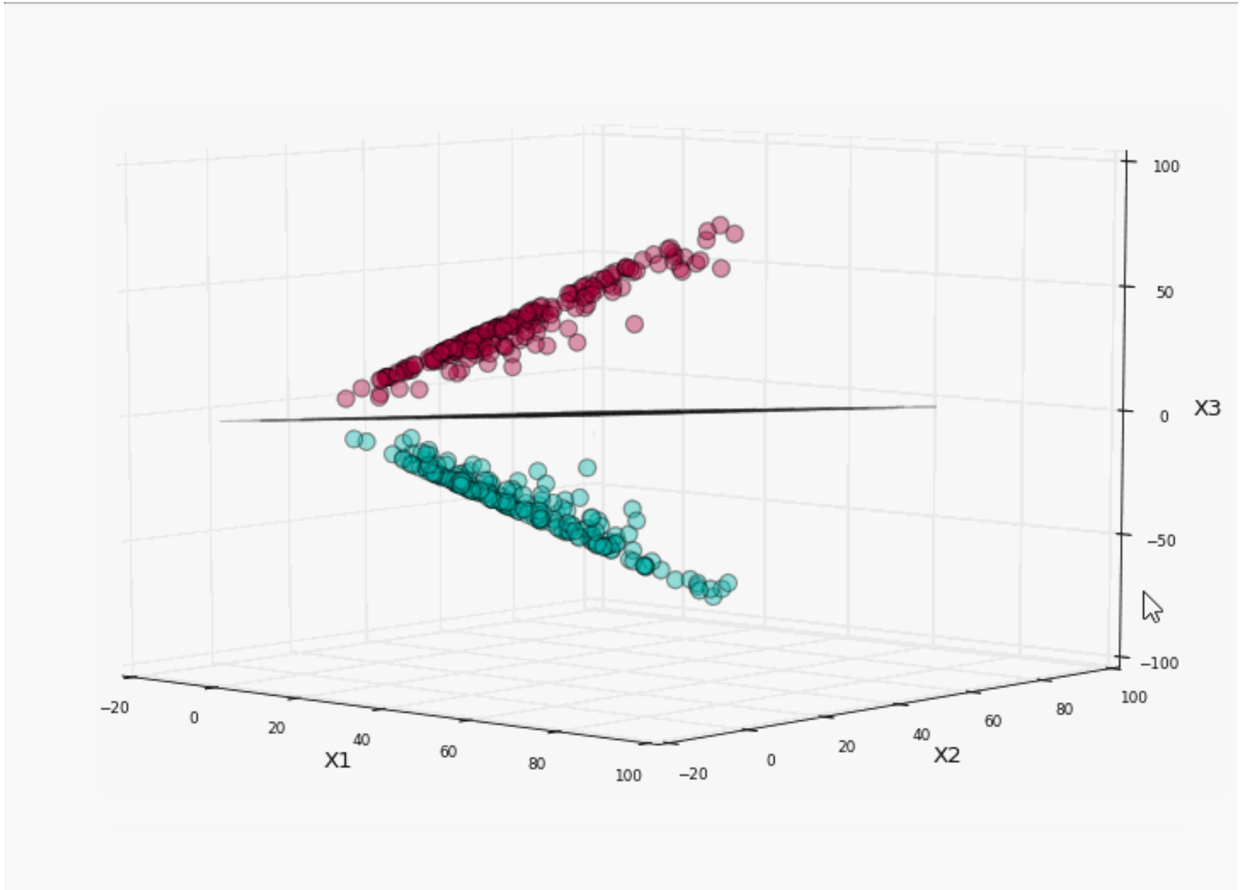

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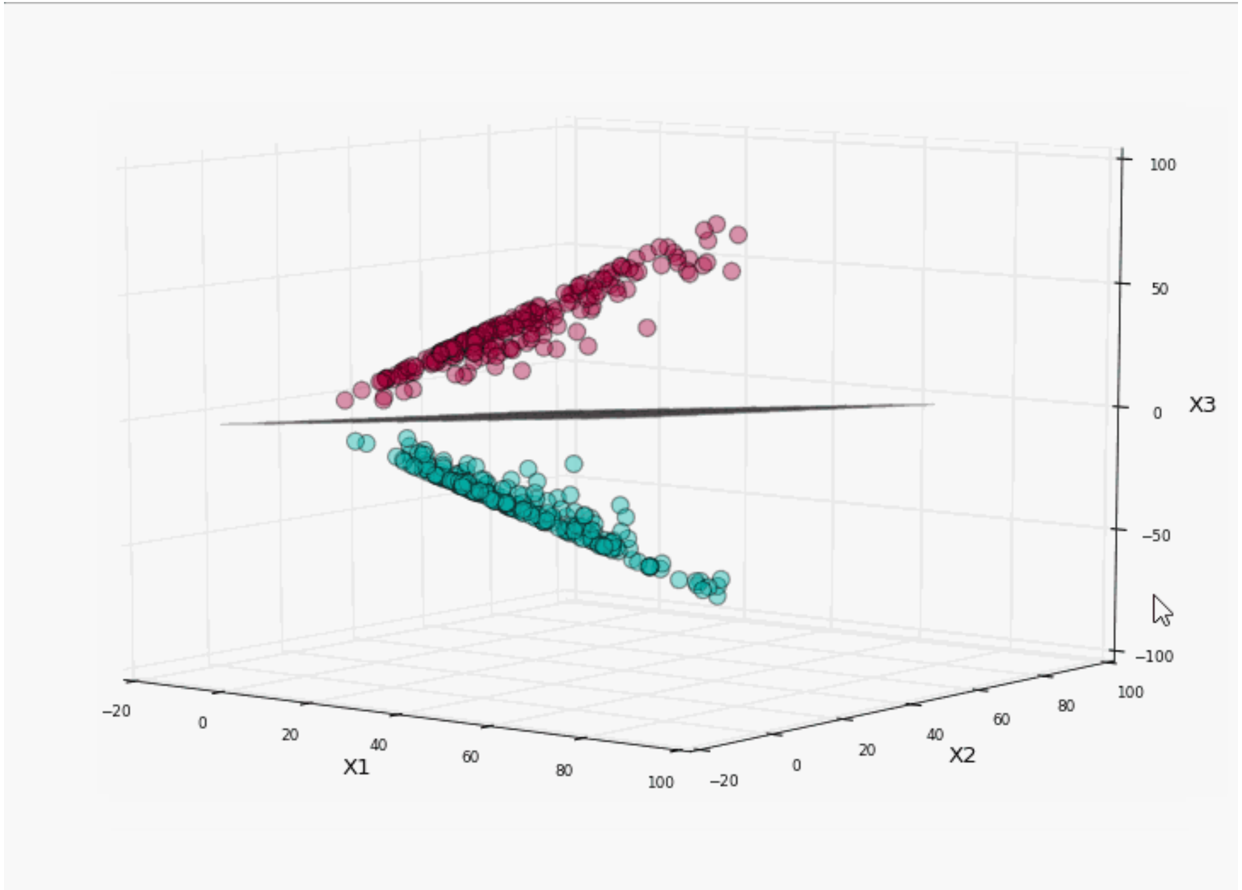
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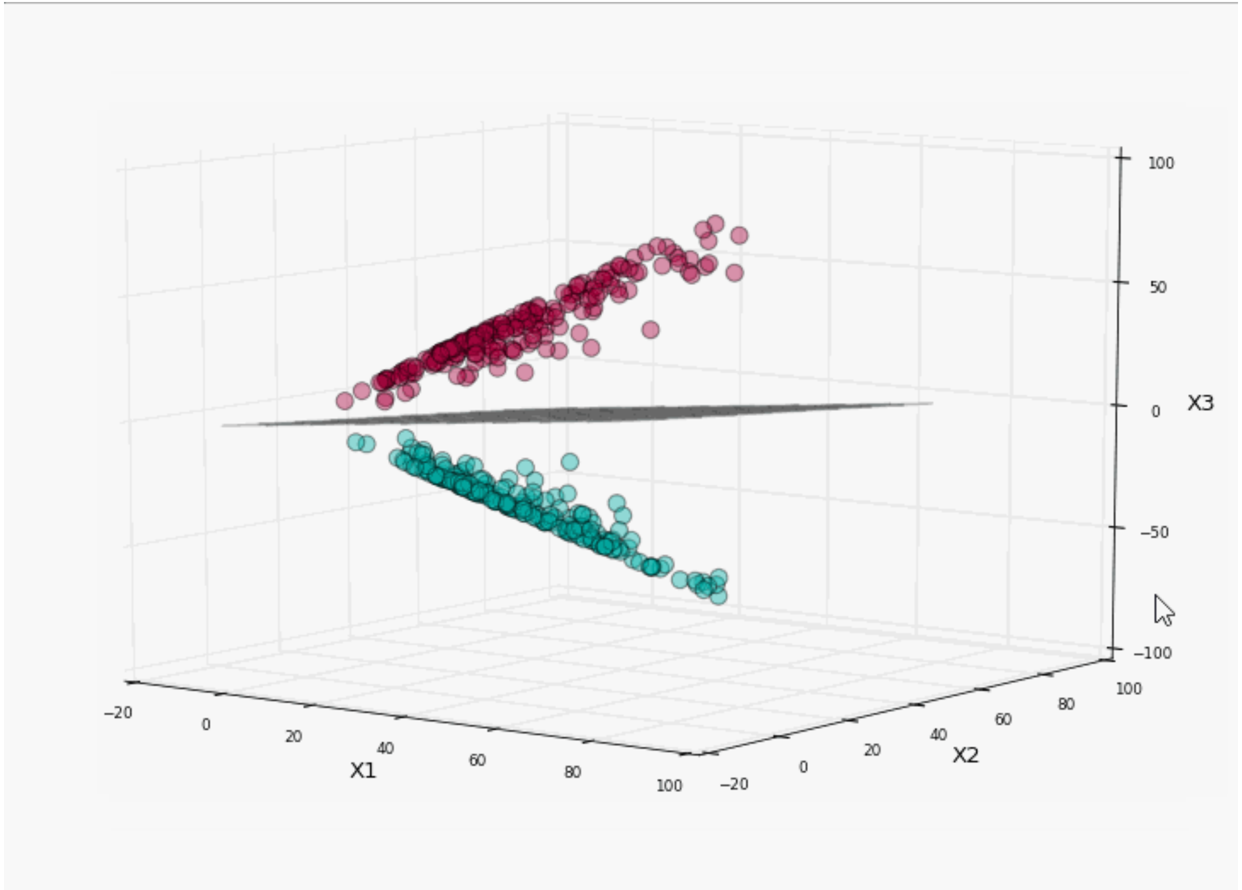

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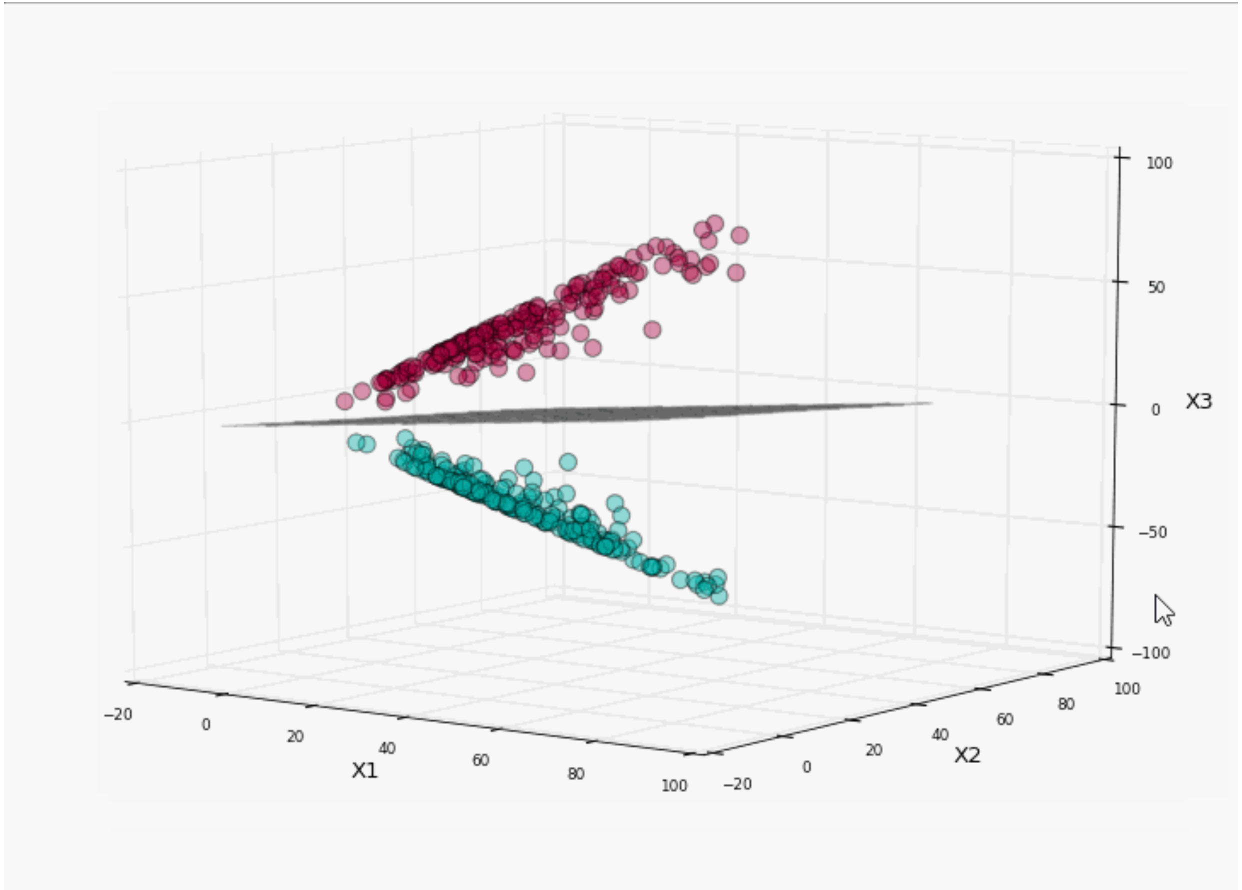

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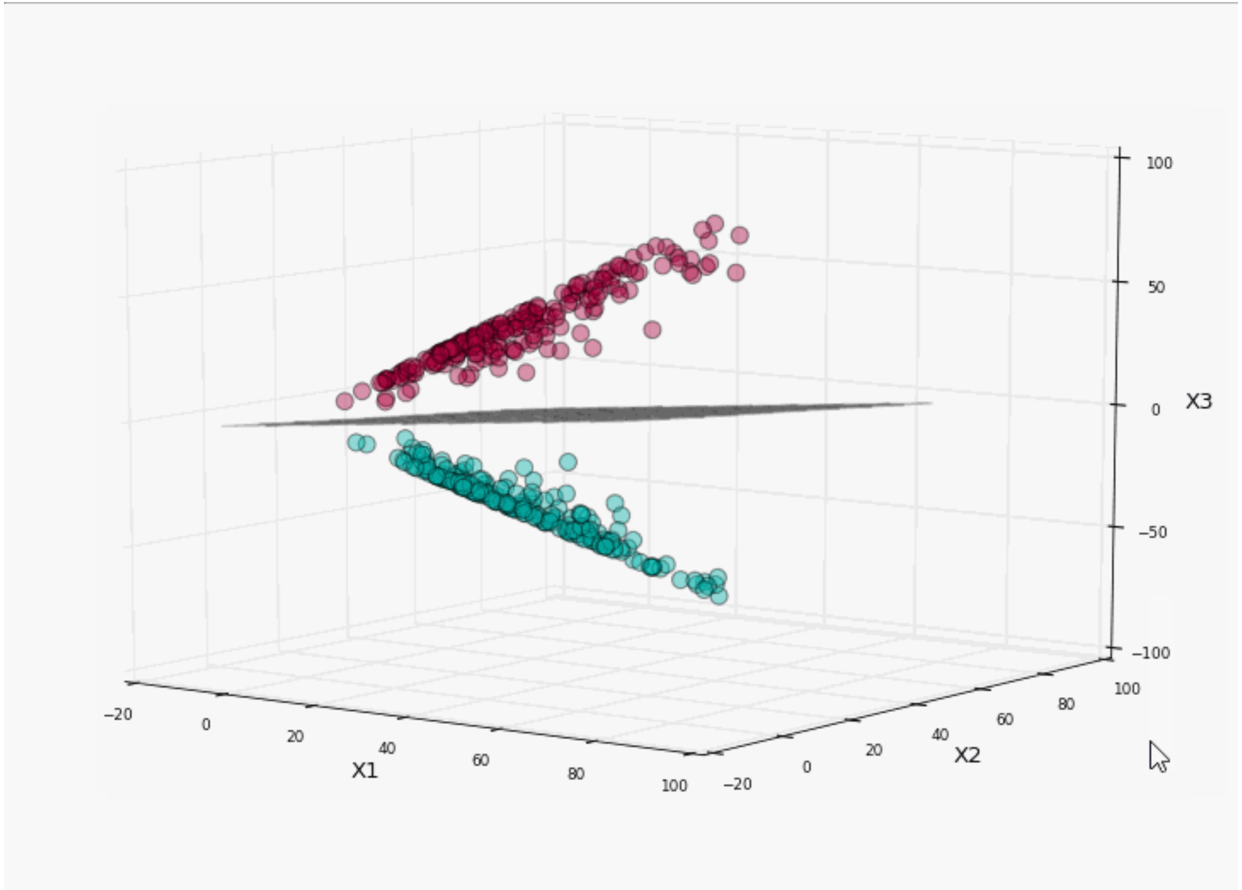

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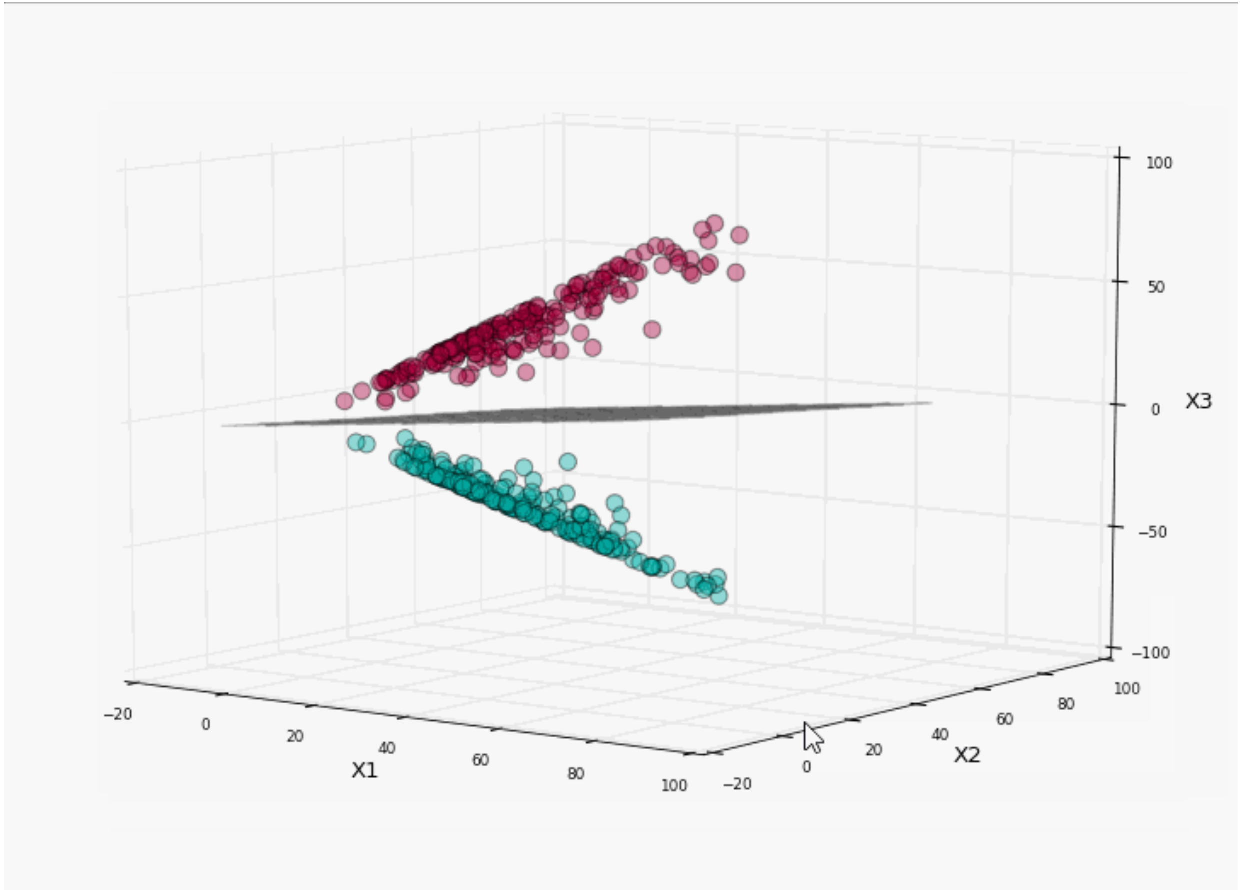

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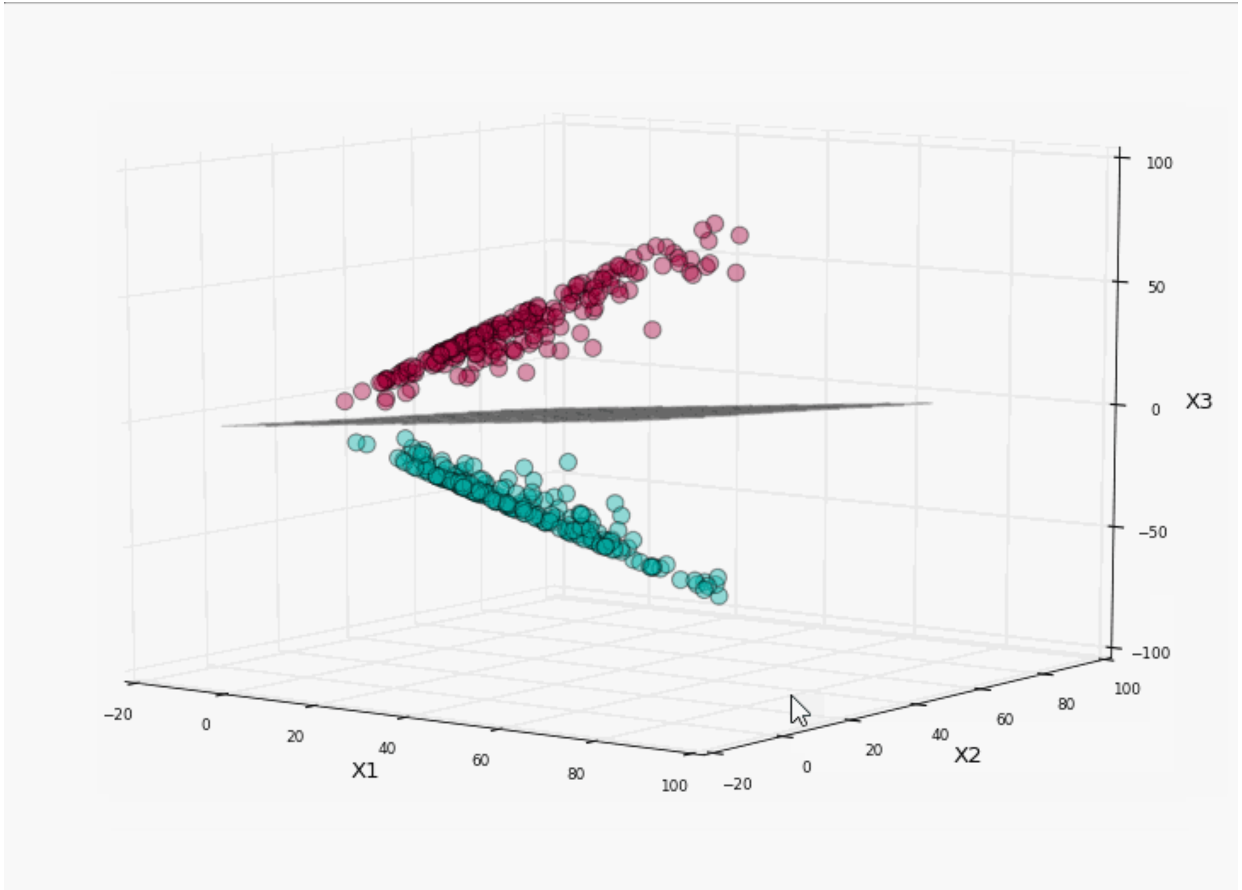

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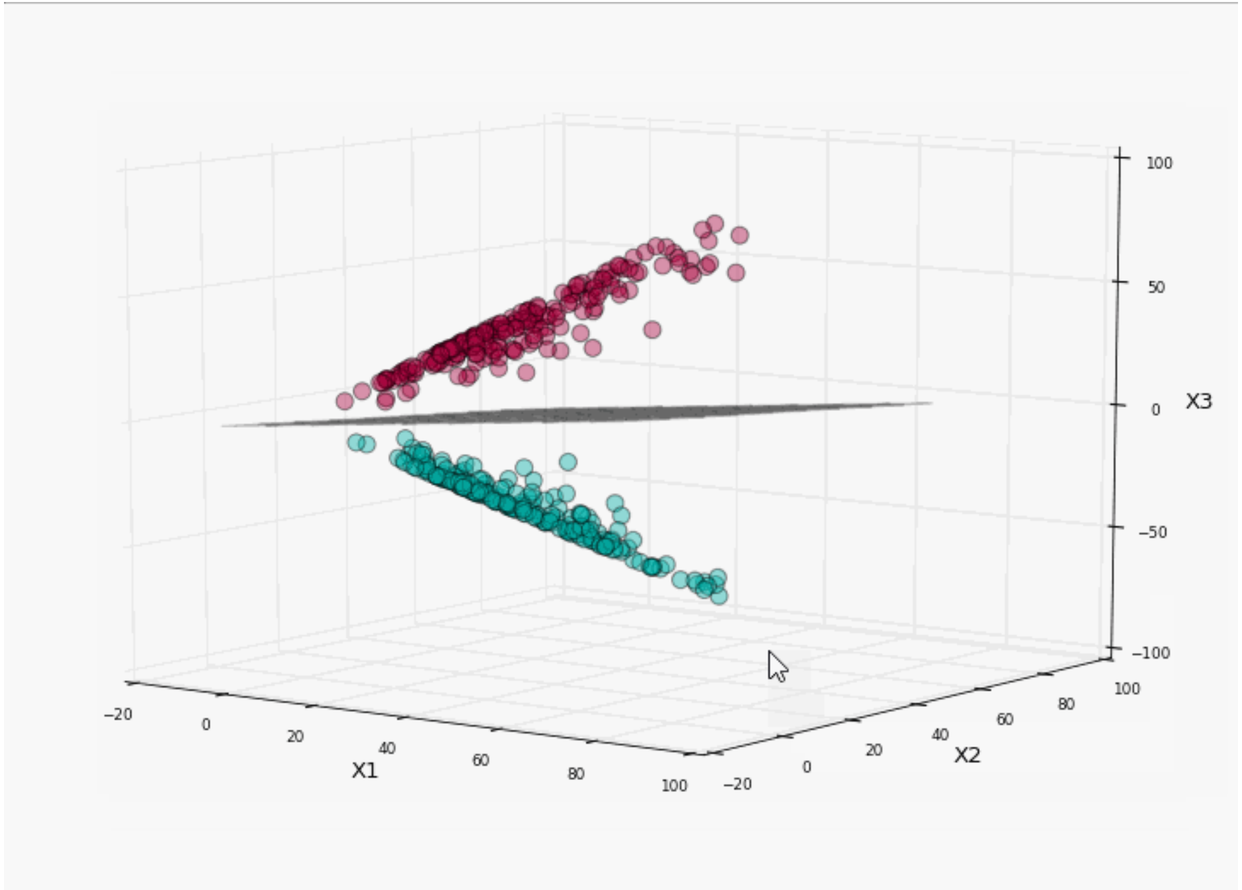
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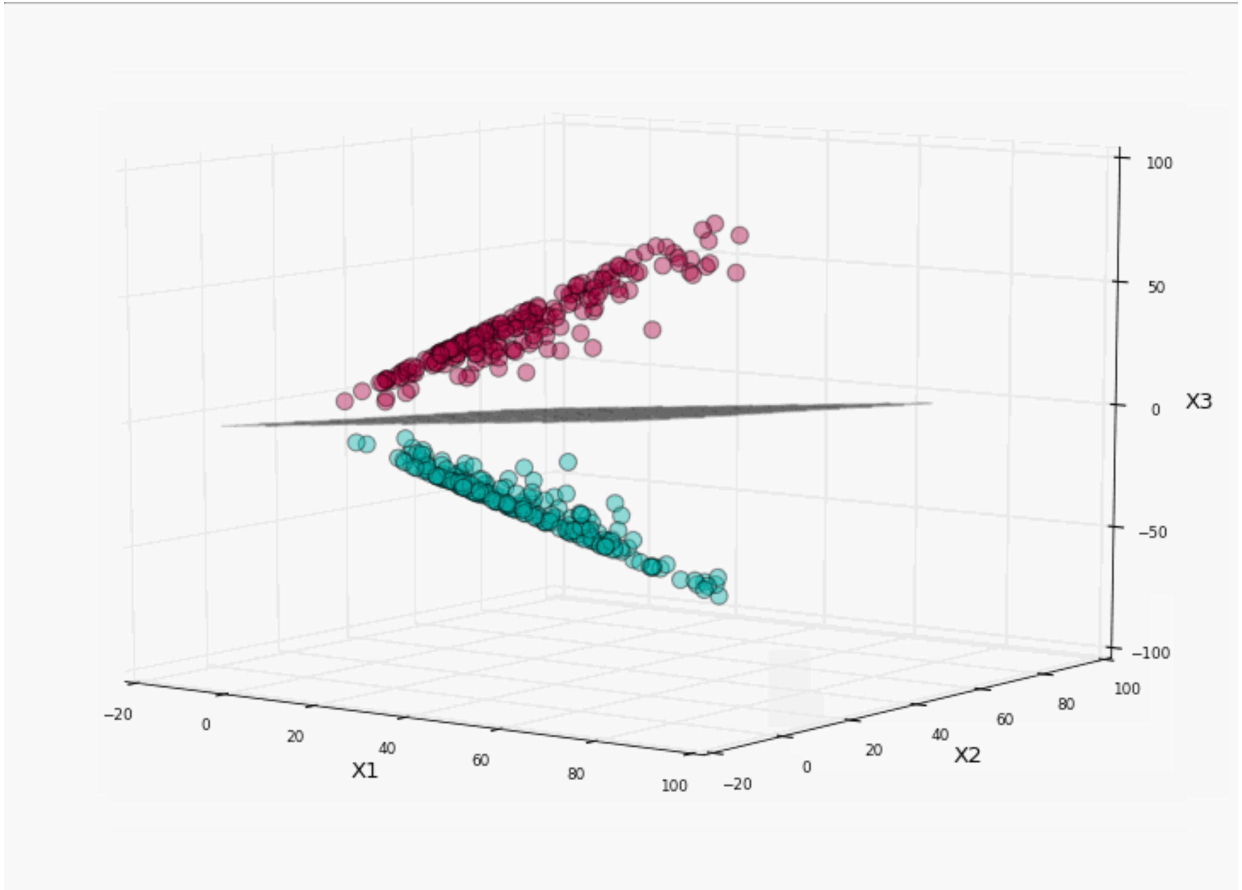

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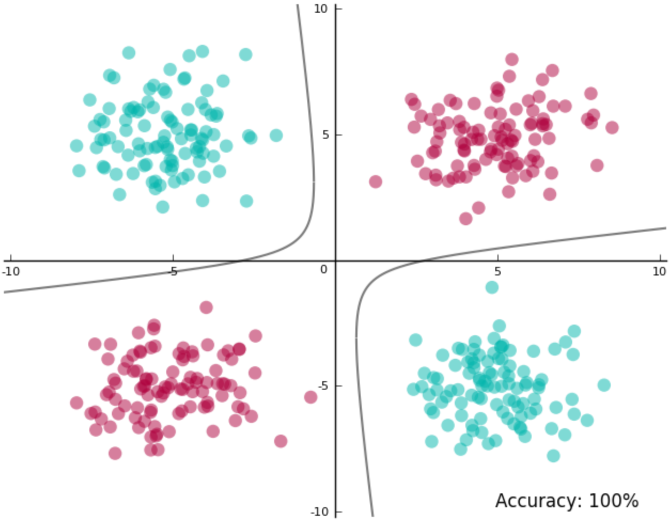
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Accuracy: 100%

The kernel-based function is exactly equivalent to preprocessing the data by applying similarity function to all inputs, then learning a linear model in the new transformed space.

Commonly used kernels

- Homogeneous polynomials

$$k(x, y) = (\langle x, y \rangle)^d$$

- Inhomogeneous polynomials

$$k(x, y) = (\langle x, y \rangle + 1)^d$$

- Gaussian Kernel

$$k(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$

- Sigmoid Kernel

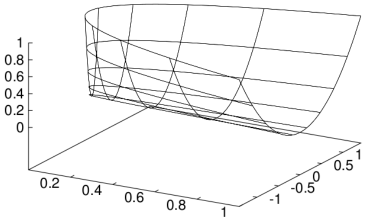
$$k(x, y) = \tanh(\eta \langle x, y \rangle + v)$$

Polynomial kernel

$$k(x, y) = (\langle x, y \rangle)^d$$

Example: $n = 2, d = 2, x = (x_1, x_2)$

- $\Phi(x) = \left(x_1^2, \sqrt{2}x_1x_2, x_2^2\right)$

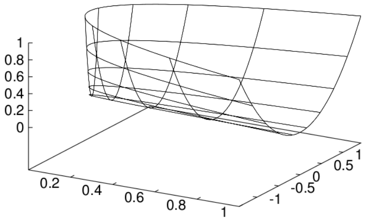


Polynomial kernel

$$k(x, y) = (\langle x, y \rangle)^d$$

Example: $n = 2, d = 2, x = (x_1, x_2)$

- $\Phi(x) = \left(x_1^2, \sqrt{2}x_1x_2, x_2^2\right)$



- Neither the mapping Φ nor the feature space is unique
  - $\Phi(x) = \left(x_1^2, x_1x_2, x_1x_2, x_2^2\right)$
  - $\Phi(x) = \frac{1}{\sqrt{2}}\left(x_1^2 - x_2^2, 2x_1x_2, x_1^2 + x_2^2\right)$