

Information Gain & Decision Trees



Slides adopted from
Data Mining for Business Analytics

Lecture 3: Supervised Classification

**Stern School of Business
New York University
Spring 2014**

Supervised Classification

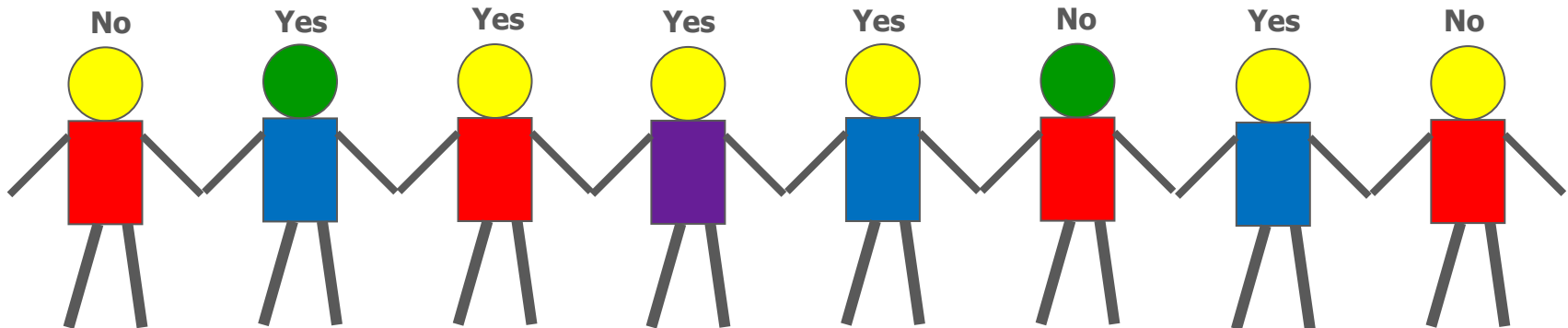
- How can we classify the population into groups that differ from each other with respect to some quantity of interest?
- Informative attributes
 - Find **knowable** attributes that correlate with the target of interest
 - Increase accuracy
 - Alleviate computational problems
 - E.g., *tree induction*

Supervised Classification

- How can we judge whether a variable contains important information about the target variable?
 - How much?

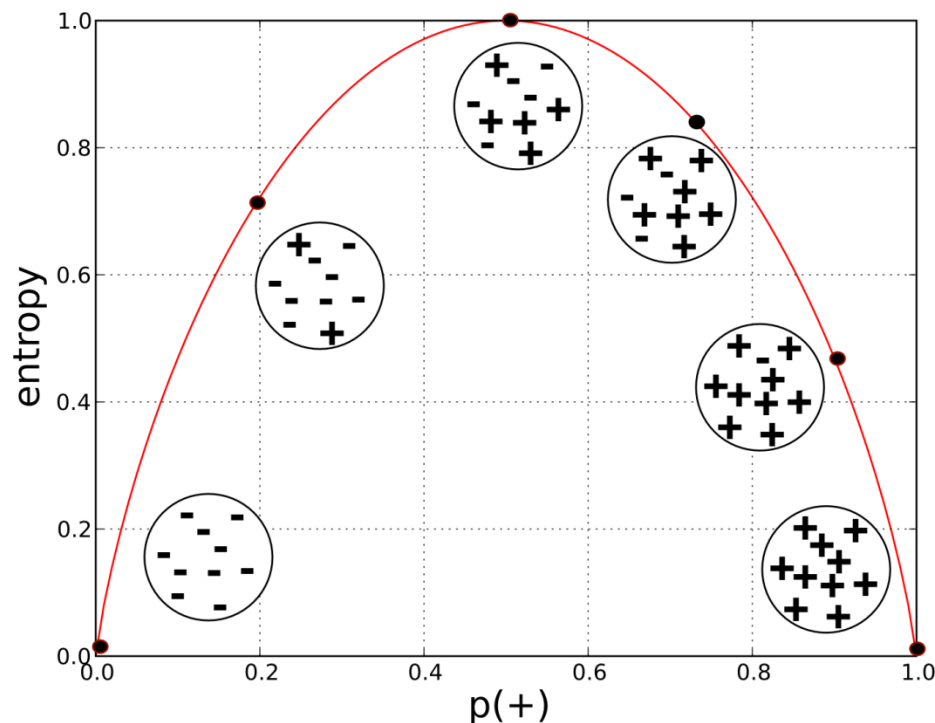
Selecting Informative Attributes

Objective: Based on customer attributes, partition the customers into subgroups that are less impure – with respect to the class (i.e., such that in each group as many instances as possible belong to the same class)



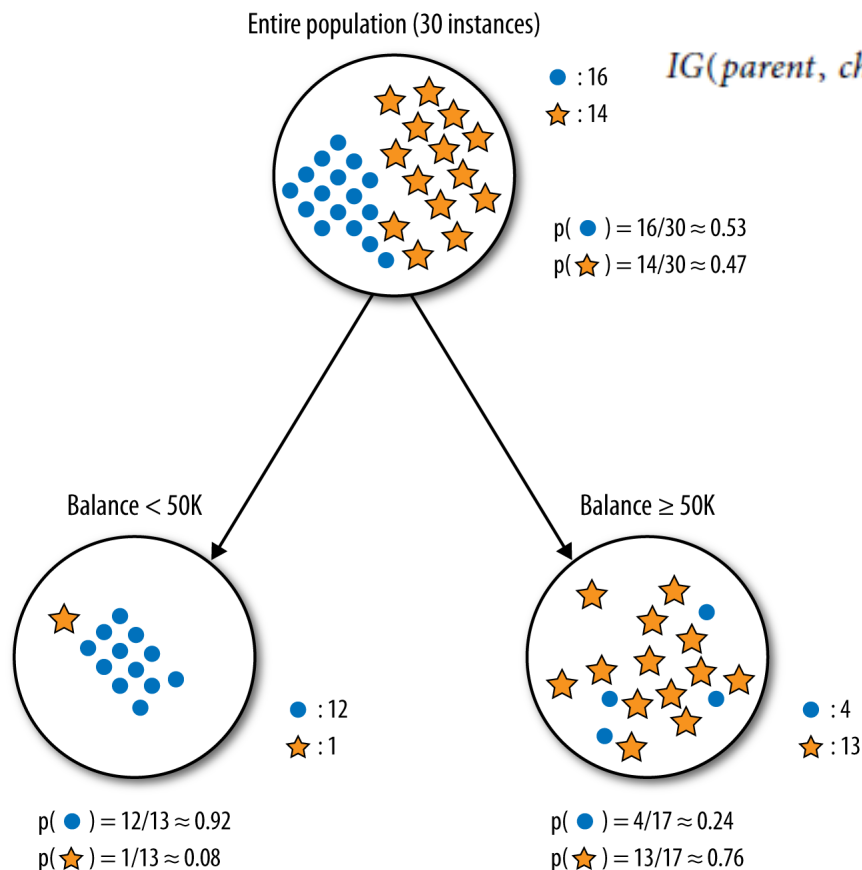
Selecting Informative Attributes

- The most common splitting criterion is called **information gain (IG)**
 - It is based on a **purity measure** called **entropy**
 - $entropy = -p_1 \log_2(p_1) - p_2 \log_2(p_2) - \dots$
 - Measures the general disorder of a set



Information Gain

- Information gain measures the *change* in entropy due to any amount of new information being added



$$IG(\text{parent}, \text{children}) = \text{entropy}(\text{parent}) - [p(c_1) \times \text{entropy}(c_1) + p(c_2) \times \text{entropy}(c_2) + \dots]$$

Information Gain

Entire population (30 instances)

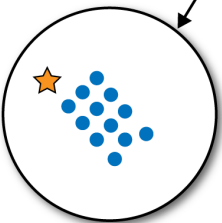


● : 16
★ : 14

$p(\bullet) = 16/30 \approx 0.53$
 $p(\star) = 14/30 \approx 0.47$

$$\begin{aligned} \text{entropy}(\text{parent}) &= -[p(\bullet) \times \log_2 p(\bullet) + p(\star) \times \log_2 p(\star)] \\ &\approx -[0.53 \times -0.9 + 0.47 \times -1.1] \\ &\approx 0.99 \quad (\text{very impure}) \end{aligned}$$

Balance < 50K



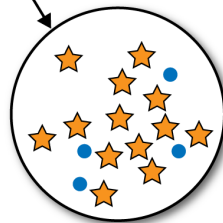
● : 12
★ : 1

$p(\bullet) = 12/13 \approx 0.92$
 $p(\star) = 1/13 \approx 0.08$

The entropy of the *left* child is:

$$\begin{aligned} \text{entropy}(\text{Balance} < 50K) &= -[p(\bullet) \times \log_2 p(\bullet) + p(\star) \times \log_2 p(\star)] \\ &\approx -[0.92 \times (-0.12) + 0.08 \times (-3.7)] \\ &\approx 0.39 \end{aligned}$$

Balance ≥ 50K



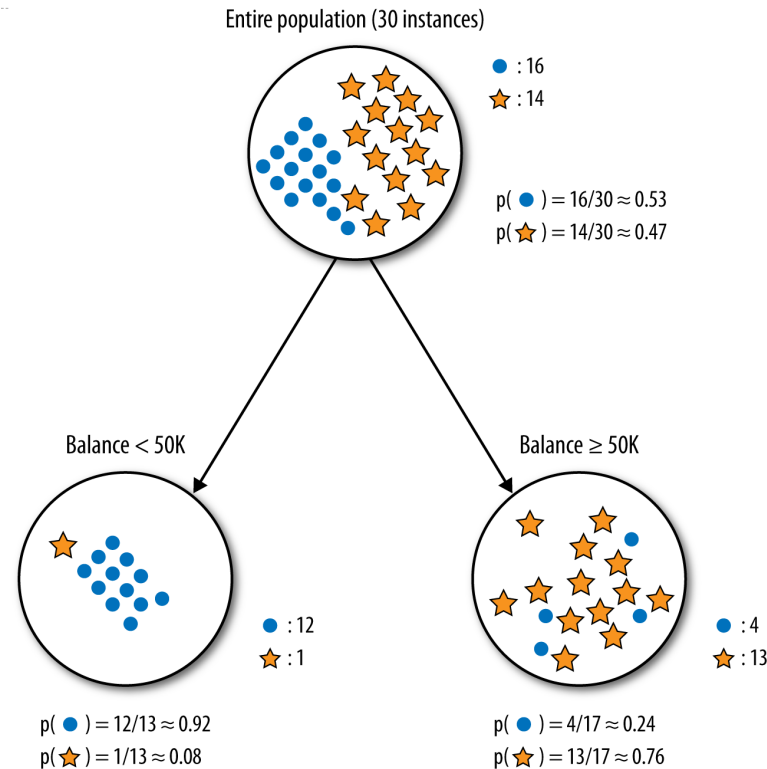
● : 4
★ : 13

$p(\bullet) = 4/17 \approx 0.24$
 $p(\star) = 13/17 \approx 0.76$

The entropy of the *right* child is:

$$\begin{aligned} \text{entropy}(\text{Balance} \geq 50K) &= -[p(\bullet) \times \log_2 p(\bullet) + p(\star) \times \log_2 p(\star)] \\ &\approx -[0.24 \times (-2.1) + 0.76 \times (-0.39)] \\ &= 0.79 \end{aligned}$$

Information Gain



$$\begin{aligned}
 IG &= \text{entropy}(\text{parent}) - [p(\text{Balance} < 50\text{K}) \times \text{entropy}(\text{Balance} < 50\text{K}) \\
 &\quad + p(\text{Balance} \geq 50\text{K}) \times \text{entropy}(\text{Balance} \geq 50\text{K})] \\
 &\approx 0.99 - [0.43 \times 0.39 + 0.57 \times 0.79] \\
 &\approx 0.37
 \end{aligned}$$

$$\text{Relative IG} = \text{IG}/\text{entropy}(\text{parent}) = 0.37/0.99 = 0.37$$

Attribute Selection

Reasons for selecting only a subset of attributes:

- Better insights and business understanding
- Better explanations and more tractable models
- Reduced cost
- Faster predictions
- Better predictions!
 - Over-fitting (*to be continued..*)

and also determining the most informative attributes.

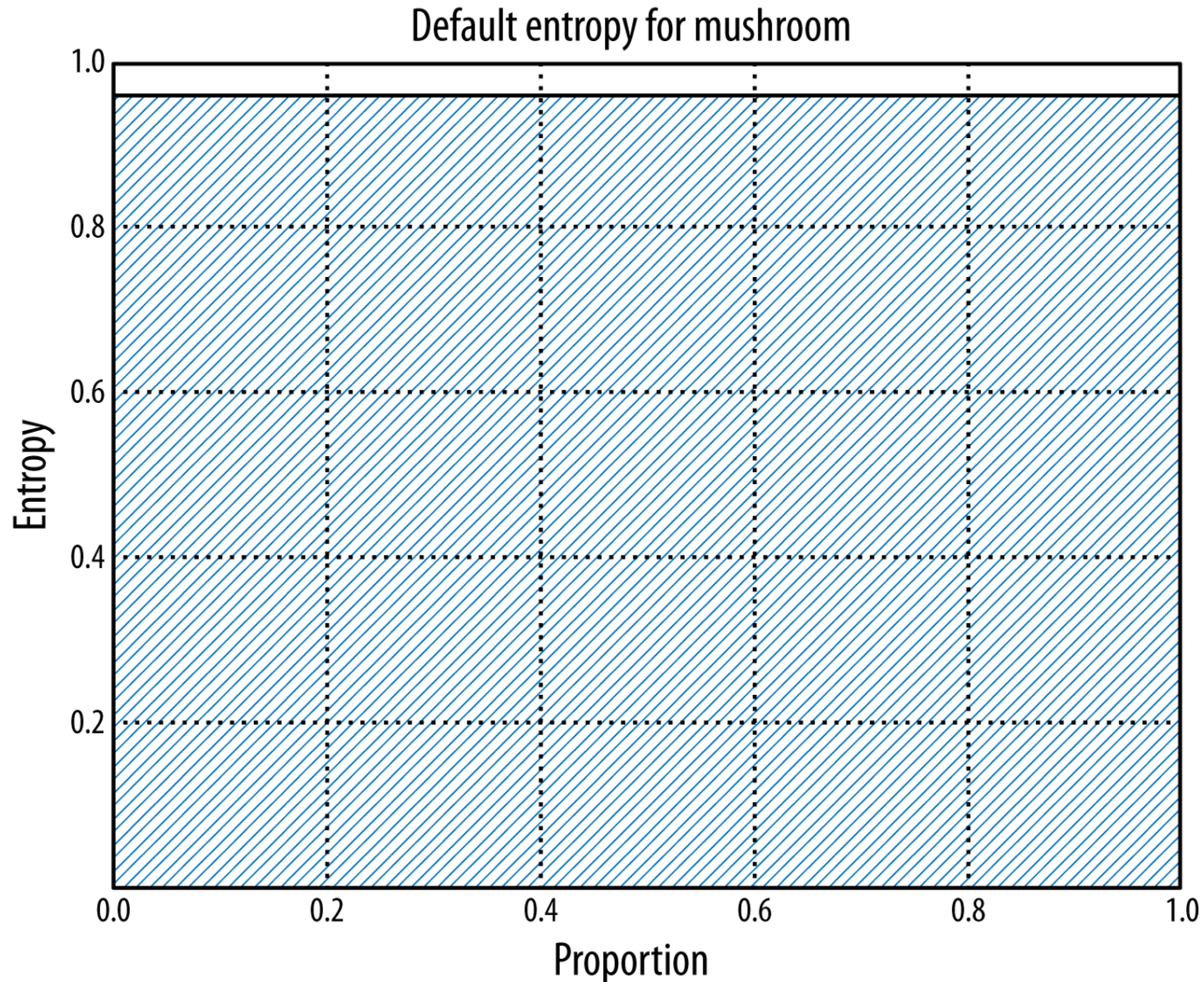
Example: Attribution Selection with Information Gain

- This dataset includes descriptions of hypothetical samples corresponding to 23 species of gilled mushrooms in the Agaricus and Lepiota Family
- Each species is identified as definitely edible, definitely poisonous, or of unknown edibility and not recommended
 - This latter class was combined with the poisonous one
- The Guide clearly states that there is no simple rule for determining the edibility of a mushroom; no rule like “leaflets three, let it be” for Poisonous Oak and Ivy

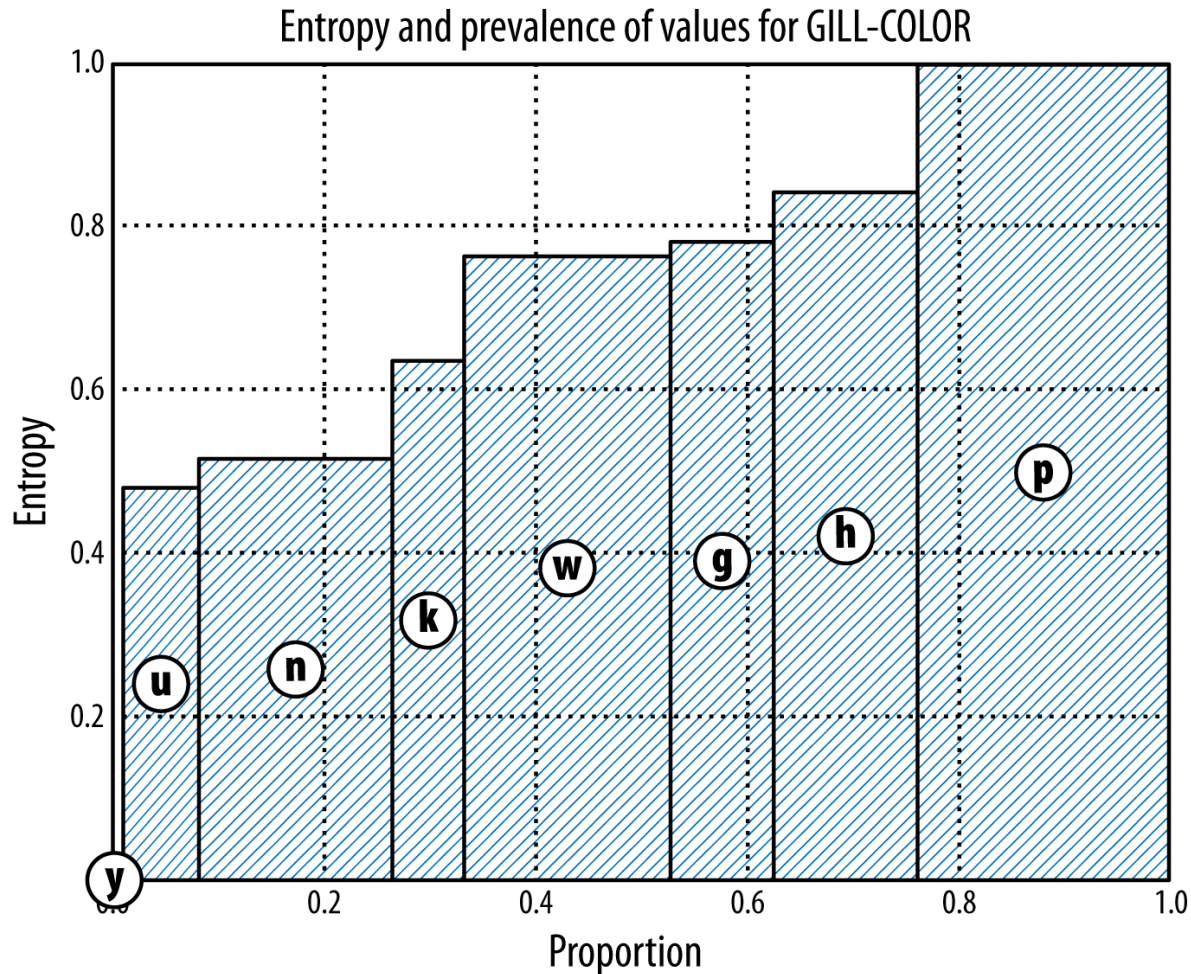
Example: Attribution Selection with Information Gain

Attribute name	Possible values	
CAP-SHAPE	bell, conical, convex, flat, knobbed,	<p>MUSHROOM</p> <p>cap, gills, ring, volva, scale, tubes, pores, stipe, stalk, scales</p> <p>www.infovisual.info</p>
CAP-SURFACE	fibrous, grooves, scaly, smooth	
CAP-COLOR	brown, buff, cinnamon, gray, green, pink, white, yellow	
BRUISES?	yes, no	
ODOR	almond, anise, creosote, fishy, foul, pungent, spicy	
GILL-ATTACHMENT	attached, descending, free, notched	
GILL-SPACING	close, crowded, distant	
GILL-SIZE	broad, narrow	
GILL-COLOR	black, brown, buff, chocolate, gray, green, orange, pink, purple, red, white, yellow	
STALK-SHAPE	enlarging, tapering	
STALK-ROOT	bulbous, club, cup, equal, rhizomorphs, rooted, missing	
STALK-SURFACE-ABOVE-RING	fibrous, scaly, silky, smooth	
STALK-SURFACE-BELOW-RING	fibrous, scaly, silky, smooth	

Example: Attribution Selection with Information Gain



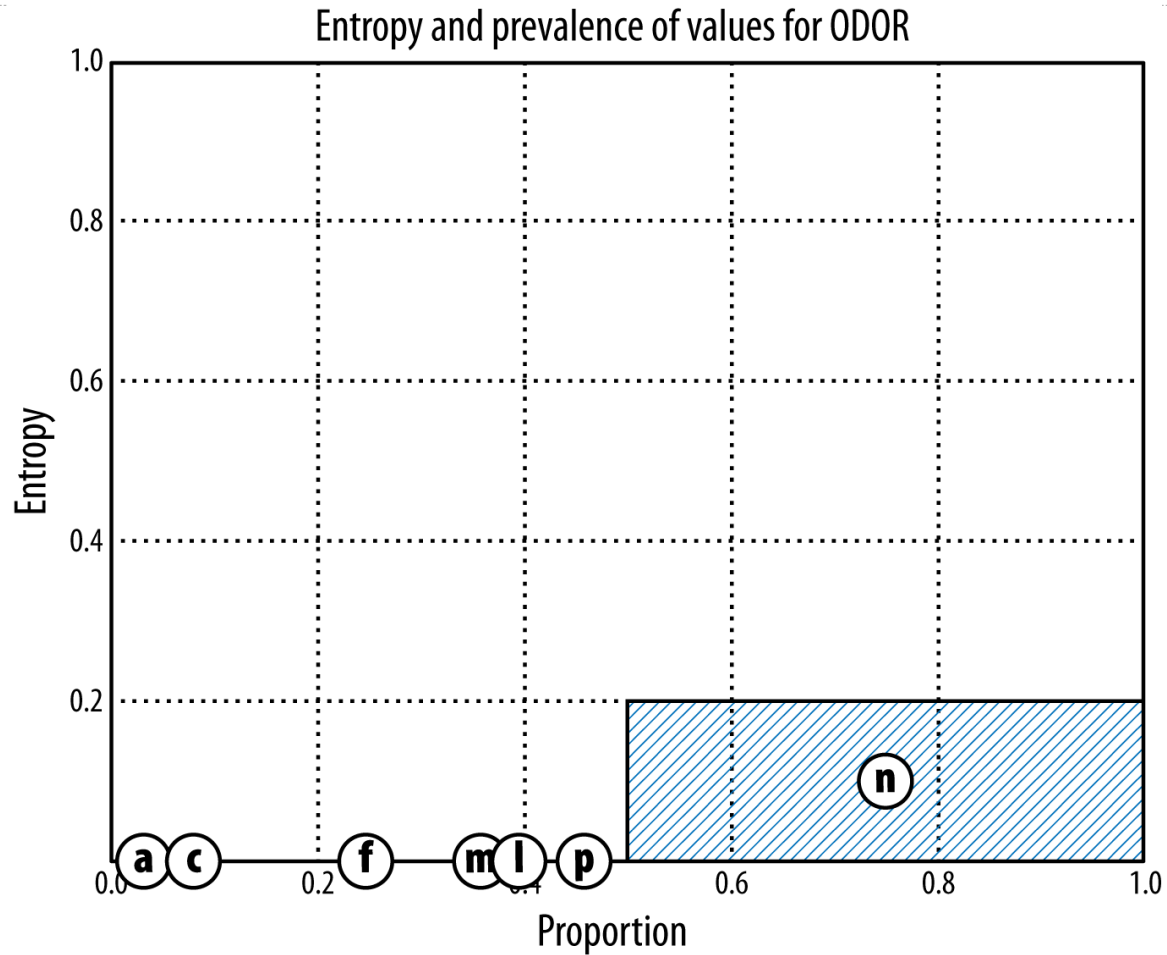
Example: Attribution Selection with Information Gain



GILL-COLOR

black, brown, buff, chocolate, gray, green, orange, pink,
purple, red, white, yellow

Example: Attribution Selection with Information Gain



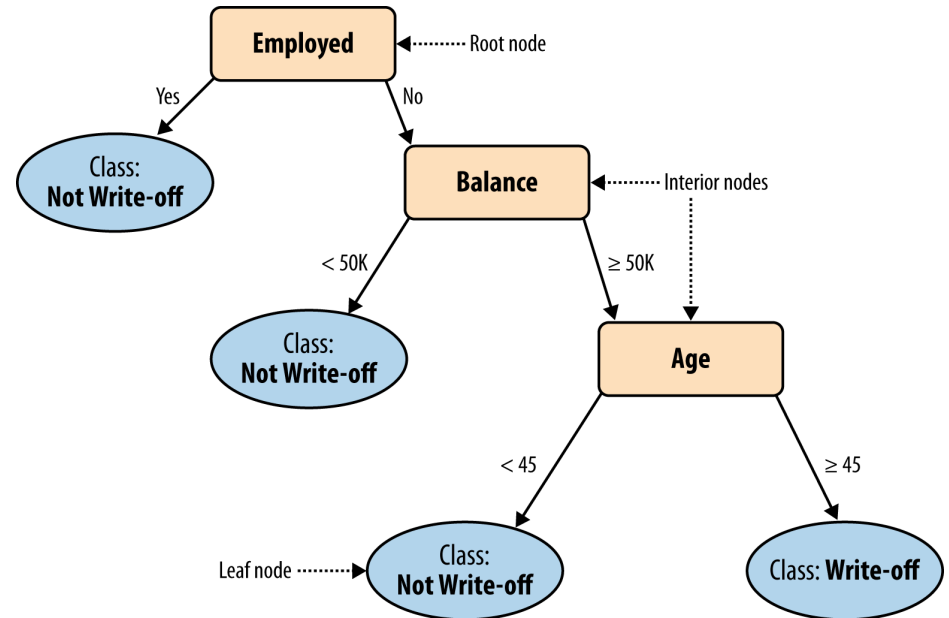
ODOR

almond, anise, creosote, fishy, foul, musty, none, pungent, spicy

Multivariate Supervised Classification

- If we select the *single* variable that gives the most information gain, we create a very *simple* classification
- If we select multiple attributes each giving some information gain, how do we put them together?

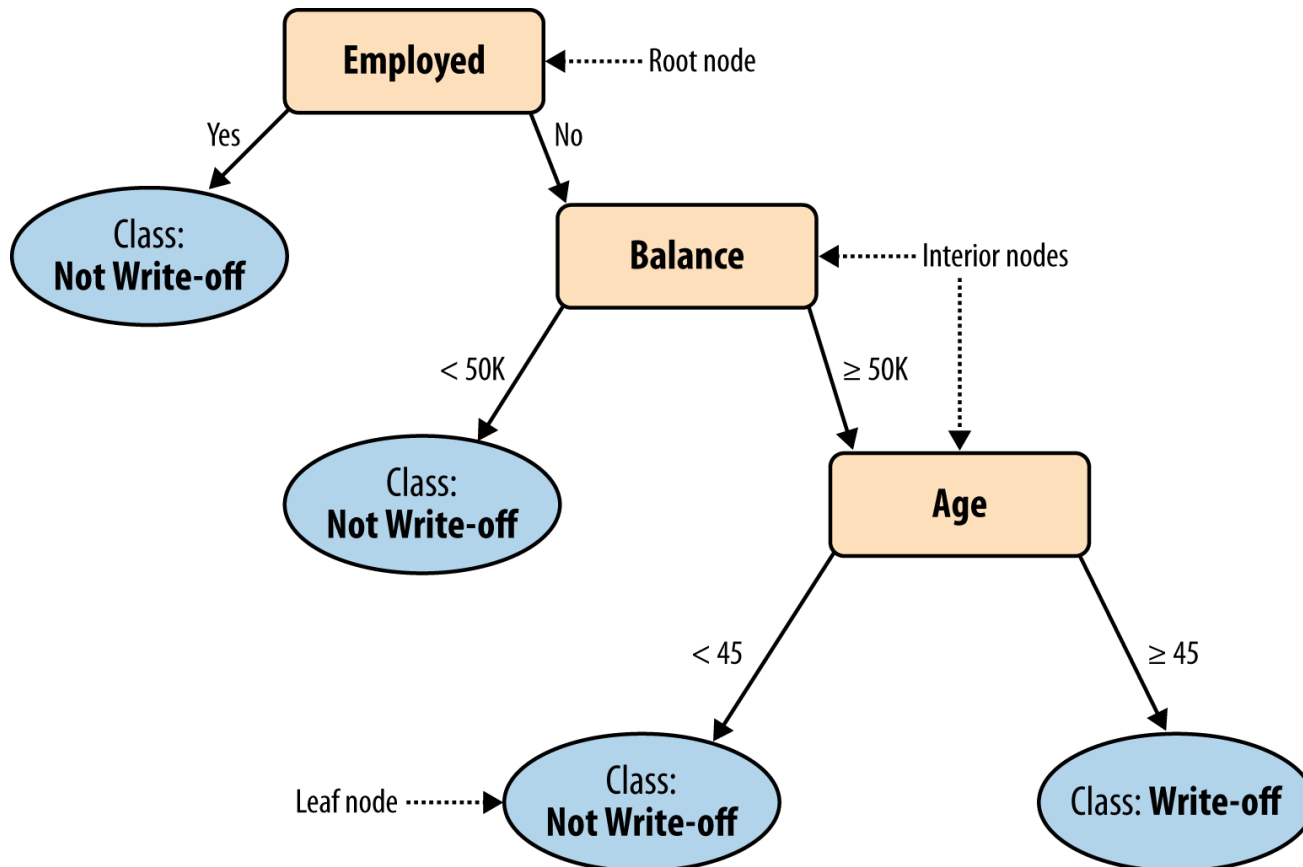
Tree-Structured Models



Write-off: not to pay off their account balances. i.e., defaulting on one's phone bill or credit card balance

Tree-Structured Models

- Classify 'John Doe'
 - Balance=115K, Employed=No, and Age=40

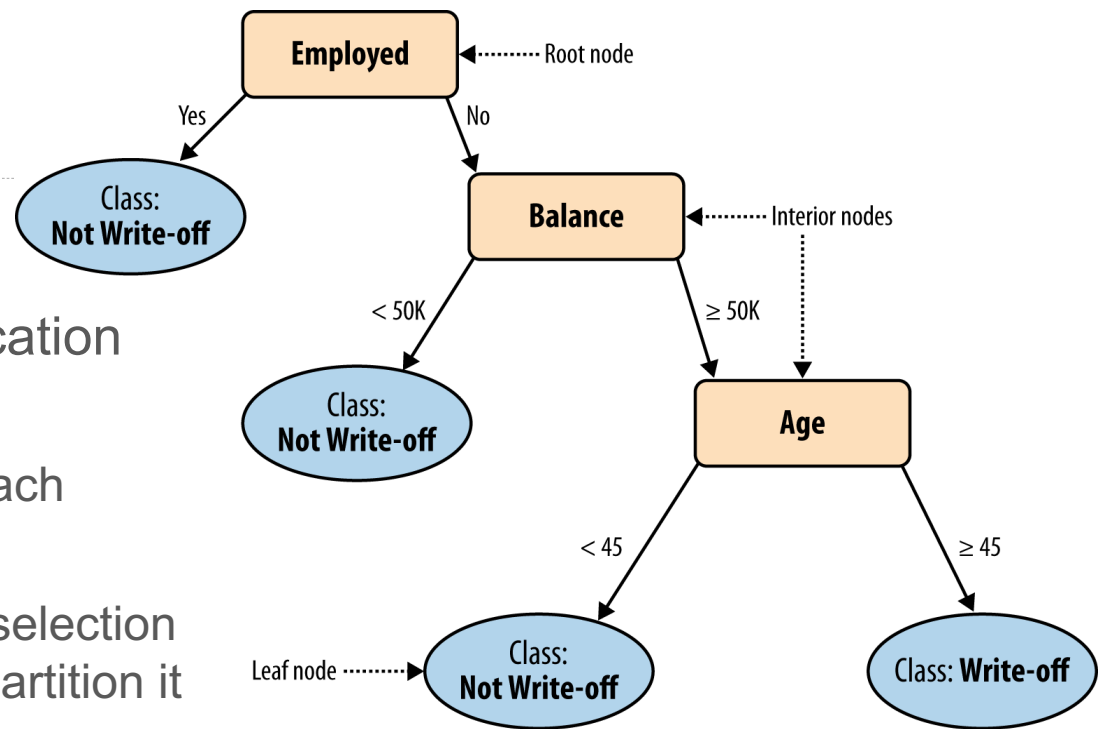


Tree-Structured Models: “Rules”

- No two parents share descendants
- There are no cycles
- The branches always “point downwards”
- Every example always ends up at a leaf node with some specific class determination
 - Probability estimation trees, regression trees (*to be continued..*)

Tree Induction

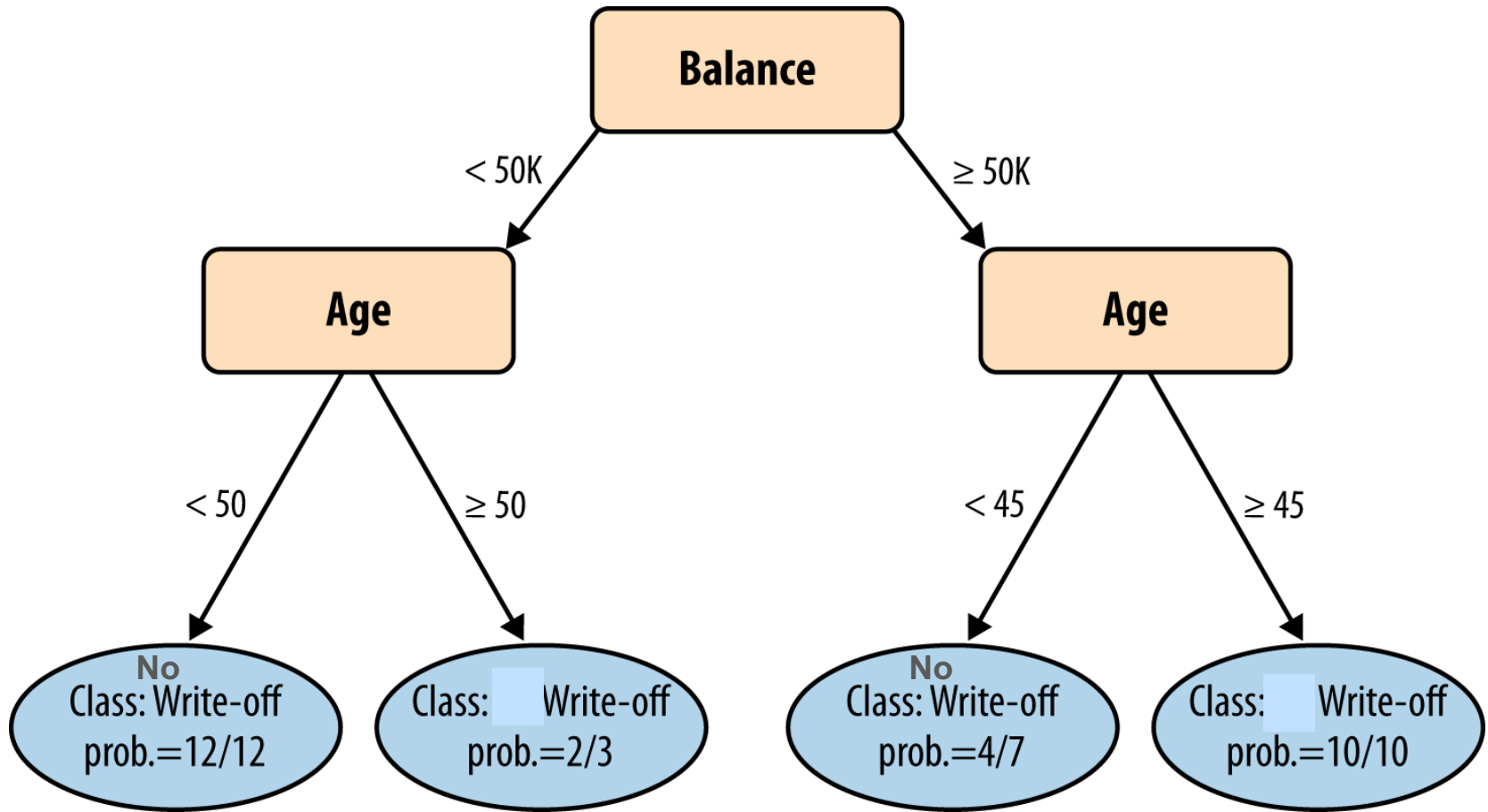
- How do we create a classification tree from data?
 - **divide-and-conquer** approach
 - take each data subset and **recursively** apply attribute selection to find the best attribute to partition it
- When do we stop?
 - The nodes are pure,
 - there are no more variables, or
 - even earlier (over-fitting – *to be continued..*)



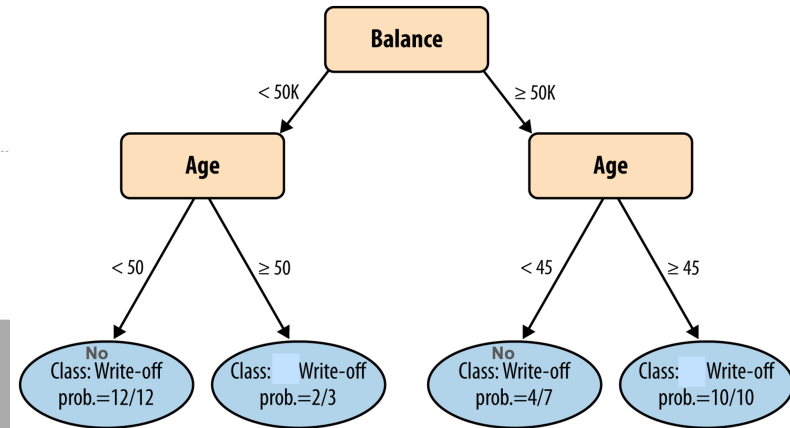
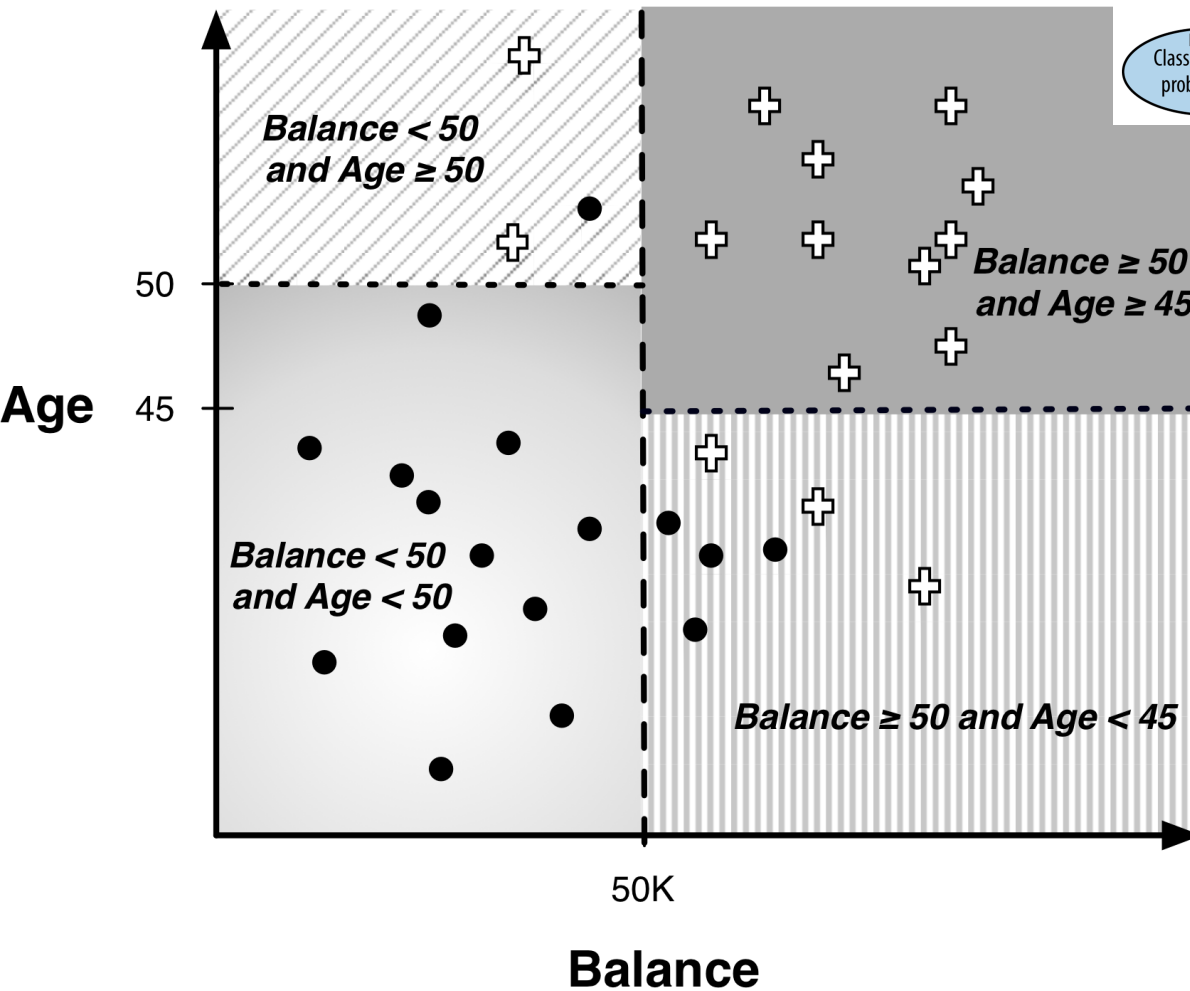
Why trees?

- Decision trees (DTs), or classification trees, are one of the most popular data mining tools
 - (along with linear and logistic regression)
- They are:
 - Easy to understand
 - Easy to implement
 - Easy to use
 - Computationally cheap
- Almost all data mining packages include DTs
- They have advantages for model comprehensibility, which is important for:
 - model evaluation
 - communication to non-DM-savvy stakeholders

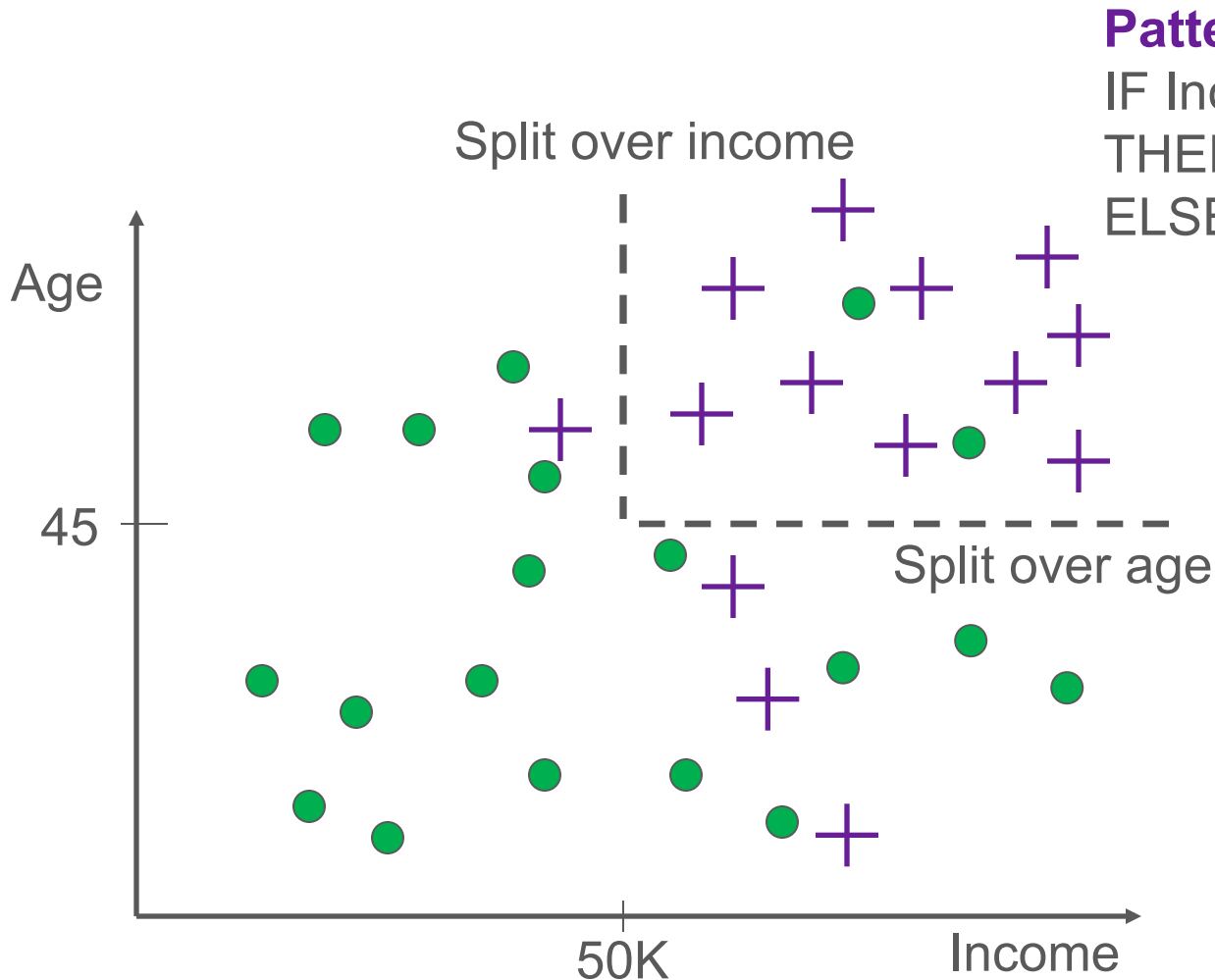
Visualizing Classifications



Visualizing Classifications



Geometric interpretation of a model



Pattern:

IF $\text{Income} \geq 50\text{K}$ & $\text{Age} > 45$

THEN Buy_Insurance = 'yes' +

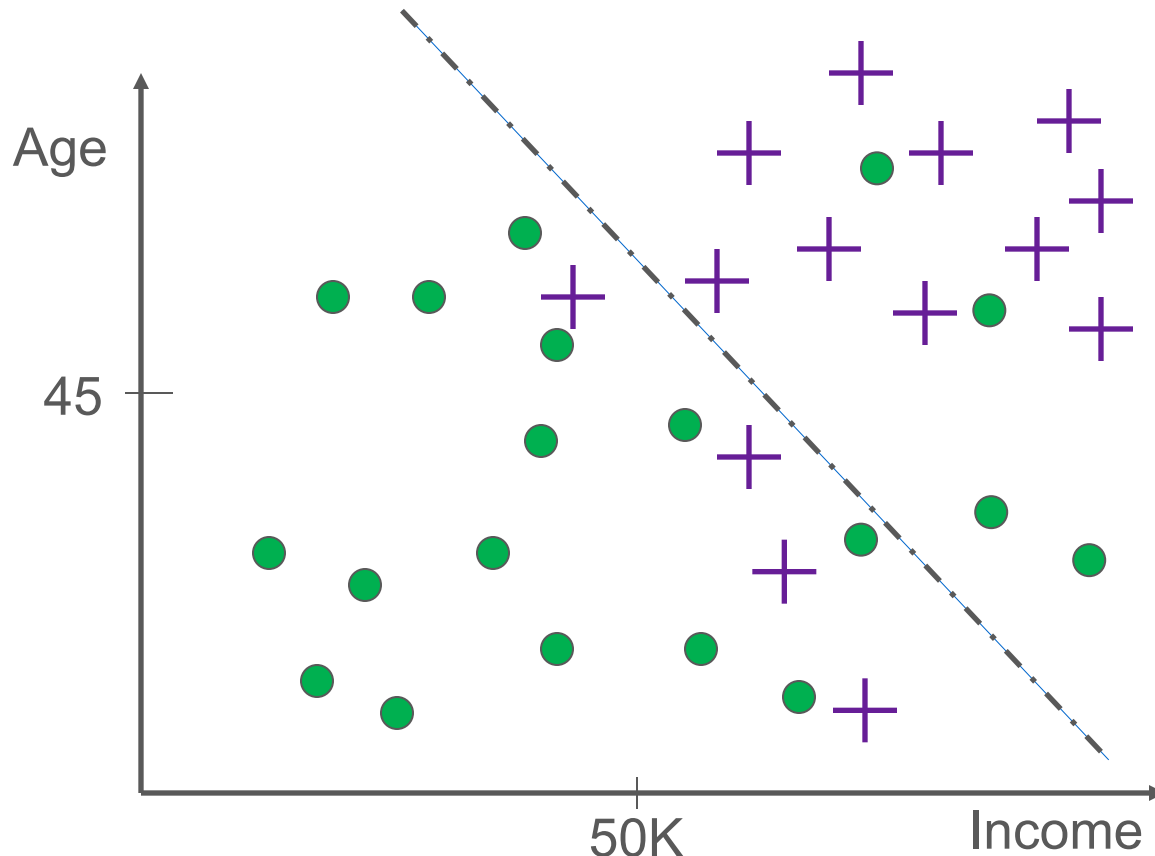
ELSE Buy_Insurance = 'no' ●

● Did not buy life insurance

| Bought life insurance

Geometric interpretation of a model

What alternatives are there to partitioning this way?



“True” boundary may not be closely approximated by a linear boundary!

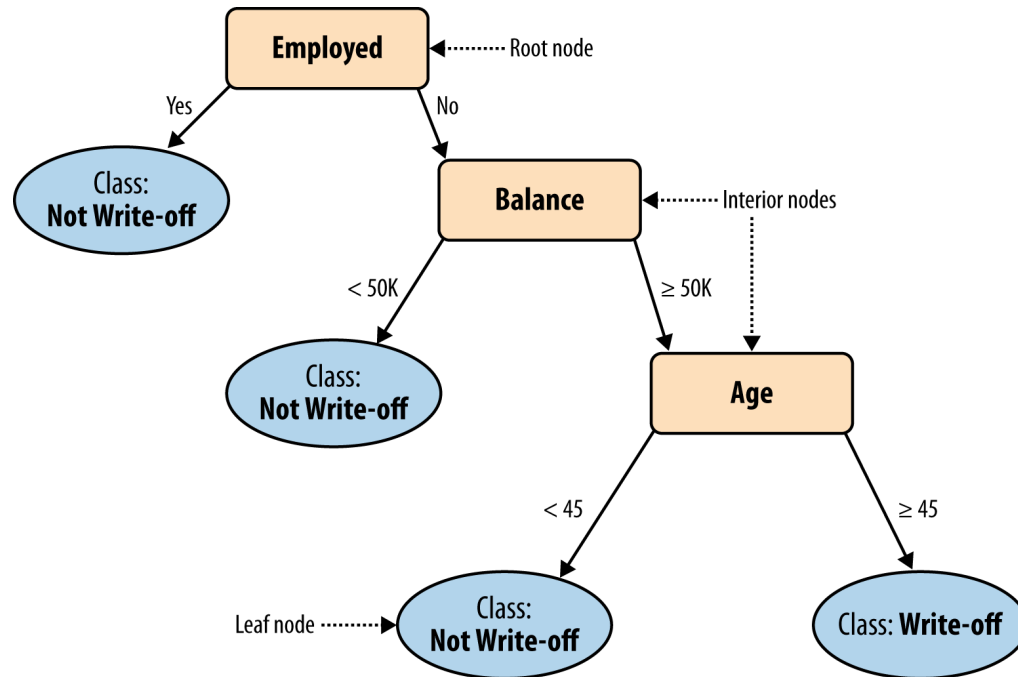
● Did not buy life insurance

+ Bought life insurance

Trees as Sets of Rules

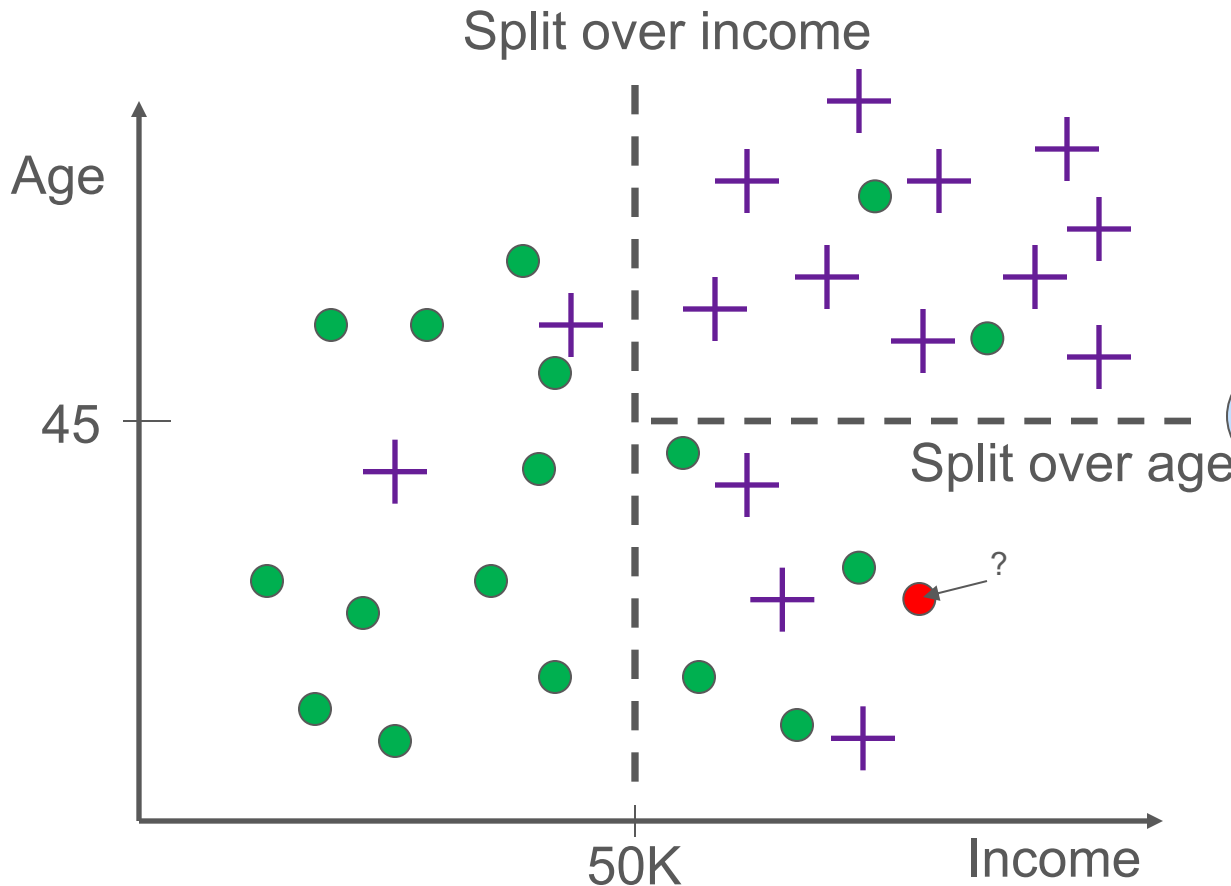
- The classification tree is equivalent to this rule set
- Each rule consists of the attribute tests along the path connected with **AND**

Trees as Sets of Rules: Interpretability

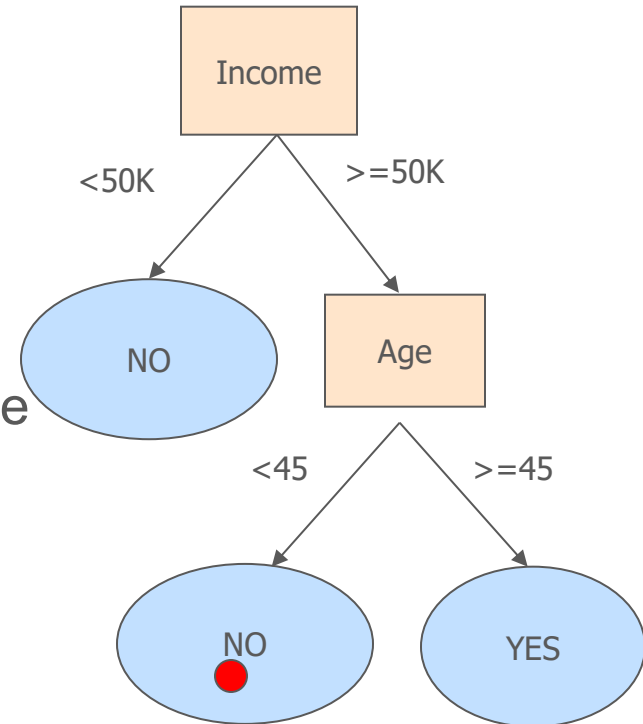


- IF (Employed = Yes) THEN Class=No Write-off
- IF (Employed = No) AND (Balance < 50k) THEN Class=No Write-off
- IF (Employed = No) AND (Balance ≥ 50k) AND (Age < 45) THEN Class=No Write-off
- IF (Employed = No) AND (Balance ≥ 50k) AND (Age ≥ 45) THEN Class=Write-off

What are we predicting?



Classification tree

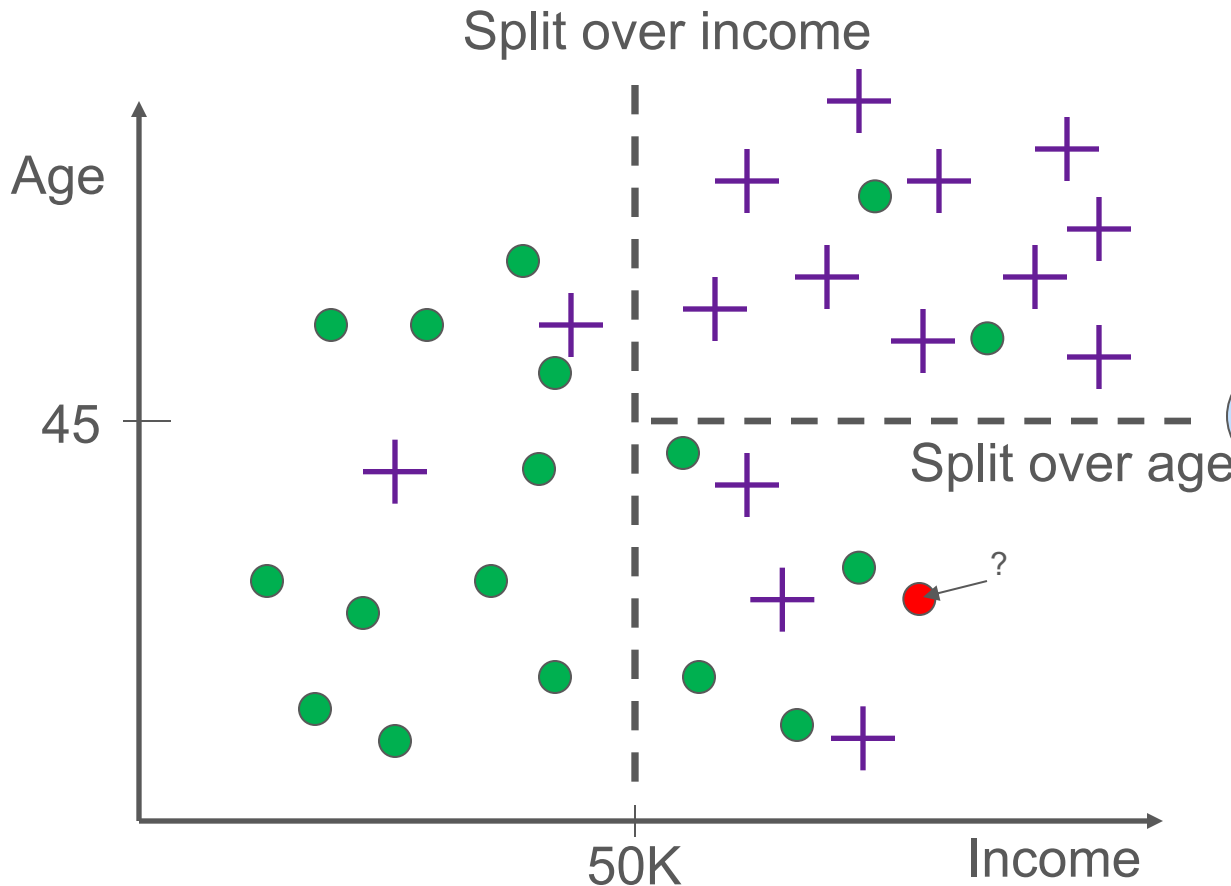


● Did not buy life insurance

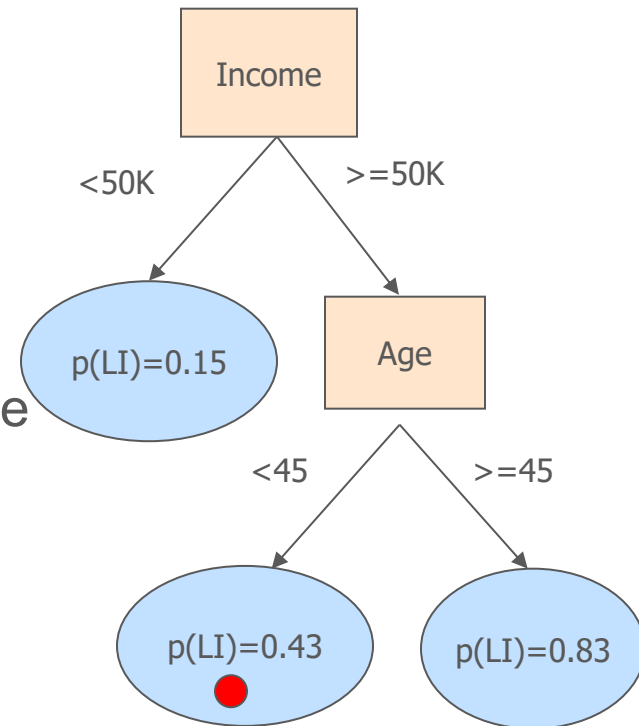
+ Bought life insurance

● Interested in LI? = NO

What are we predicting?



Classification tree



● Did not buy life insurance

+ Bought life insurance

● Interested in LI? = 3/7

Questions?