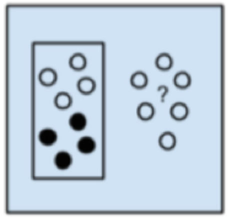
Deep Reinforcement Learning
MIT 6.S191

Alexander Amini

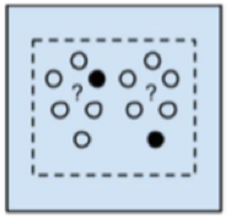
January 30, 2019

Play Video@ 01:00

Types of Deep Learning

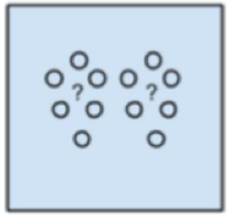


Supervised Learning



Semi-Supervised Learning

Reinforcement Learning



Unsupervised Learning

Classes of Learning Problems

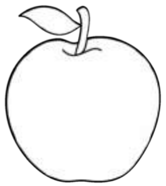
Supervised Learning

Data: (x, y)
x is data, y is label

Goal: Learn function to map

$$x \rightarrow y$$

Apple example:



This thing is an apple.

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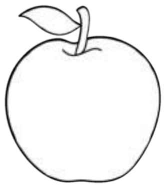

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Unsupervised Learning

Data: x
x is data, no labels!

Goal: Learn underlying structure

Apple example:

This thing is like
the other thing.

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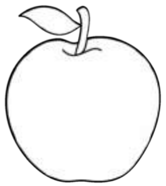

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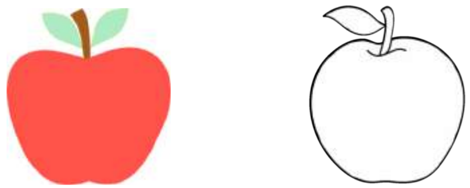

This thing is like the other thing.

Reinforcement Learning

Data: state-action pairs

Goal: Maximize future rewards over many time steps

Apple example:

Eat this thing because it will keep you alive.

Classes of Learning Problems

Supervised Learning

Data: (x, y)
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Goal: Learn function to map
$x \to y$

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This is an apple.

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Reinforcement Learning

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RL: our focus today.

Reinforcement Learning (RL): Key Concepts

AGENT

Agent: takes actions.

Reinforcement Learning (RL): Key Concepts

AGENT

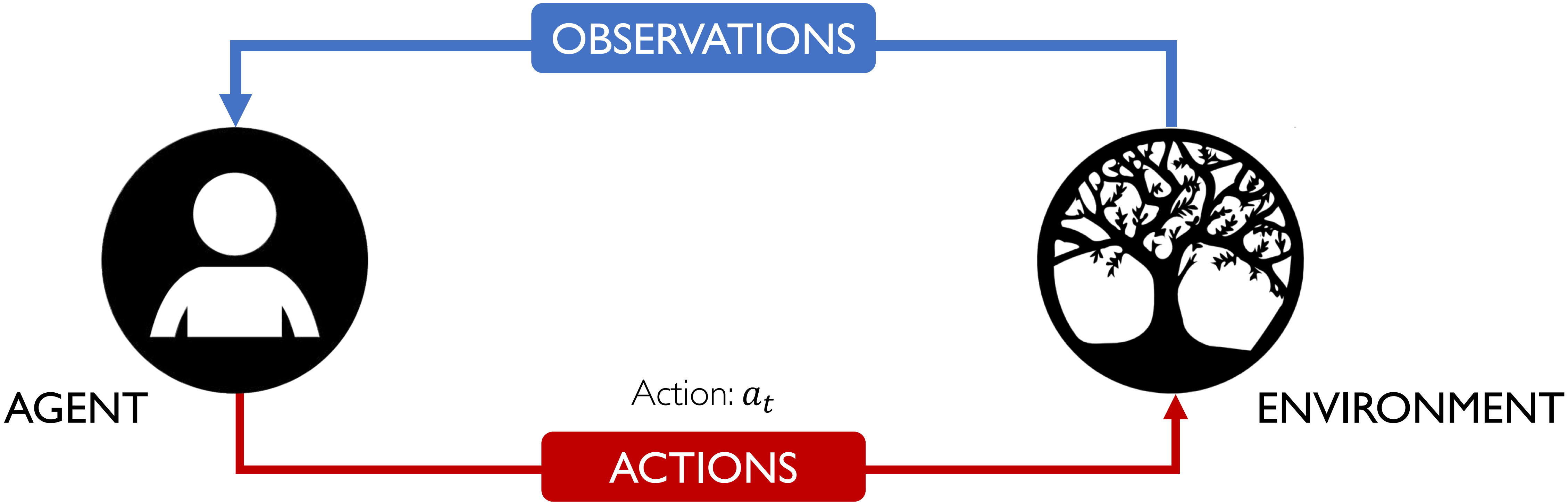
ENVIRONMENT

Environment: the world in which the agent exists and operates.

Reinforcement Learning (RL): Key Concepts

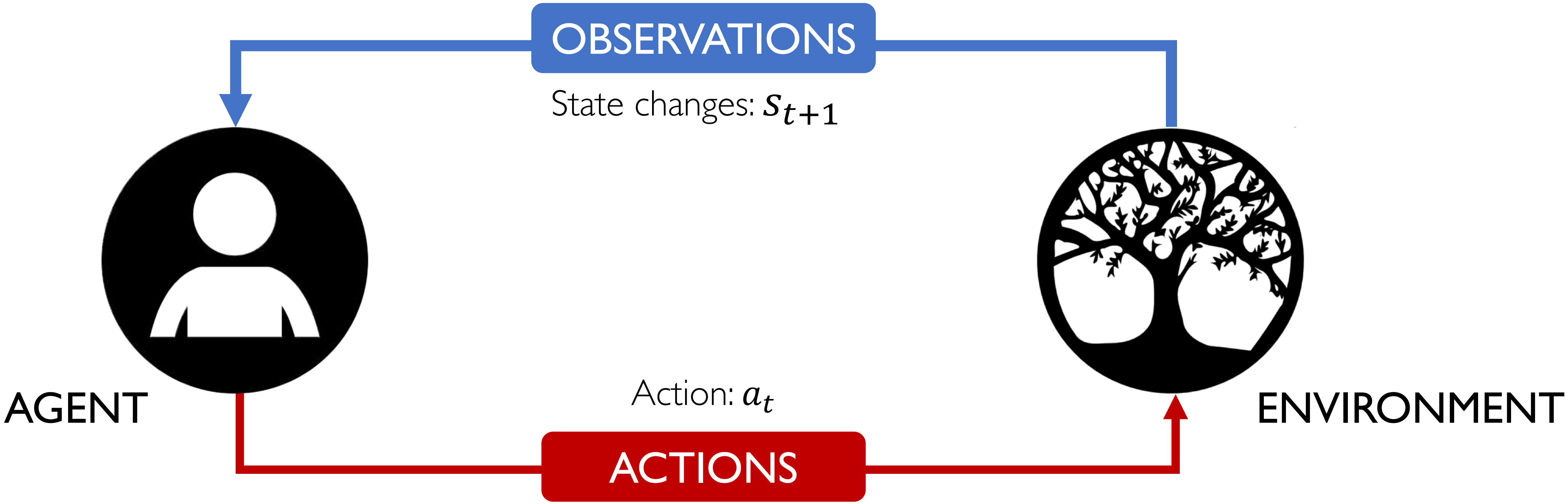
Action: a move the agent can make in the environment.

Reinforcement Learning (RL): Key Concepts



Observations: of the environment after taking actions.

Reinforcement Learning (RL): Key Concepts



State: a situation which the agent perceives.

Reinforcement Learning (RL): Key Concepts

Reward: feedback that measures the success or failure of the agent's action.

Reinforcement Learning (RL): Key Concepts

Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i$$

Reinforcement Learning (RL): Key Concepts

Total Reward

$$R_t = \sum_{i=t}^{\infty} r_i = r_t + r_{t+1} \ldots + r_{t+n} + \cdots$$

Reinforcement Learning (RL): Key Concepts

Discounted Total Reward

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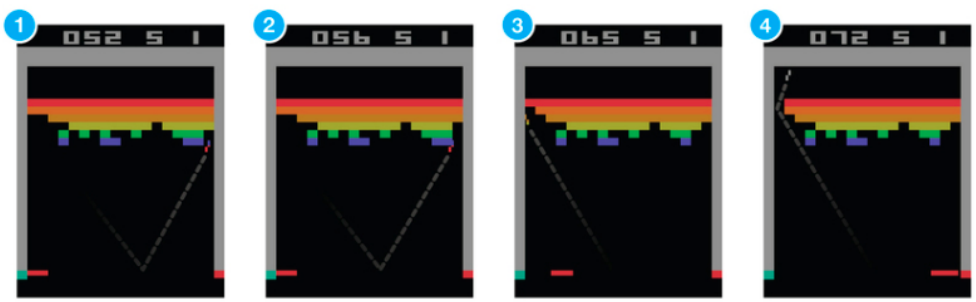
γ: discount factor

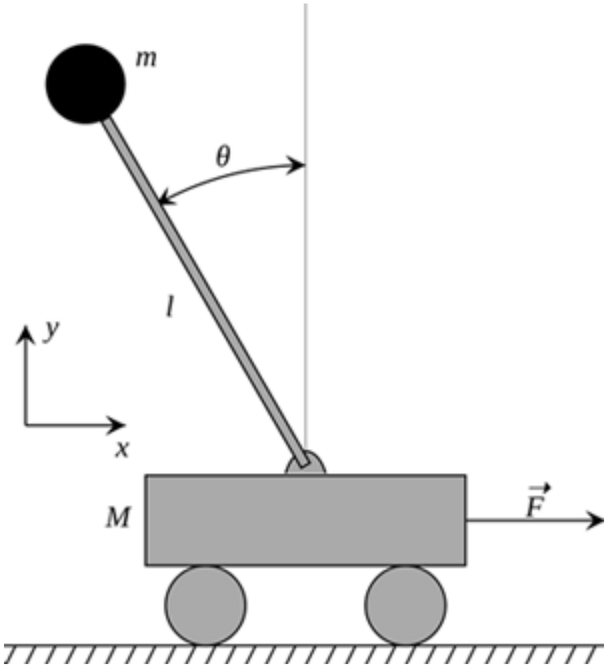
Examples of Reinforcement Learning

Reinforcement learning is a general-purpose framework for decision-making:

- An agent operates in an environment: **Atari Breakout**
- An agent has the capacity to **act**
- Each action influences the agent's **future state**
- Success is measured by a **reward** signal
- **Goal** is to select actions to maximize future reward



Examples of Reinforcement Learning



Cart-Pole Balancing

- **Goal** — Balance the pole on top of a moving cart
- **State** — Pole angle, angular speed. Cart position, horizontal velocity.
- **Actions** — horizontal force to the cart
- **Reward** — 1 at each time step if the pole is upright

Examples of Reinforcement Learning

Doom

- **Goal** — Eliminate all opponents
- **State** — Raw game pixels of the game
- **Actions** — Up, Down, Left, Right etc
- **Reward** — Positive when eliminating an opponent, negative when the agent is eliminated

Examples of Reinforcement Learning

Bin Packing

- **Goal** - Pick a device from a box and put it into a container
- **State** - Raw pixels of the real world
- **Actions** - Possible actions of the robot
- **Reward** - Positive when placing a device successfully, negative otherwise

Defining the Q-function

$$R_t = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \cdots$$

Total reward, R_t, is the discounted sum of all rewards obtained from time t

$$Q(s, a) = \mathbb{E}[R_t]$$

The Q-function captures the **expected total future reward** an agent in state, s, can receive by executing a certain action, a

How to take actions given a Q-function?

$$Q(s, a) = \mathbb{E}[R_t]$$

(state, action)

Ultimately, the agent needs a **policy $\boldsymbol{\pi(s)}$**, to infer the **best action to take** at its state, s

Strategy: the policy should choose an action that maximizes future reward

$$\pi^*(s) = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Deep Reinforcement Learning Algorithms

Value Learning

Find $Q(s, a)$

$$a = \underset{a}{\operatorname{argmax}}\, Q(s, a)$$

Policy Learning

Find $\pi(s)$

Sample $a \sim \pi(s)$

Deep Reinforcement Learning Algorithms

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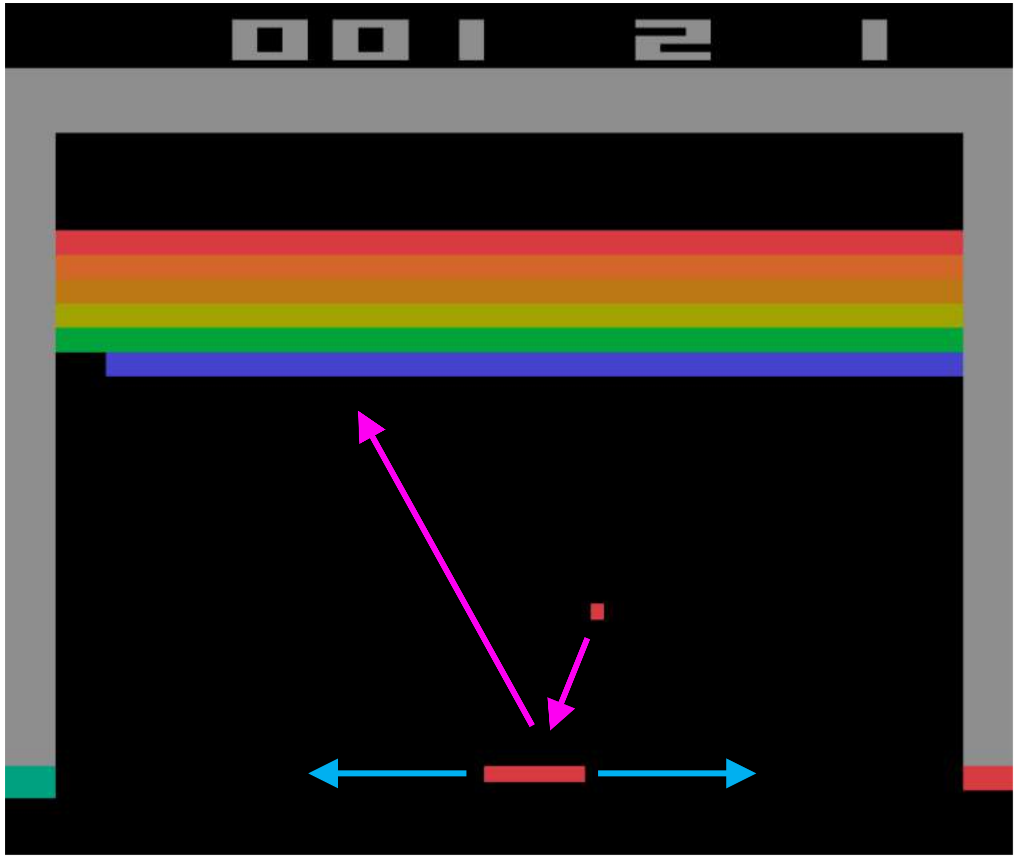
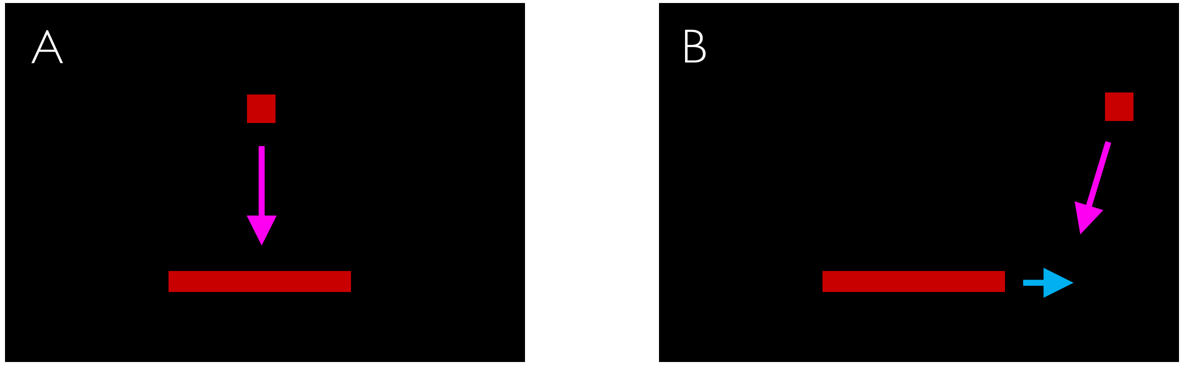
Digging deeper into the Q-function

Example: Atari Breakout



It can be very difficult for humans to accurately estimate Q-values



Which (s, a) pair has a higher Q-value?

Digging deeper into the Q-function

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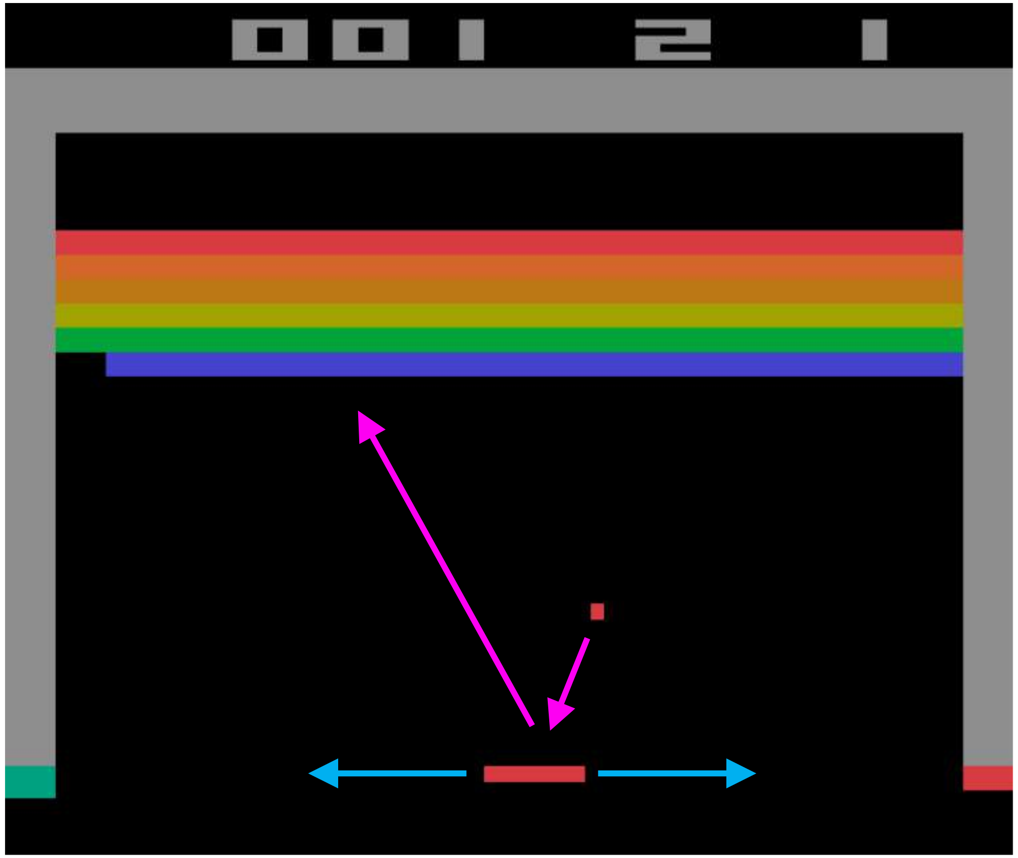

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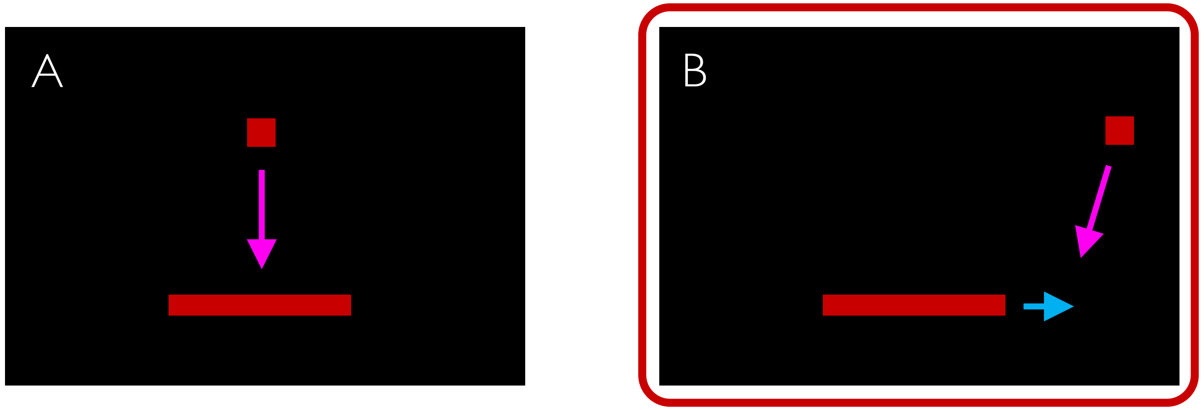
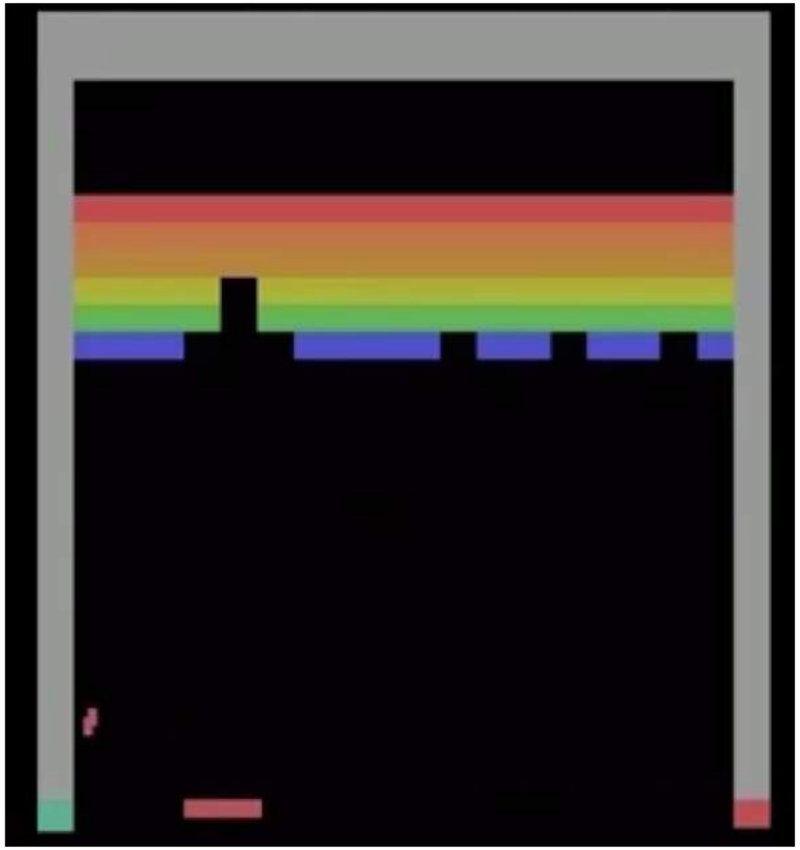


Which (s, a) pair has a higher Q-value?

Digging deeper into the Q-function

Example: Atari Breakout - Middle



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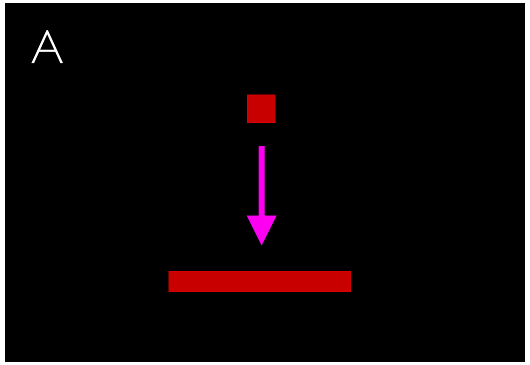
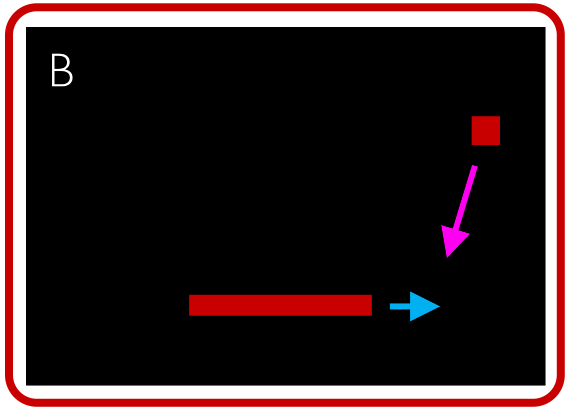
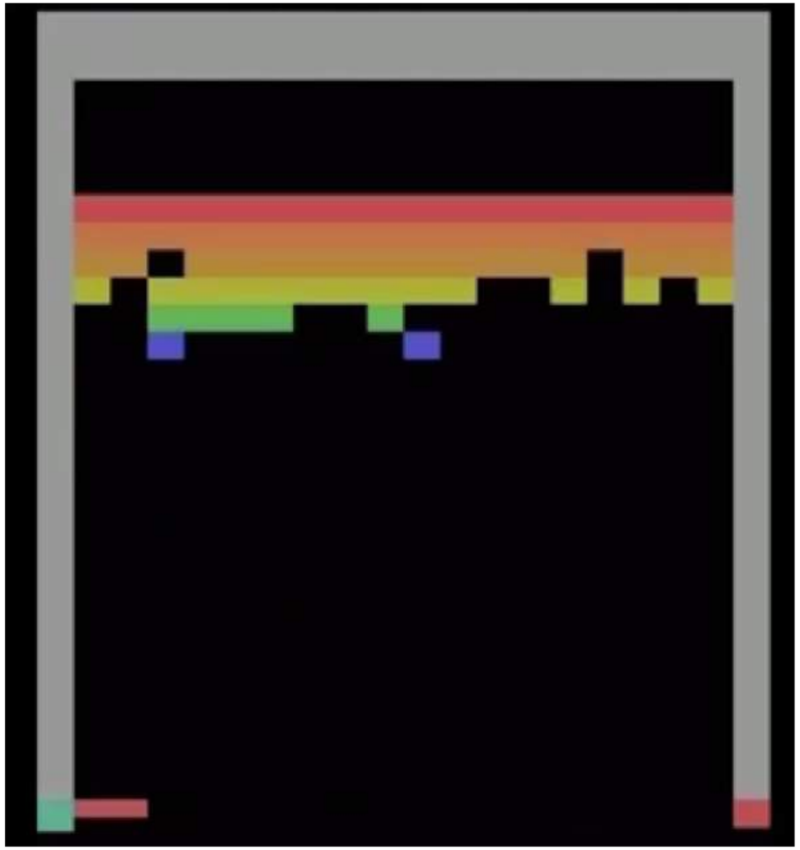




Which (s, a) pair has a higher Q-value?

Digging deeper into the Q-function

Example: Atari Breakout - Side



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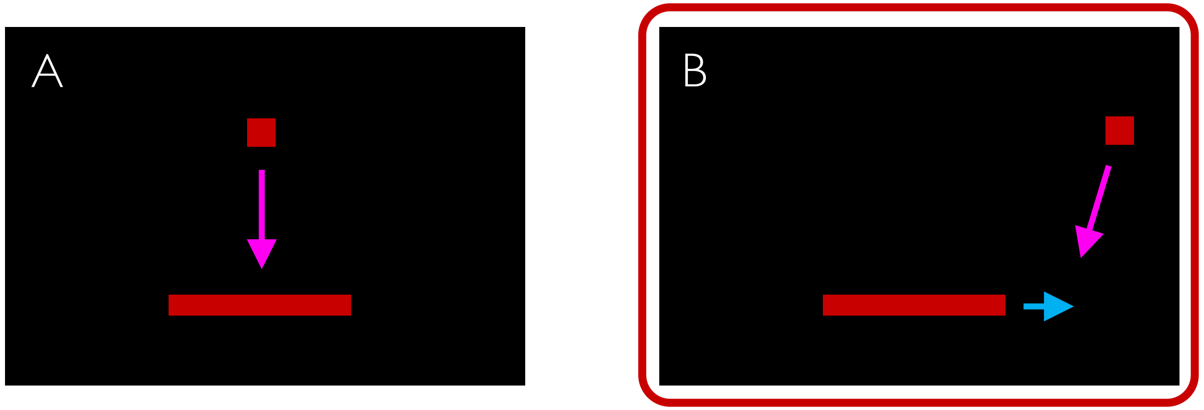

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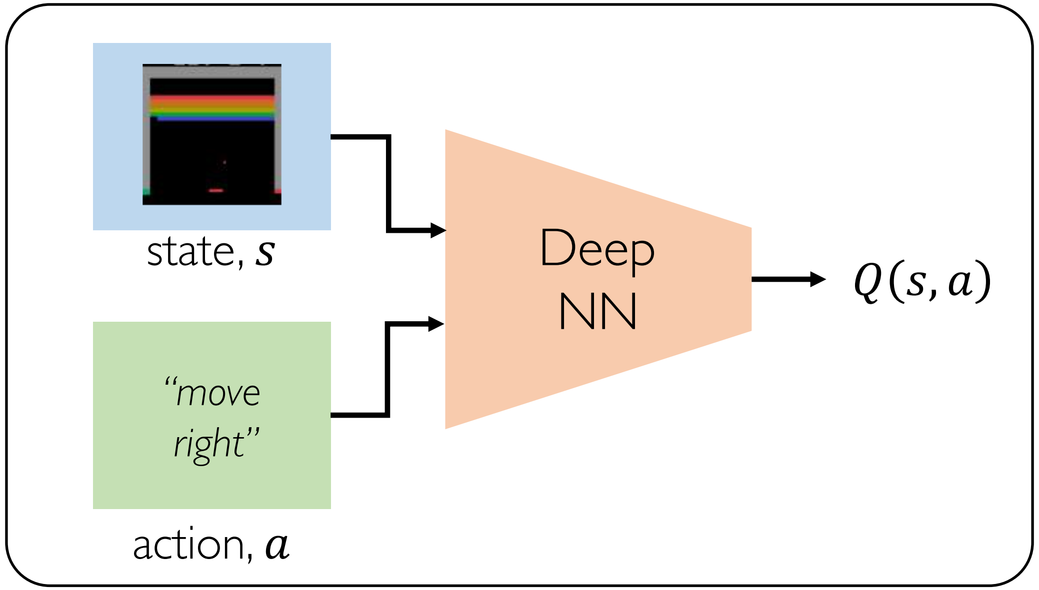
Deep Q Networks (DQN)

How can we use deep neural networks to model Q-functions?



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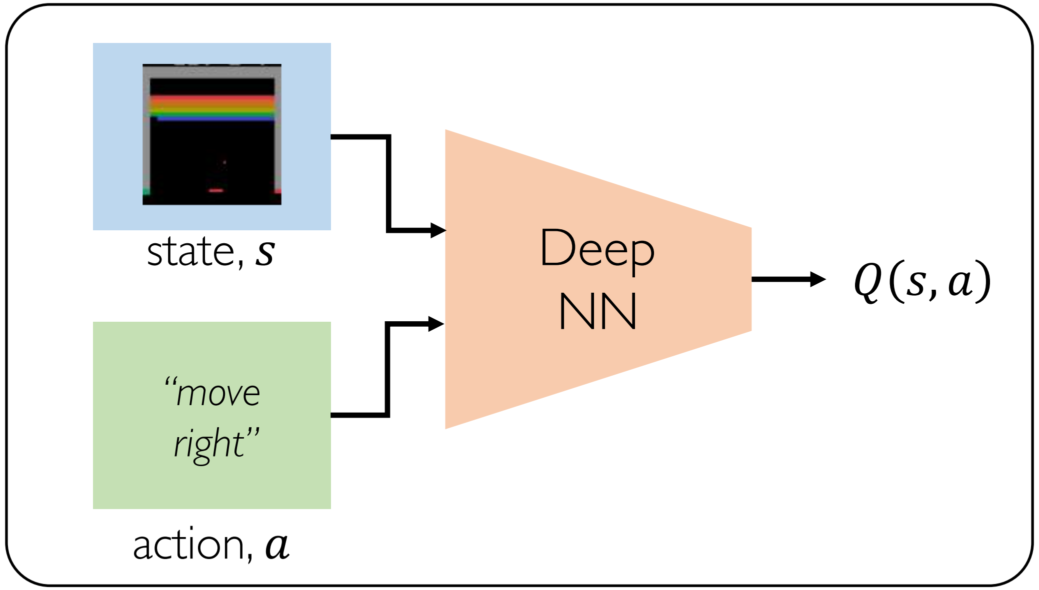

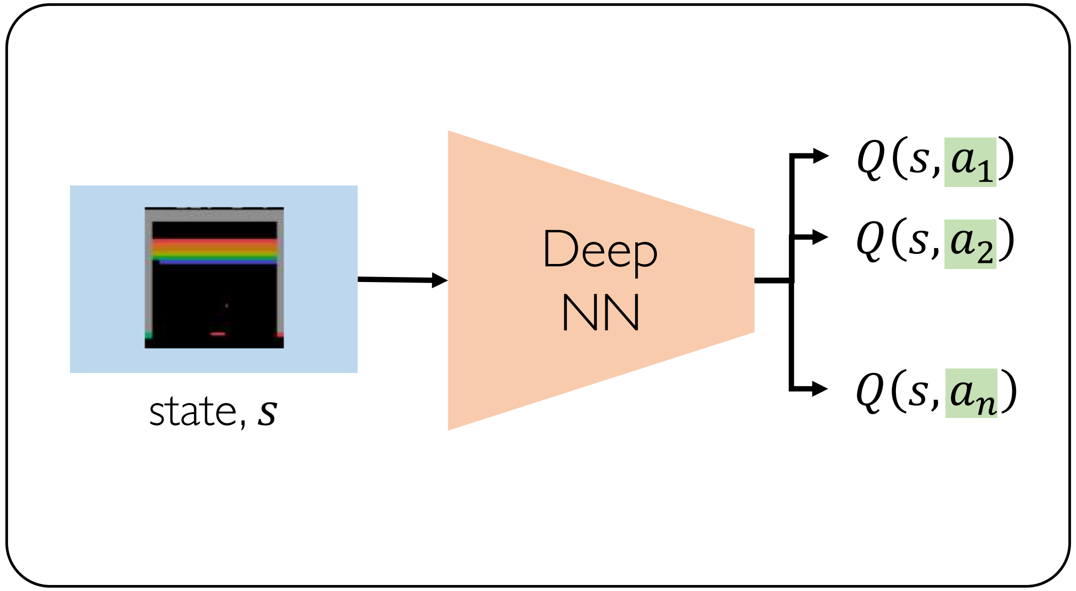

Deep Q Networks (DQN): Training

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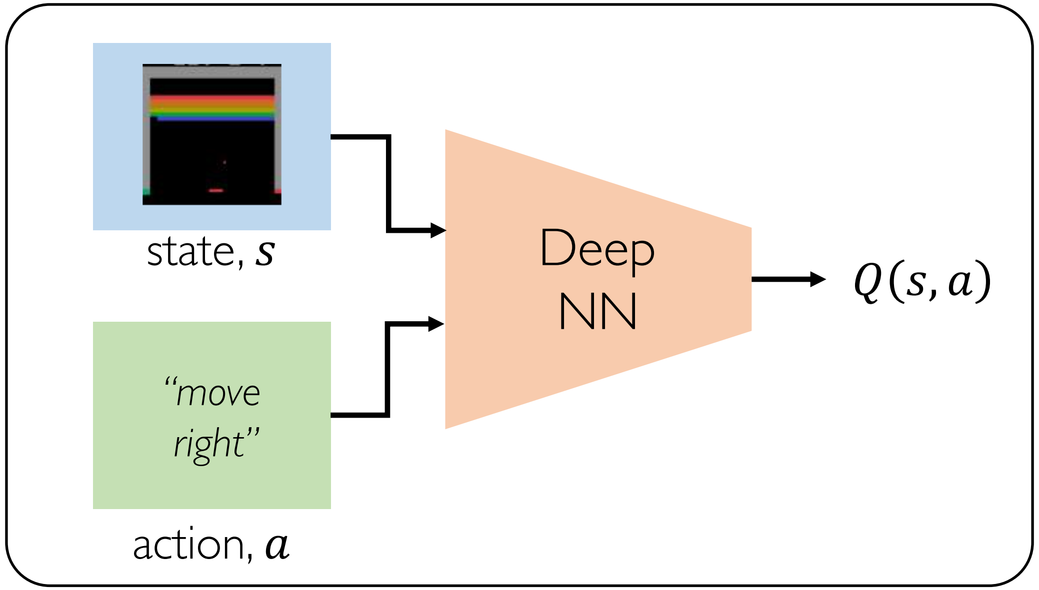

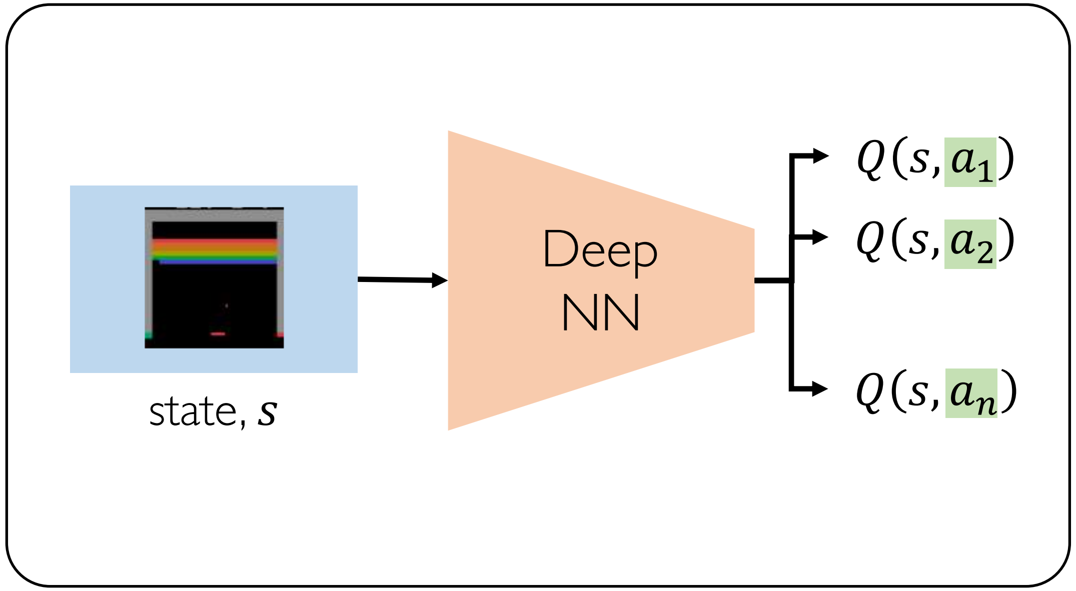

$$\mathcal{L} = \mathbb{E}\left[\left\| \left(r + \gamma \max_{a'} Q(s', a')\right) - Q(s, a) \right\|^2\right]$$

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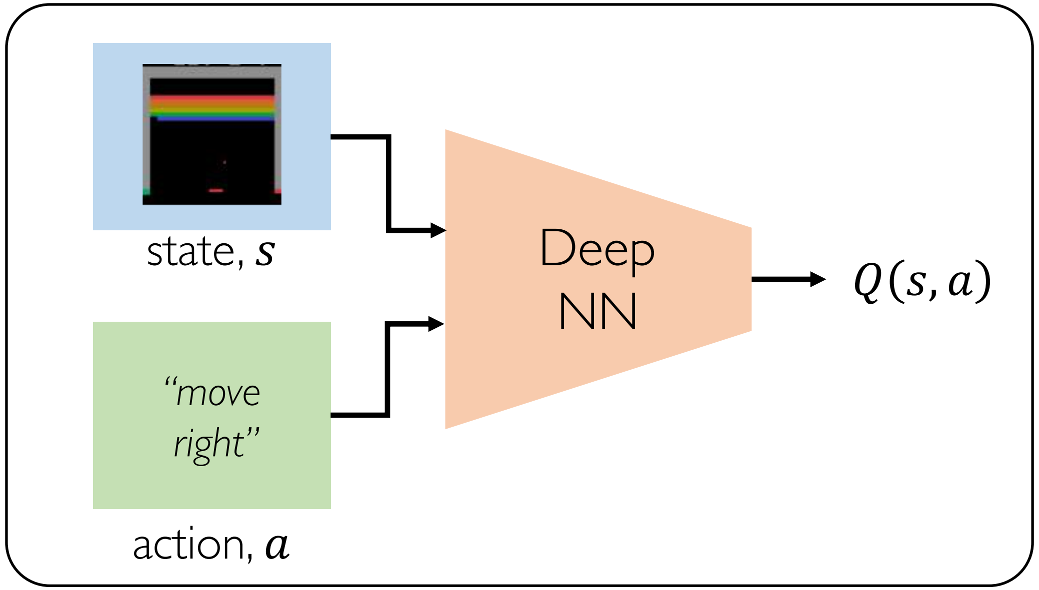

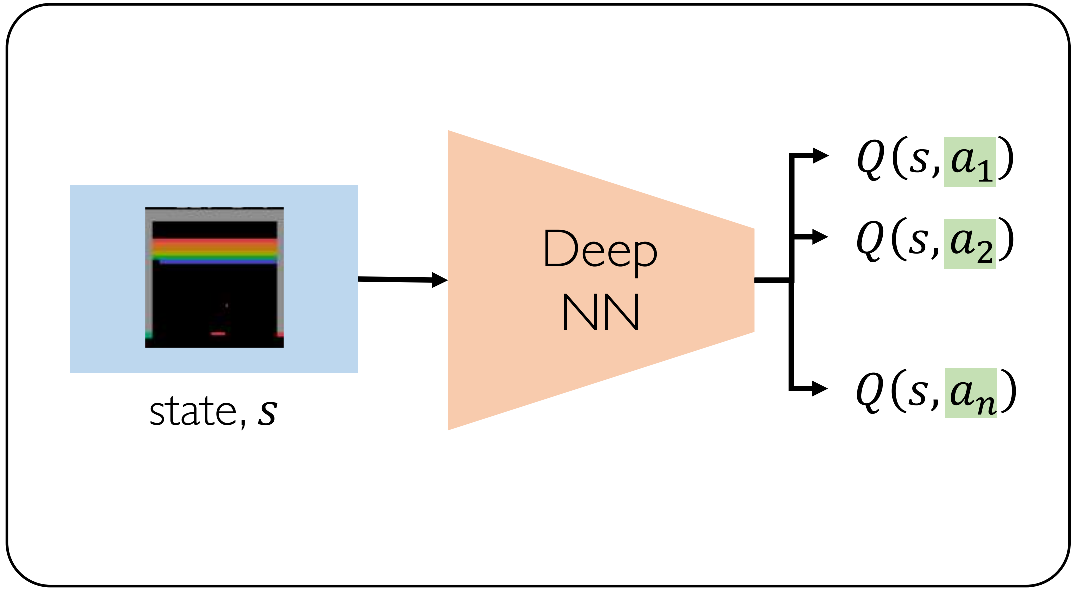

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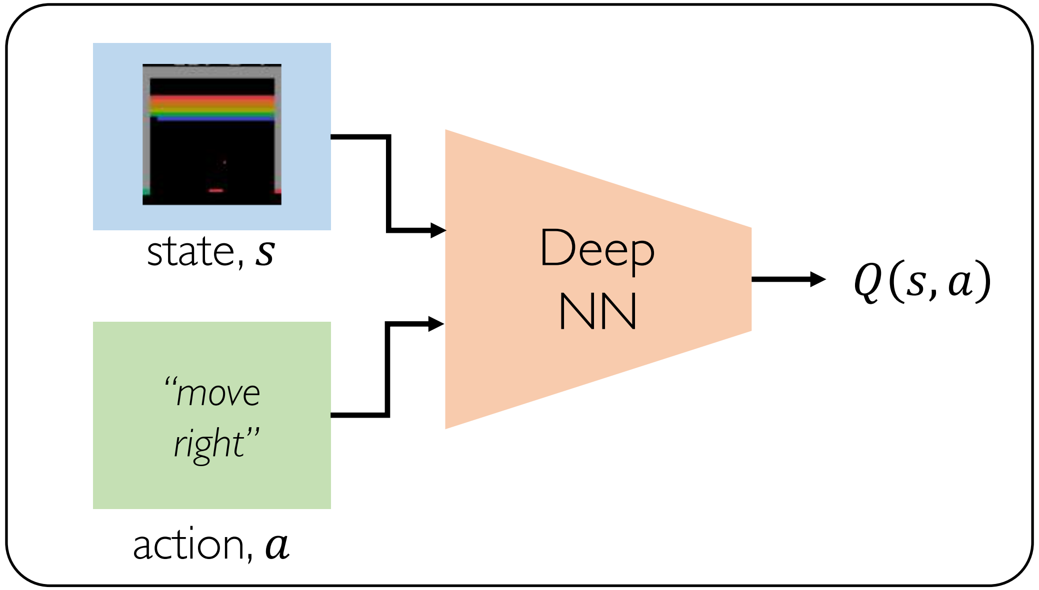

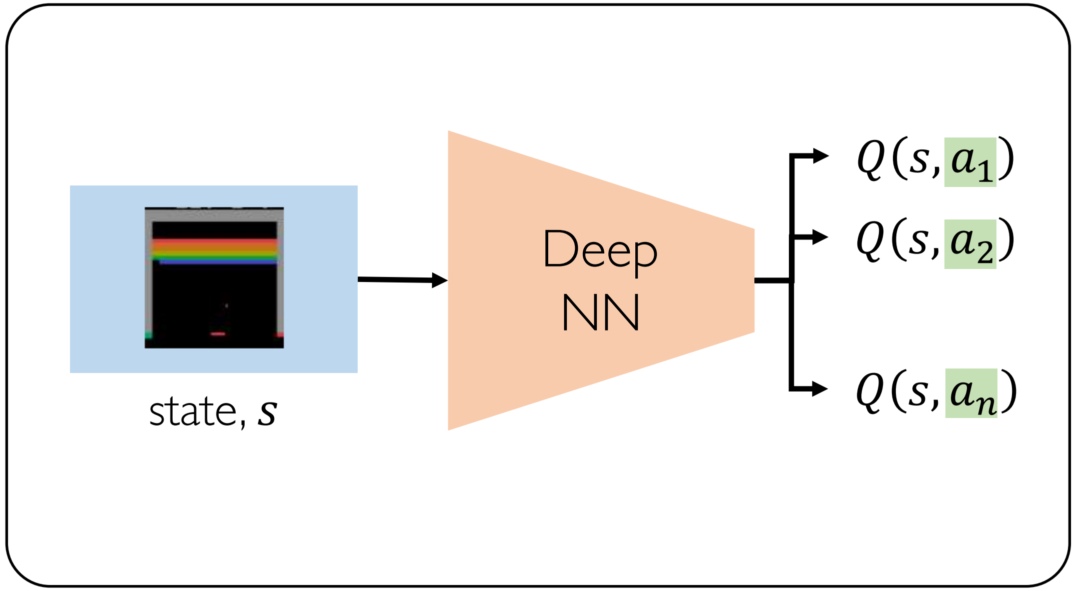

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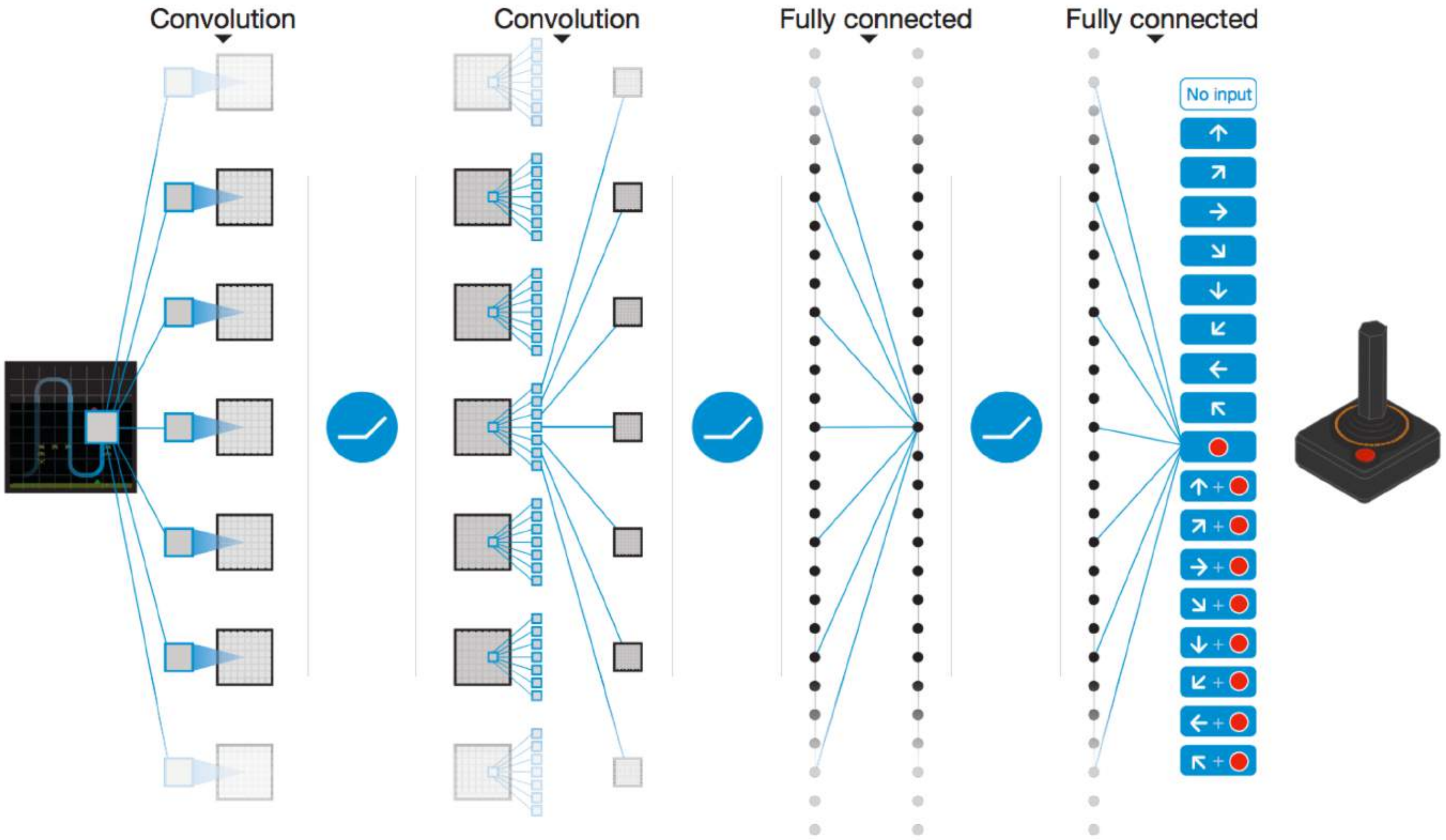
DQN Atari Results



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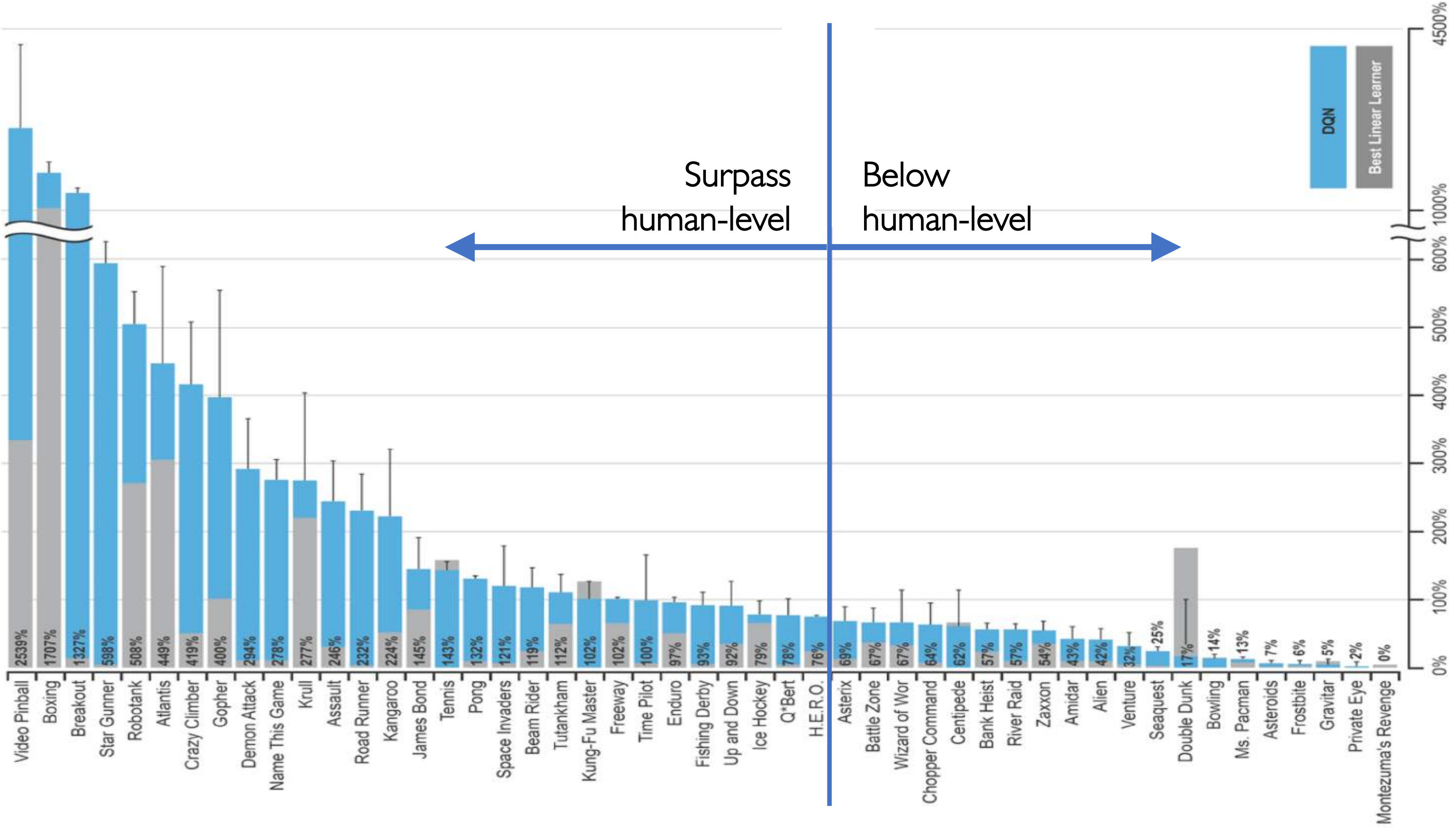


Downsides of Q-learning

Complexity:

- Can model scenarios where the action space is discrete and small
- Cannot handle continuous action spaces

IMPORTANT:
Imagine you want to predict steering wheel angle of a car!

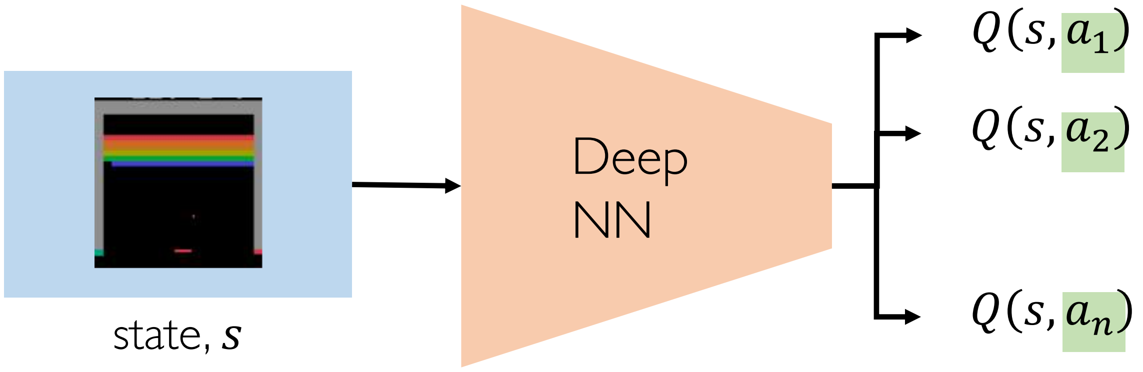
Flexibility:

- Cannot learn stochastic policies since policy is deterministically computed from the Q function

To overcome, consider a new class of RL training algorithms: Policy gradient methods

Policy Gradient (PG) : Key Idea

DQN (before): Approximating Q and inferring the optimal policy,



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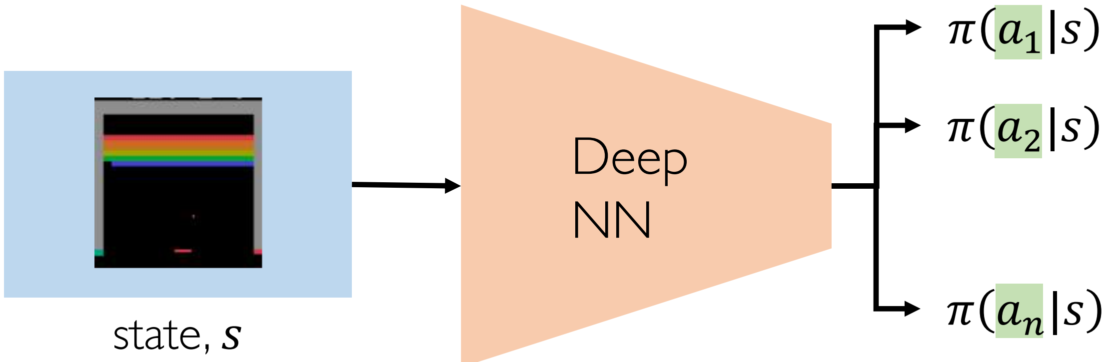
DQN (before): Approximating Q and inferring the optimal policy,

Policy Gradient: Directly optimize the policy!



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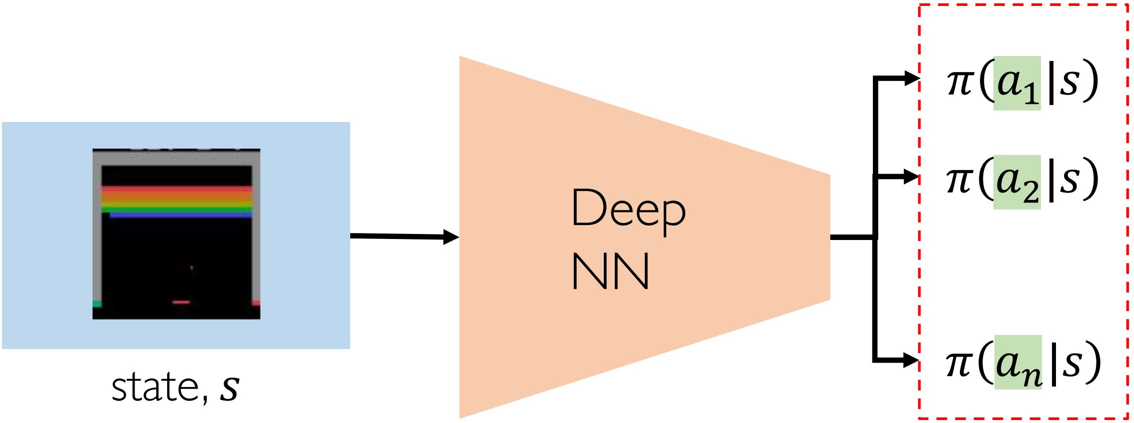

$$\sum_{a_i \in A} \pi(a_i|s) = 1$$

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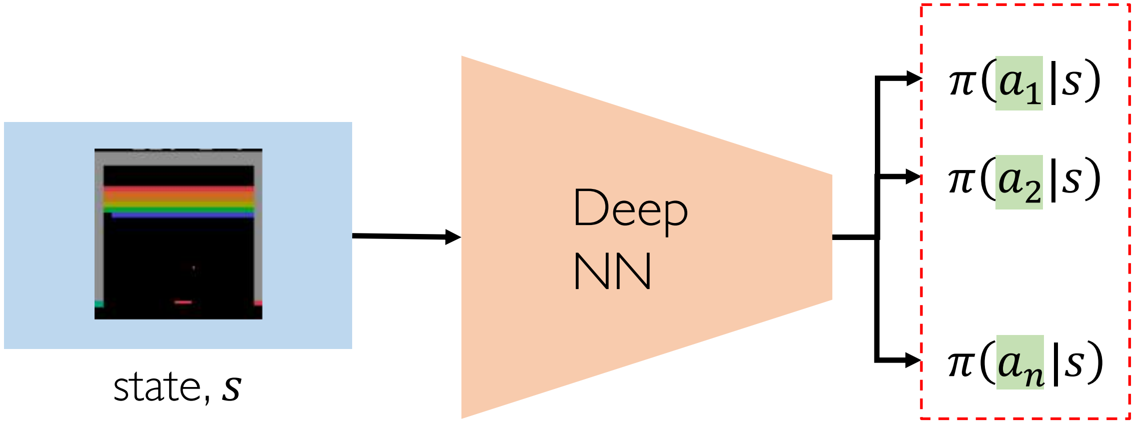

$$\sum_{a_i \in A} \pi(a_i|s) = 1$$

$$\pi(a|s) = P(action|state)$$

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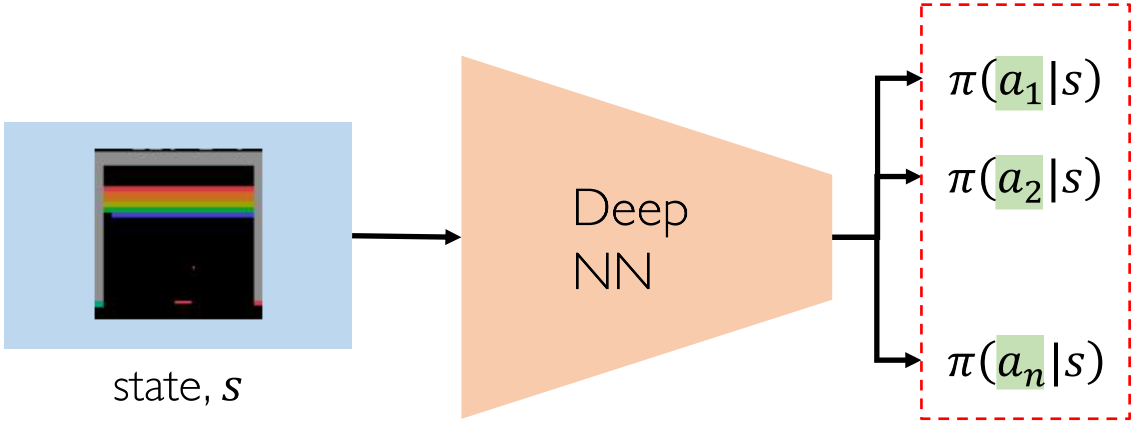

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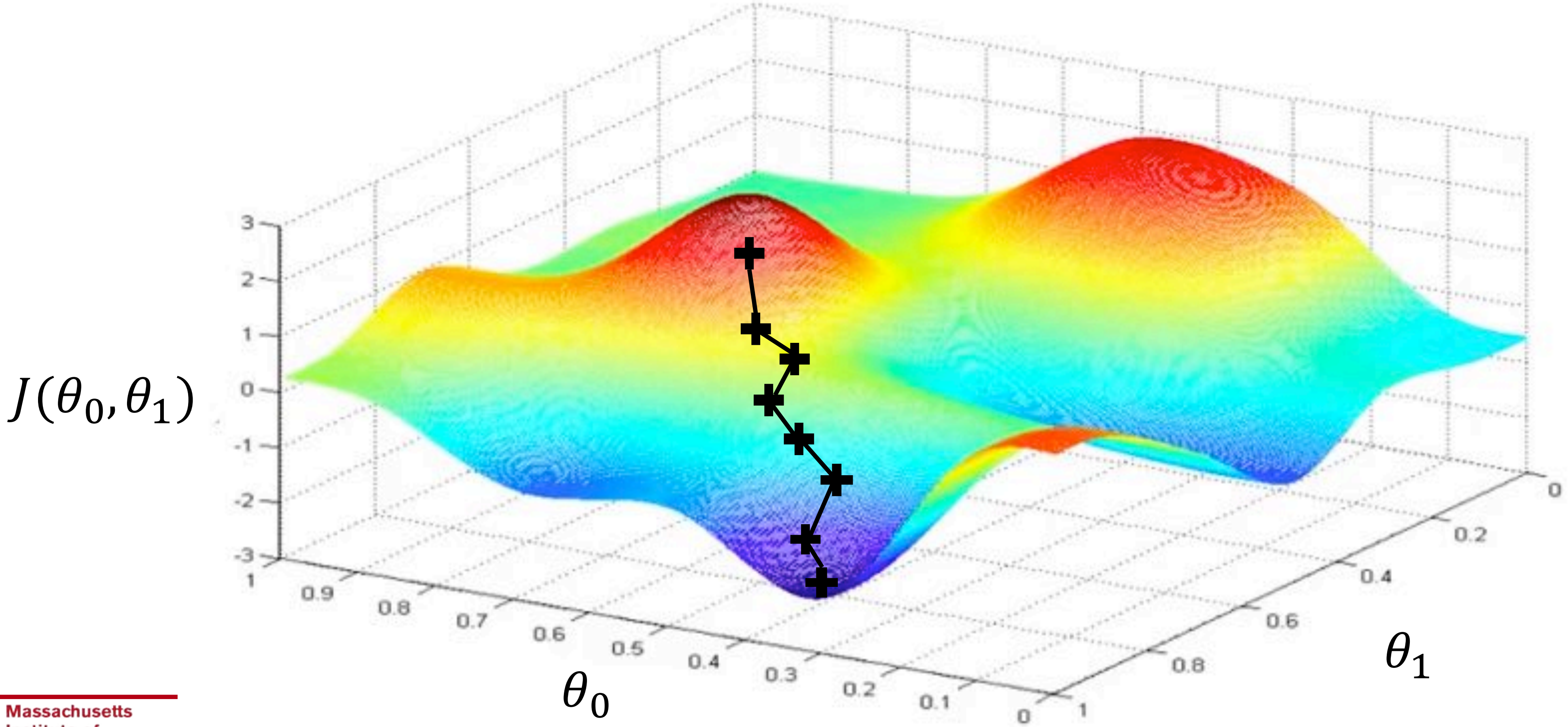
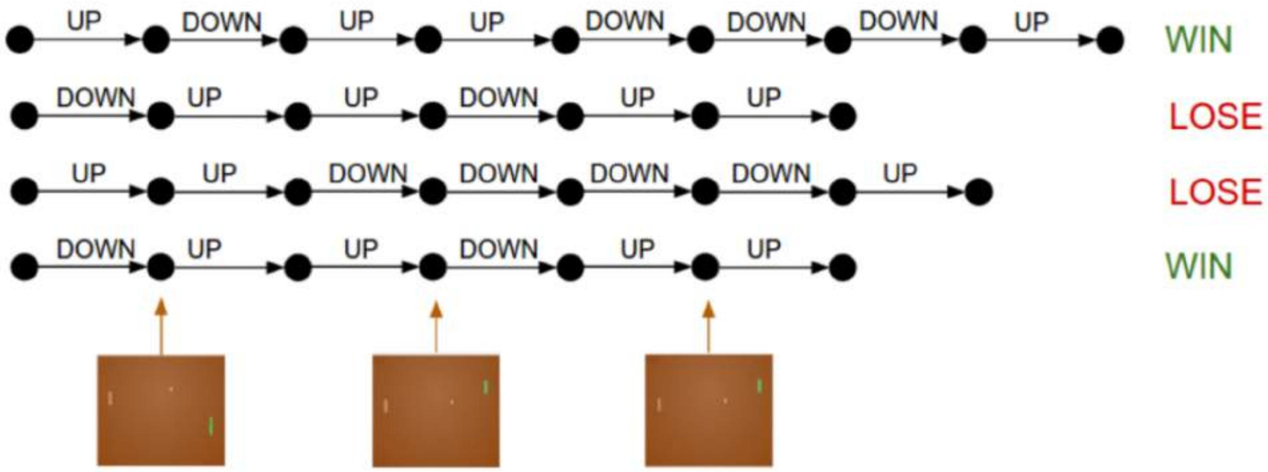
Gradient Descent in Deep Neural Network

Repeat until convergence



Massachusetts Institute of Technology

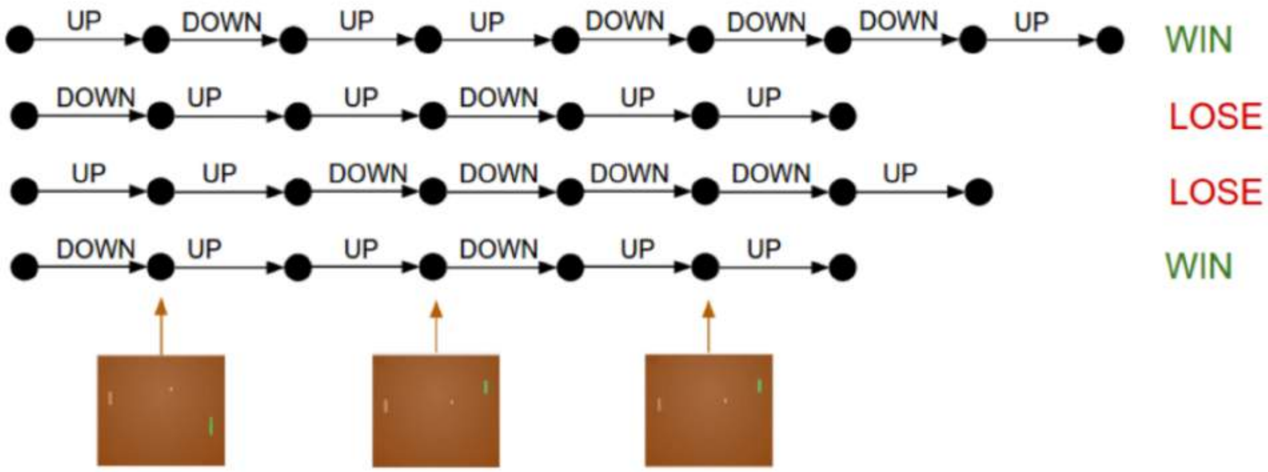
Policy Gradient (PG): Training



1. Run a policy for a while
2. Increase probability of actions that lead to high rewards
3. Decrease probability of actions that lead to low/no rewards

function REINFORCE
  Initialize θ
  for $episode \sim \pi_\theta$
    $\{s_i, a_i, r_i\}_{i=1}^{T-1} \leftarrow episode$
    for t = 1 to T-1
      $\nabla \leftarrow \nabla_\theta \log \pi_\theta(a_t|s_t)\, R_t$
      $\theta \leftarrow \theta + \alpha \nabla$
  return θ

Policy Gradient (PG): Training



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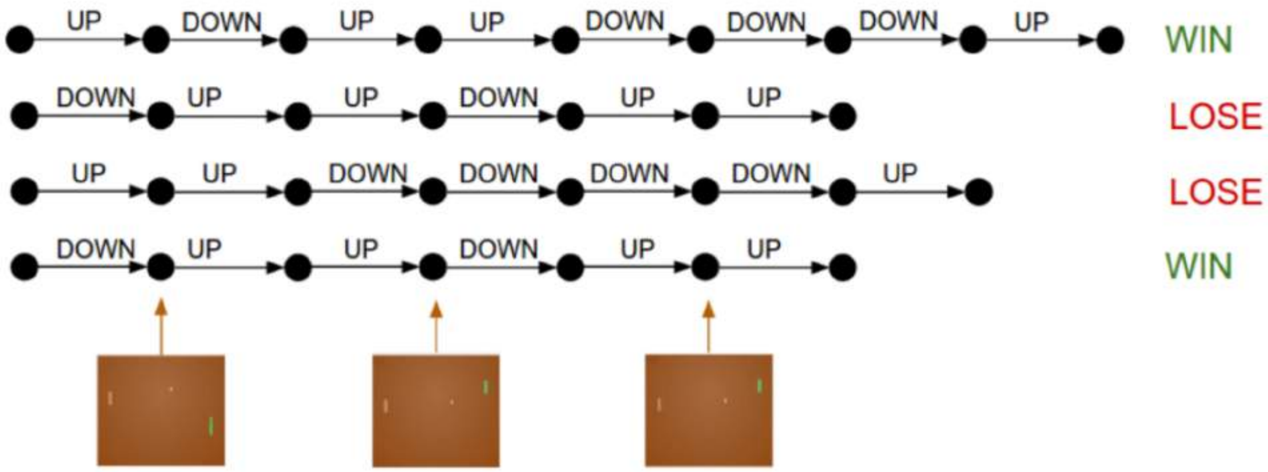

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$$\nabla_\theta \underbrace{\log \pi_\theta(a_t|s_t)}_{\text{}}\, R_t$$

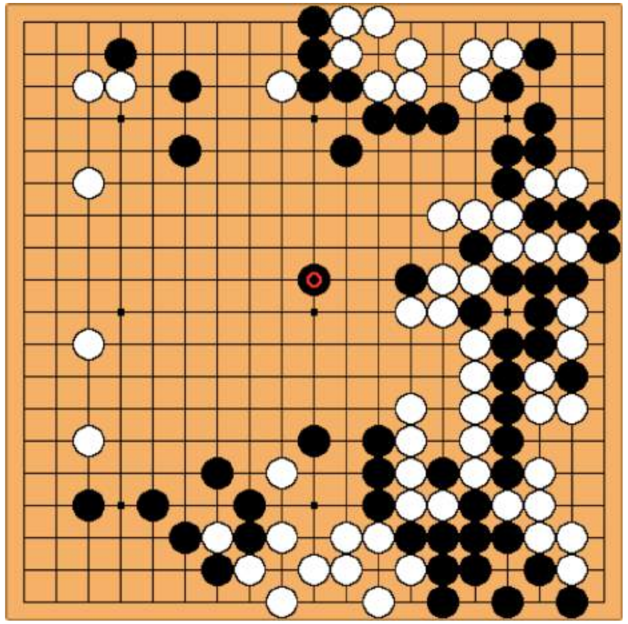
log-likelihood of action

reward

Calculate derivative to obtain the direction that gives a higher log-likihood of a_t action at s_t

The Game of Go

Aim: Get more board territory than your opponent.



Since each location on the board can be either empty, black, or white

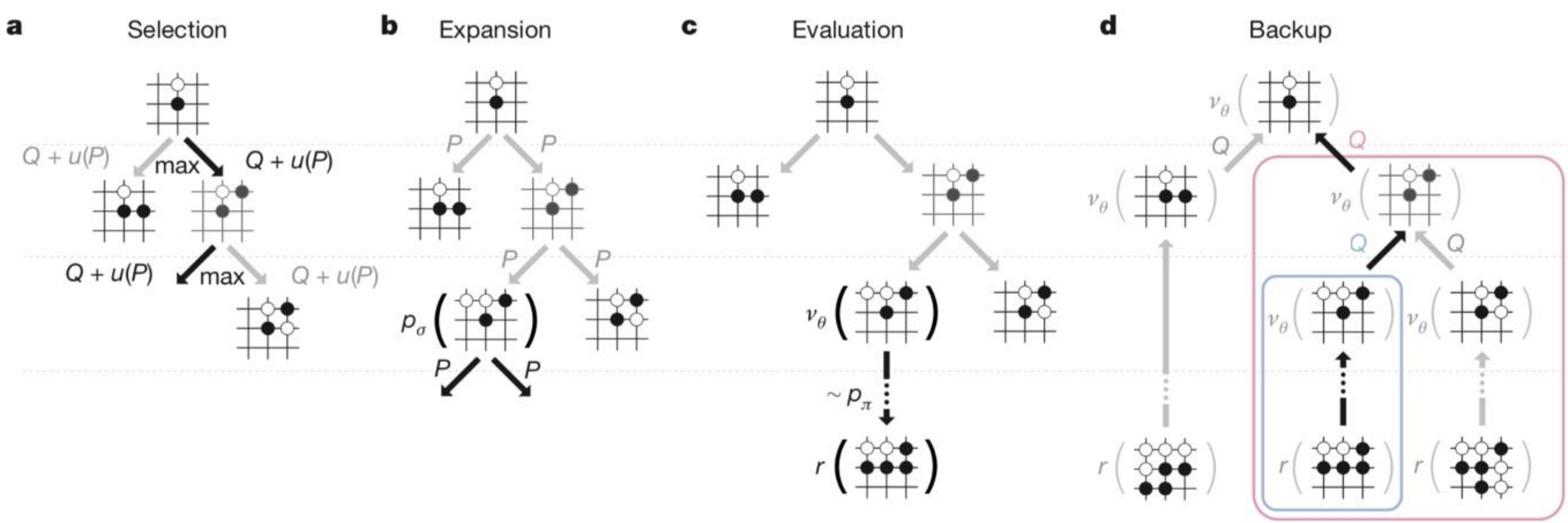
Board Size n x n	Positions 3^{n^2}	% Legal	Legal Positions
1×1	3	33.33%	1
2×2	81	70.37%	57
3×3	19,683	64.40%	12,675
4×4	43,046,721	56.49%	24,318,165
5×5	847,288,609,443	48.90%	414,295,148,741
9×9	$4.434264882 \times 10^{38}$	23.44%	$1.03919148791 \times 10^{38}$
13×13	$4.300233593 \times 10^{80}$	8.66%	$3.72497923077 \times 10^{79}$
19×19	$1.740896506 \times 10^{172}$	1.20%	$2.08168199382 \times 10^{170}$

Greater number of legal board positions than atoms in the universe.

Source: Wikipedia.

AlphaGo Approach

- Monte Carlo Tree Search (MCTS)
  - Balance exploitation/exploration (going deep on promising positions or exploring new underplayed positions)
- Use a neural network as "intuition" for which positions to expand as part of MCTS



Upper confidence bound (UCB)

Pick each node with probability proportional to:

$$\underbrace{v_i}_{\text{value estimate}} + \underbrace{C}_{\text{tunable parameter}} \times \sqrt{\frac{\ln(\overbrace{N}^{\text{parent node visits}})}{\underbrace{n_i}_{\text{number of visits}}}}$$

- probability is decreasing in the number of visits (explore)
- probability is increasing in a node's value (exploit)
- always tries every option once

UCB1 Formula (Auer et al 2002)

- Name UCB stands for Upper Confidence Bound
- Policy:

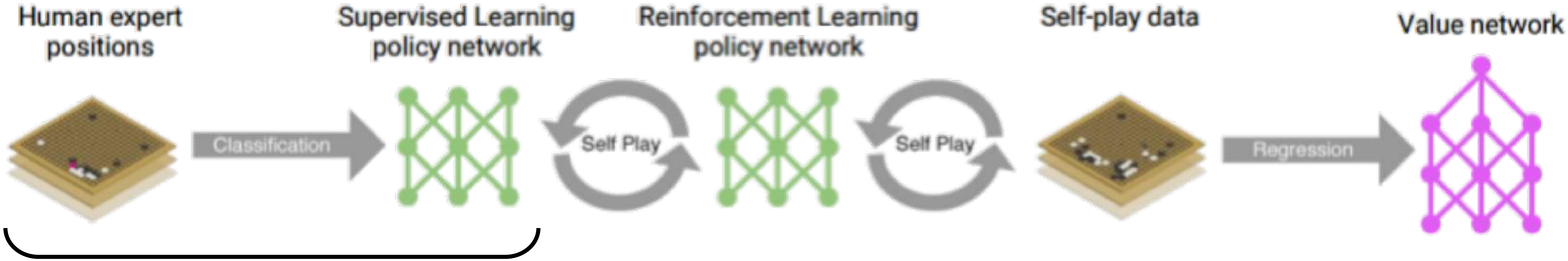
1. First, try each arm once
2. Then, at each time step:
   - choose arm *i* that maximizes the *UCB1 formula* for the upper confidence bound:

$$\bar{x}_i + \sqrt{\frac{2\ln(n)}{n_i}}$$

AlphaGo Beats Top Human Player at Go (2016)

Silver et al., *Nature* 2016.

AlphaGo Beats Top Human Player at Go (2016)



1) Initial training: human data

Silver et al., *Nature* 2016.

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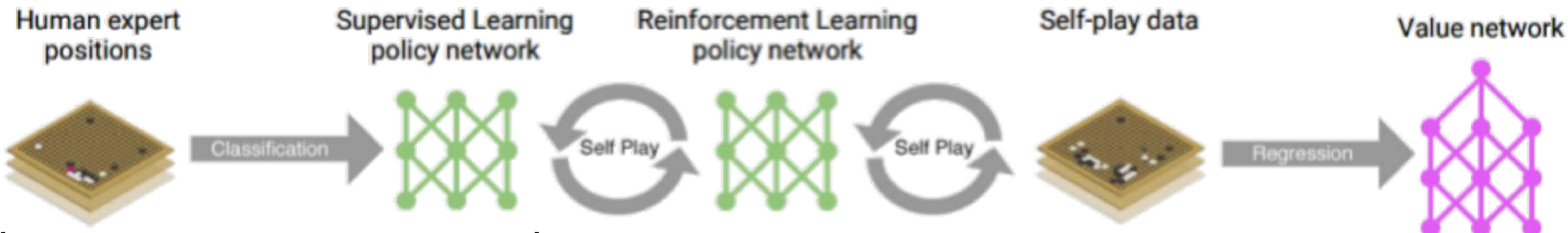

1) Initial training: human data

2) Self-play and reinforcement learning
→ super-human performance

Silver et al., *Nature* 2016.

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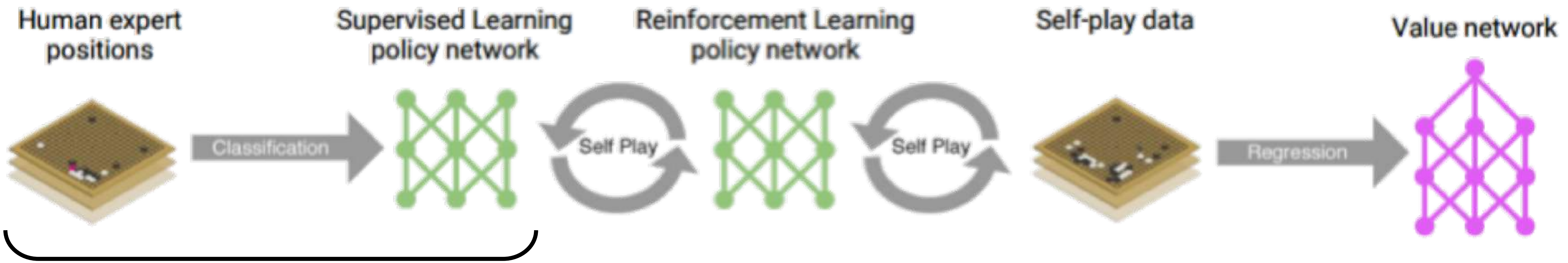

1) Initial training: human data

2) Self-play and reinforcement learning
→ super-human performance

3) "Intuition" about board state

Silver et al., *Nature* 2016.

AlphaGo Beats Top Human Player at Go (2016)

Silver et al., *Nature* 2016.

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Questions?